This question paper contains 4 printed pages.]

Your Roll No..... .....

ir. No. of Question Paper	:	**6621**	**HC**
Jnique Paper Code	:	32351101	
Name of the Paper	:	Calculus	
Name of the Course	:	**B.Sc. (Hons.) Mathematics**	
Semester	:	I	

Duration : 3 Hours | Maximum Marks : 75

nstructions for Candidates

. Write your Roll No. on the top immediately on receipt of this question paper.

. All the sections are compulsory.

. **All** questions carry equal marks.

. Use of non-programmable scientific calculator is allowed.

Section – I

Attempt any **four** *questions from* **Section I.**

. If $y = \cos(m \sin^{-1} x)$ then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

P.T.O.

2. Sketch the graph of $f(x)=\frac{x^2-x-2}{x-3}$ by finding intervals increase and decrease, critical points, points of relati maxima and minima, concavity of the graph and inflecti points.

3. Find the horizontal asymptote to the graph of the functi

$$f(x)=x^5\left[\sin\frac{1}{x}-\frac{1}{x}+\frac{1}{6x^3}\right]$$

4. A carpenter wants to make an open-topped box out of rectangular sheet of tin 24 inches wide and 45 inch long. The carpenter plans to cut congruent squares of each corner of the sheet and then bend and solder t edges of the sheet upward to form the sides of the box. F what dimension does the box have the greatest possi volume?

5. Sketch the graph of the curve in polar coordina $r = 1 - 2\sin\theta$.

Section – II

Attempt any **four** *questions from* **Section-II.**

6. Find the reduction formula for $\int \sin^m x \cos^n xdx$ where m being positive integers and hence evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^6 x$

Find the volume of the solid generated when the region enclosed by the curve $y = \sqrt{x}$, $y = 3$, and $x = 0$ is revolved about the y-axis.

Find the volume of the solid generated when the region enclosed by the curve $y = x^2 + 1$, $y = x$, $x = 0$ over the interval [0, 3] revolved about the x-axis.

Find the arc length of the parametric curve $x = \sin 2t$, $y = \cos 2t$ for $0 \le t \le \frac{\pi}{2}$.

. Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le 1$, about the x-axis.

Section – III

Attempt any **three** *questions from* **Section-III.**

. Find the equation of a ellipse with foci at (2, 3) and (2, 5) and vertices (2, 2) and (2, 6).

. Find the foci and equation of the hyperbola with vertices (0, ±2) and asymptote $y = \pm 2x$.

. Describe the graph of the equation $16x^2 - 9y^2 - 64x - 54y + 1 = 0$.

14. Trace the conic $9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$ rotating the coordinate axes to remove the xy term.

Section – IV

Attempt any **four** *questions from* **Section-IV**.

15. A particle moves with position function

$$\vec{F}(t) = (t \ln t)\hat{i} + (\sin t)\hat{j} + e^{-t}\hat{k}.$$

Find the velocity, speed and acceleration of the particle

16. A shell is fired with muzzle speed 150 m/s and angle elevation 45° from a position that is 10 m above the grou level. Where does the projectile hit the ground and w what speed?

17. Find the tangential and normal components of accelerati of an object that moves with position vector $R(t) = (t^3, t^2,$

18. An object moves along the curve

$r = \sin\theta$ and $\theta = 2t$

Find its velocity and acceleration in terms of unit po vectors u_r and u_θ.

19. Find the curvature and radius of curvature of the twist cubic for a curve

$r(t) = \{t, t^2, t^3)$ at a general point and at (0, 0, 0).

'his question paper contains 6 printed pages.]

Your Roll No..............

. No. of Question Paper : 6622 **HC**

nique Paper Code : 32351102

ame of the Paper : Algebra

ame of the Course : **B.Sc. (Hons.) Mathematics**

emester : I

uration : 3 Hours Maximum Marks : 75

<u>nstructions for Candidates</u>

Write your Roll No. on the top immediately on receipt of this question paper.

All **Six** questions are compulsory.

Do any **two** parts from each question.

(a) Find all complex numbers z, such that $|z| = 1$ and

$$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1. \qquad (6)$$

(b) Find the fourth roots of unity and represent them in the complex plane. Show that they form a square inscribed in the unit circle. (6)

P.T.O.

(c) Solve the equation

$$z^6 + iz^3 + i - 1 = 0.$$

2. (a) For $a, b \in Z$, define $a \sim b$ if and only if $3\ a+b$ i multiple of 4.

(i) Prove that $\sim$ defines an equivalence relation

(ii) Find the equivalence class of 0 and 2.

(b) Let $\sim$ denote an equivalence relation on a set and $a \in A$. Prove that for any $x \in A$, $x \sim a$ if and or if $\bar{x} = \bar{a}$, where $\bar{x}$ denotes the equivalence class x.

(c) Show that Z and 3Z have the same cardinality.

3. (a) Using Euclidean algorithm find g.c.d [1004, –24) a express it as an integral linear combination of the giv integers.

(b) Find $(1017)^{12}$ (mod 7).

(c) Using Principle of Mathematical Induction, pro that for every positive integer n, $n^3 + 2n$ is divisil by 3.

(a) Find the general solution to the linear system whose augmented matrix is

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 & -3 & 2 \\ 1 & 1 & 1 & 2 & -3 & 3 \\ 2 & 1 & 0 & 2 & -3 & 4 \\ 4 & 3 & 1 & 1 & -9 & 9 \end{bmatrix}$$

by row reducing the matrix to Echelon Form. Encircle the leading entries, list the basic variables and free variables. Write the general solution in Parametric Vector Form. (6½)

(b) Define Linearly Dependent Set.

Let $v_1 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -5 \\ 10 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix}$ for what value(s) of h, the set $\{v_1, v_2, v_3\}$ is

(i) Linearly Independent

(ii) Linearly Dependent. (6½)

(c) Let $v_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$

Do the vectors v_1, v_2, v_3 span $\mathcal{R}^3$? Justify. Hence otherwise express $v = \begin{bmatrix} 8 \\ -4 \\ 2 \end{bmatrix}$ as linear combination v_1, v_2, v_3. (6

5. (a) Boron sulphide reacts violently with water to form bo acid and hydrogen sulphide gas. The unbalanced equat is

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Balance the chemical equation using the vector equati approach. (6

(b) Let T: $\mathcal{R}^2 \rightarrow \mathcal{R}^2$ be a linear transformation such t T first rotates through $\frac{\pi}{2}$-radians in the anti-clockw direction and then reflects through the line $x_1 = x_2$. F the Standard matrix of T. (6

(c) Let T: $\mathcal{R}^2 \to \mathcal{R}^2$ be defined as $T(x_1, x_2) = (x_2 - x_1, 2x_2 + x_1)$ be a linear transformation. Prove that T is invertible and find a rule for T^{-1}. (6½)

(a) Let

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix} \text{ and } u = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$$

Is u in Nul A ? Is u in Col A ? Justify each answer. (6½)

(b) (i) Suppose a 4 × 7 matrix A has three pivot columns. Is Col A = $\mathcal{R}^3$? What is the dimension of Nul A ? Explain your answer.

(ii) Consider the basis $B = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ for $\mathcal{R}^2$. If $[x]_B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, find the vector x. (3½,3)

(c) For the matrix given below, find the characteristic equation and the eigen values with their multiplicities. Also, find a basis for the eigenspace corresponding to any one of the eigenvalues.

$$A = \begin{bmatrix} 5 & 8 & 0 & 1 \\ 0 & -4 & 7 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This question paper contains 2 printed pages

Roll No.

Unique paper code : 2351301

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : Algebra II (Group Theory - I)

Semester : III

Duration : 3 Hours

Maximum marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt **any two parts** from each question.
3. All questions are compulsory.

1(a) (i) For any elements a and b from a group and any integer n prove that $(a^{-1}ba)^n = a^{-1}b^n a$.

(ii) Give an example of a non-cyclic group all of whose proper subgroups are cyclic.

(b) Define center of a group. Prove that the center of a group G is a subgroup of G.

(c) If $G = \langle a \rangle$ is a cyclic group of order n then prove that $G = \langle a^k \rangle$ *iff* g.c.d $(k, n) = 1$.

(6 × 2 = 12)

2(a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0 \right\}$. Show that G is a group under matrix multiplication.

(b) (i) Let H be a non empty finite subset of a group G. Then prove that H is a subgroup of G if H is closed under the operation of G.

(ii) Let G be a group and let a be any element of G. Then prove that $\langle a \rangle$ is subgroup of G.

(c) (i) How many subgroup does Z_{30} have. List a generator for each of these.

(ii) Prove that $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \mid n \in \mathbb{Z} \right\}$ is a cyclic subgroup of $GL(2, \mathbb{R})$ (6 × 2 = 12)

3(a) Prove that the order of a permutation of a finite set written as a product of disjoint cycles, is the least common multiple of the lengths of the cycles.

(b) State and prove Lagrange's Theorem. Is the converse true? Justify your answer.

(c) Let G be a group and H a normal subgroup of G. Prove that the set $G/_H = \{aH | a \in G\}$ is a group under the operation $(aH)(bH) = abH$. (6 × 2 = 12)

4. (a) Show that if H is a subgroup of S_n ($n \geq 2$) then either every member of H is an even permutation or exactly half of them are even.

(b) State and prove Fermat's Little Theorem.

(c)(i) Prove that a subgroup H of a group G is a normal subgroup of G if and only if

$ghg^{-1} \in H$ for all $g \in G$ and for all $h \in H$.

(ii) Suppose G is a group and $H = \{g^2 : g \in G\}$ is a subgroup of G. Prove that H is a normal subgroup of G. (6.5 × 2 = 13)

5. (a) Let G be a group and Z (G) be the centre of G. If G/Z (G) is cyclic then prove that G is Abelian.

(b) Show that any infinite cyclic group is isomorphic to (Z, +) the group of integers under addition.

(c) Let G be a group of permutation and {1,-1} be the multiplicative group. For each $\sigma \in G$, define a mapping

$\varphi : G \rightarrow \{1, -1\}$.

by

$$\varphi(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is an even;} \\ -1 & \text{if } \sigma \text{ is an odd.} \end{cases}$$

Prove that φ is a group homomorphism. Also, find Ker φ. (6.5 × 2 = 13)

6. (a) Suppose that φ is an isomorphism from a group G onto a group G*. Prove that G is cyclic if and only if G* is cyclic. Hence show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition.

(b) If M and N are normal subgroups of a group G and N ≤ M, prove that (G/N) / (M/N) ≈ G/M.

(c) Let φ be a group homomorphism from G onto G* then prove that G/Ker φ ≈ G*. (6.5 × 2 = 13) is question paper contains 4 printed pages.]

Your Roll No.............

No. of Question Paper :	**6623**		**HC**
que Paper Code	:	32351301	
ne of the Paper	:	Theory of Real Functions	
ne of the Course	:	**B.Sc. (Hons.) Mathematics**	
nester	:	III	

ation : 3 Hours Maximum Marks : 75

ructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

All questions are compulsory.

Attempt any **three** parts from each question.

(a) Use the $\in$-δ definition of the limit to find $\lim_{x\to 2} f(x)$ where $f(x) = \frac{1}{1-x}$. (5)

(b) State and prove Sequential Criterion for Limits. (5)

(c) State Squeeze Theorem. For $n \in N$, $n \geq 3$, derive the inequality, $-x^2 \leq x^n \leq x^2$ for $-1 < x < 1$. Hence prove that $\lim_{x\to 0} x^n = 0$ for $n \geq 3$, assuming that $\lim_{x\to 0} x^2 = 0$. (5)

P.T.O.

(d) Let f, g be defined on $A \subseteq R$ to R, and le
a cluster point of A. Suppose that f is bound
a neighbourhood of c and that $\lim_{x \to c} g = 0$. Prov
$\lim_{x \to c} fg = 0$.

2. (a) Let $c \in R$ and let f be defined for $x \in (c, \infty$
$f(x) > 0$ for all $x \in (c, \infty)$. Show that $\lim_{x \to c} f = \infty$
only if $\lim_{x \to c} 1/f = 0$.

(b) Prove that

(i) $\lim_{x \to 0} \frac{1}{\sqrt{|x|}} = +\infty, \; x \neq 0$

(ii) $\lim_{x \to 0-} e^{1/x} = 0, \; x \neq 0$.

(c) Let A = R and let f be Dirichlet's function defir

$$g(x) = \begin{cases} 1, & \text{for x rational} \\ -1, & \text{for x irrational} \end{cases}$$

Show that f is discontinuous at any point of R.

(d) Let $f : R \to R$ be continuous at c and let $f(c) > 0$.
that there exists a neighbourhood $V_\delta(c)$ of c suc
if $x \in V_\delta(c)$ then $f(x) > 0$.

(a) Determine the points of continuity of the function $f(x) = x - [x]$, $x \in R$, where $[x]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. (5)

(b) Let $A, B \subseteq R$, let $f: A \to R$ be continuous on A, and let $g: B \to R$ be continuous on B. If $f(A) \subseteq B$, show that the composite function $gof: A \to R$ is continuous on A. (5)

(c) Let f be a continuous real valued function defined on [a, b]. Show that f is a bounded function. (5)

(d) Prove that a polynomial of odd degree has at least one real root. (5)

(a) Define uniform continuity of a function on a set $A \subseteq R$. Show that every uniformly continuous function on A is continuous on A. Is the converse true? Justify your answer. (5)

(b) Show that the function $\sqrt{x}$ is uniformly continuous on $[0, \infty)$. (5)

(c) Let I, J be intervals in R, let $g: I \to R$ and $f: J \to R$ be functions such that $f(J) \subseteq I$ and let $c \in J$. If f is differentiate at c and if g is differentiate at f(c), show that the composite function gof is differentiate at c and $(gof)'(c) = g'(f(c)).f'(c)$. (5)

(d) Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable at $x = 0$ and $f'(0)$.

5. (a) Let f be continuous on [a, b] and differentiate on (a
Prove that f is increasing on [a, b] if and onle $f'(x) \geq 0 \ \forall x \in [a, b]$.

(b) State Darboux's Theorem. Suppose that if f : [0,2] is continuous on [0, 2] and differentiate on (0, 2), that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that there exists $c_1 \in (0,1)$ such $f'(c_1) = 1$

(ii) Show that there exists $c_2 \in (1,2)$ such $f'(c_2) = 0$

(iii) Show that there exists $c \in (0,2)$ such $f'(c) = 1/3$.

(c) Find the Taylor series for cos x and indicate wh converges to cos x $\forall x \in R$.

(d) Define a convex function on [a, b]. Check the conve of the following functions on given intervals :

(i) $f(x) = x - \sin x$, $x \in [0, \pi]$.

(ii) $g(x) = x^3 + 2x$, $x \in [-1, 1]$.

his question paper contains 4 printed pages.]

Your Roll No..............

. **No. of Question Paper : 6624** HC

ique Paper Code : 32351302

me of the Paper : Group Theory 1

me of the Course : **B.Sc. (Hons.) Mathematics**

mester : III

ration : 3 Hours Maximum Marks : 75

structions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Attempt any **two** parts from each question.

All questions are compulsory.

(a) For a fixed point (a, b) in $\mathbb{R}^2$, define $T_{(a,b)} : \mathbb{R}^2 \to \mathbb{R}^2$ by $(x, y) \to (x + a, y + b)$.

Show that $T(\mathbb{R}^2) = \{T_{a,b} \mid a, b \in \mathbb{R}\}$

is a group under function composition. (6)

(b) (i) Find the inverse of $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL(2, \mathbb{Z}_{11})$. (4)

(ii) Let G be an Abelian group under multiplication wi identity e. Show that

$H = \{x^2 \mid x \in G\}$ is a subgroup of G. (2

(c) (i) Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$ where Z(G) is the Center of G and C(a) is th Centralizer of a. (4

(ii) Let G be the group of nonzero real numbers unde multiplication. Show that

$H = \{x \in G \mid x = 1 \text{ or } x \text{ is irrational}\}$

and $K = \{x \in G \mid x \geq 1\}$ are not subgroups of (

(2

2. (a) Let G be a group and let $a \in G$. If $|a| = n$, prove tha $\langle a \rangle = \{e, a, a^2, \ldots, a^{n-1}\}$ and $a^i = a^j$ if and only if divides $i - j$. (6

(b) Suppose that $|a| = 24$. Find a generator for $\langle a^{21} \rangle \cap \langle a^{10} \rangle$ (6

(c) If $|a| = n$, show that

$$\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$$

and that $$|a^k| = \frac{n}{\gcd(n,k)}$$

3. (a) Define the Alternating Group A_n. Show that it forms a subgroup of the Permutation Group S_n and $|A_n| = \frac{n!}{2}$. (6)

(b) Prove that every group is isomorphic to a group of permutations. (6)

(c) Prove that U(10) is not isomorphic to U(12). (6)

4. (a) State and prove Orbit Stabilizer Theorem. (6½)

(b) (i) Prove that aH = H if and only if $a \in H$. (3)

(ii) Prove that aH = bH or $aH \cap bH = \phi$. (3½)

(c) (i) Prove that order of U(n) is even when $n > 2$. (3)

(ii) Prove that a group of prime order is cyclic. (3½)

5. (a) Let H and K be subgroups of a finite group G and let

$$HK = \{hk \mid h \in H,\ k \in K\}$$

and

$$KH = \{kh \mid k \in K,\ h \in H\}.$$

Prove that HK is a group if and only if HK = KH. (6½)

(b) Let φ be a homomorphism from a group G to a gro
$\bar{G}$ and let g be an element of G. Prove that

(i) If $\varphi(g) = g'$, then $\varphi^{-1}(g') = \{x \in G | \varphi(x) = g'\} = gKe$
(4

(ii) If $|Ker\varphi| = n$, then φ is an n – to – 1 mappi
from G onto φ(G). (2½

(c) (i) Prove that A_n is normal in S_n. (3½

(ii) If G is a non-Abelian group of order p^3 (p is prim
and $Z(G) \neq \{e\}$, prove that $|Z(G)| = p$. (3

6. (a) State and prove The First Isomorphism Theorem.
(6½

(b) Let G be a group and let Z(G) be the center of G
Prove that if $G/Z(G)$ is cyclic, then G is Abelian.
(6½

(c) Let $4Z = (0, \pm 4, \pm 8, \cdots\}$. Find Z/4Z. (6½

(3

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6625**

Unique Paper Code : **32351303** **HC**

Name of the Paper : **Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *five* questions from each Section.

All questions carry equal marks.

Section I

1. Let f be the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

Is f continuous at $(0, 0)$? Explain.

P.T.O.

2. Find the equation for each horizontal tangent plane to the surface :

$$z = 5 - x^2 - y^2 + 4y.$$

3. Let f and g be twice differentiable functions of one variable and let $u(x, t) = f(x + ct) + g(x - ct)$ for a constant c. Show that :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

4. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0(1, 2)$ has magnitude 50 and is attained in the direction from P_0 towards $Q(3, -4)$. Use this information to find $\nabla f(1, 2)$.

5. Find the absolute extrema of $f(x, y) = x^2 + xy + y^2$ on the closed bounded set S where S is the disk $x^2 + y^2 \leq 1$.

6. Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

Section II

7. Find the area of the region D by setting double integral, where D is bounded by the parabola $y = x^2 - 2$ and the line $y = x$.

8. Write an equivalent integral with the order of integration reversed and then compute the integral :

$$\int_0^4 \int_0^{4-x} xy \, dy dx.$$

9. Calculate the Jacobian of transformation from rectangular to polar coordinates and hence evaluate the integral :

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}} dx dy.$$

10. Find the volume V of the solid bounded above by the cylinder $y^2 + z = 4$ and below by $x^2 + 3y^2 = z$.

11. Evaluate the integral below, where D is the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$:

$$\iiint_D z \, dx \, dy \, dz.$$

12. Let D be the region in the xy-plane that is bounded by the co-ordinate axes and the line $x + y = 1$. Use the suitable change of variable to compute the integral :

$$\iint_D \left(\frac{x - y}{x + y}\right)^6 dy\, dx.$$

Section III

13. State Green's theorem for simply connected regions. Use Green's theorem to find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\boldsymbol{i} + (\cos y + x^3)\boldsymbol{j}$ along the circle $x^2 + y^2 = 1$ in anticlockwise direction.

14. Give the geometrical interpretation of the surface integral $\iint ds$ over piecewise smooth surface S. Evaluate the surface integral $\iint xz\, ds$ over the surface S which is the part of the plane $x + y + z = 1$ that lies in the first octant.

15. Verify Stokes' theorem for the vector field $\mathbf{F}(x, y, z) = 2z\boldsymbol{i} + 3x\boldsymbol{j} + 5y\boldsymbol{k}$ taking surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

16. State and prove Divergence theorem.

17. Verify that the vector field $\mathbf{F}(x, y) = (e^x \sin y - y)\boldsymbol{i} + (e^x \cos y - x - 2)\boldsymbol{j}$ is conservative using cross partial test. Use a line integral to find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

18. Let E be the solid unit cube with opposing corners at the origin and (1, 1, 1) with faces parallel to co-ordinate planes. Let S be the boundary surface of E oriented with the outward pointing normal. If $\mathbf{F}(x, y, z) = 2xy\boldsymbol{i} + 3ye^z\boldsymbol{j} + x \sin z\, \boldsymbol{k}$, find the integral $\iint \mathbf{F.n}\, ds$ over surface S using divergence theorem.

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6626**

Unique Paper Code : **32351501** HC

Name of the Paper : **C11-Metric Spaces**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Let $d_p (p \geq 1)$ on the set $\mathbf{R}^n$, be given by

$$d_p(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^p\right)^{1/p},$$

for all $x = (x_1, x_2, \ldots\ldots\ldots, x_n)$, $y = (y_1, y_2, \ldots\ldots\ldots, y_n)$ in $\mathbf{R}^n$.

Show that $(\mathbf{R}^n, d_p)$ is a metric space. Does d_p define a metric on $\mathbf{R}^n$, when $0 < p < 1$? 4+2=6

P.T.O.

(*b*) Let S be any non-empty set and B(S) denote the set of all real- or complex-valued functions on S, each of which is bounded. Define the uniform metric d on B(S). Show that (B(S), d) is a complete metric space. 6

(*c*) (*i*) Let X = **N**, the set of natural numbers. Define

$$d(m, n) = \left|\frac{1}{m} - \frac{1}{n}\right|, \quad m, n \in X.$$

Show that (X, d) is an incomplete metric space.

(*ii*) Prove that metric spaces, **R** with the usual metric and $(0, \infty)$ with the usual metric induced from **R** are homeomorphic. 4+2=6

2. (*a*) (*i*) Let (X, d) be a metric space. Prove that the closed ball $\bar{S}(x, r)$, where $x \in X$ and $r > 0$, is a closed subset of X.

(*ii*) Is the set $A = \{(x, y) : x + y = 1\}$ open in the metric space (R^2, d_2) ? Justify your answer. 4+2=6 (*b*) Let (X, d) be a metric space and Y a subspace of X. Let Z be a subset of Y. Prove that Z is closed in Y if and only if there exists a closed set F of X such that $X = F \cap Y$. 6

(*c*) (*i*) Let (X, d) be a metric space and F_1 and F_2 be subsets of X. Prove that :

$$cl(F_1 \cup F_2) = cl(F_1) \cup cl(F_2).$$

(*ii*) Define a separable metric space. Is the discrete metric space (X, d) separable ? Justify your answer. 3+3=6

3. (*a*) Let (X, d) be a metric space and for every nested sequence $\{F_n\}$, $n \geq 1$ of non-empty closed subsets of X such that $d(F_n) \to 0$ as $n \to \infty$, the intersection $\bigcap_{n=1}^{\infty} F_n$

contains one and only one point. Prove that (X, d) is a complete metric space. Further, show that the condition : $d(F_n) \to 0$ as $n \to \infty$ in the above statement can't be dropped. 4+2=6

(*b*) Let (X, d) be a metric space and $F \subseteq X$. Prove that a point x_0 is a limit point of F if and only if it is possible to select from the set F a sequence, $\{x_n\}$, $n \geq 1$, of distinct points such that $\lim_n d(x_n, x_0) = 0$. 6

(*c*) (*i*) Let F be subset of the metric space (X, d). Prove that the set of limit points of F is a closed subset of (X, d).

(*ii*) Let F be a non-empty bounded closed subset of **R**, with usual metric and $a = \sup F$. Show that $a \in F$. 3+3=6

4. (*a*) (*i*) Let (X, d) be any metric space and $f : (X, d) \to (R^n, d_2)$, be defined by :

$f(x) = (f_1(x), f_2(x), \ldots\ldots, f_n(x))$, for $x \in X$.

Show that if f is continuous, so is each $f_k : X \to \mathbf{R}$, $k = 1, 2, \ldots\ldots, n$.

(*ii*) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \to Y$ be continuous on X. Show that $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, for all subsets B of Y.

2½+4=6½

(*b*) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \to Y$ be uniformly continuous. Show that if $\{x_n\}$, $n \geq 1$, is a Cauchy sequence in X, then so is $\{f(x_n)\}$, $n \geq 1$, in Y. Is this result true, if $f : X \to Y$ is continuous on X ?

4+2½=6½

(c) Let X be the set of all continuous functions defined on [0, 1]. For $f, g \in X$, define the metrics 'd' and 'e' on X by :

$$d(f, g) = \sup\{|f(x) - g(x)| : x \in [0, 1]\}, \text{ and}$$

$$e(f, g) = \int_0^1 |f(x) - g(x)|\, dx.$$

Show that these metrics are not equivalent. 6½

5. (a) (i) Let (X, d) be the complete metric space and $T : X \to X$ be a contraction mapping and let $x_0 \in X$ and $\{x_n\}$, $n \geq 1$, be the sequence defined iteratively by $x_{n+1} = T x_n$ for $n = 0, 1, 2, ...$. Show that the sequence $\{x_n\}$, $n \geq 1$, is convergent in X.

(*ii*) Let $T : X \to X$, where (X, d) is a complete metric space, satisfy the inequality ;

$d(Tx, Ty) < d(x, y)$ for all $x, y \in X$.

Show that T need not have a fixed point.

4+2½=6½

(*b*) Let $(\mathbf{R}, d)$ be the space of real numbers with the usual metric. Show that a subset, I, of $\mathbf{R}$ is connected if and only if I is an interval. 6½

(*c*) (*i*) Show that the metric space (X, d) is disconnected if and only if there exists a proper subset of X that is both open and closed in X.

(*ii*) Let A be a subset of $\mathbf{R}^2$ defined by

$$A = \{(x, y) : x^2 - y^2 \geq 4\}.$$

Show that A is disconnected. 4+2½=6½

6\. (*a*) Let (X, d_X) be a metric space and every continuous function $f : (X, d_X) \to (\mathbf{R}, d_u)$, where d_u is the usual metric of **R**, has the intermediate value property. Prove that (X, d_X) is a connected space. 6½

(*b*) Define the finite intersection property. Prove that a metric space (X, d) is compact if, and only if every collection of closed sets in (X, d) with the finite intersection property has non-empty intersection. 6½

(*c*) Let f be a continuous function from a *compact* metric space (X, d_X) into a metric space (Y, d_Y). Prove that the range $f(X)$ is also compact. 6½

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: **6627** **HC**

Unique Paper Code **: 32351502**

Name of Paper **: Group Theory – II**

Name of Course **: B.Sc. (Hons.) Mathematics**

Semester **: V**

Duration **: 3 hours**

Maximum Marks **: 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory. Question No. 1 has been divided into 10 parts and each part is of 1.5 marks. Each question from Q. Nos. 2 to 6 has 3 parts and each part is of 6 marks. Attempt any two parts from each question..

(1) State true (T) or false (F). Justify your answer in brief.

(a) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ is isomorphic to $\mathbb{Z}_4$ here $\mathbb{Z}_n$ is used for group $\{0, 1, 2, \ldots, n-1\}$under the addition modulo n..

(b) The dihedral group D_8 of order 8 is not isomorphic to the quaternion group Q_8 of order 8.

(c) The external direct product $G \oplus H$ is cyclic if and only if groups G and H are cyclic.

(d) $U(165)$ can be written as an external direct product of cyclic additive groups of the form $\mathbb{Z}_n$ where $U(n)$ denotes the group of units under multiplication modulo n.

(e) Translations $z \mapsto z + a$ are the only automorphisms of the additive group of integers $\mathbb{Z}$.

(f) A subgroup N of a group G is called a characteristic subgroup if $\phi(N) = N$ for all isomorphism of G onto itself.

(g) The number of isomorphism types (classes) of a group of order 9 is 3.

(h) If G is a finite group of order n, then G is isomorphic to a subgroup of $D_n n$.

(i) If a group G acts trivially on a set A containing more than 1 elements then there is an element a in A whose stabilizer is proper subgroup of the group.

(j) $U(8)$ is isomorphic to $U(10)$.

(2) (a) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$.

(b) Prove that for any group G, $G/Z(G)$ is isomorphic to the group of inner automorphism $Inn(G)$ where $Z(G)$ is centre of the group G.

(c) Classify groups of order 6.

(3) (a) (i) Suppose that G is a group of order 4 with identity e and $x^2 = e$ for all x in G. Prove that G is isomorphic to $\mathbb{Z}_2 \oplus \mathbb{Z}_2$.

(ii) Find two groups G_1 and G_2 such that G_1 is isomorphic to G_2 but $Aut(G_1)$ is not isomorphic to $Aut(G_2)$ where $Aut(G_i)$ is the group of automorphisms of G_i.

(b) (i) Suppose that N is a normal subgroup of a finite group G. If G/N has an element of order n, show that G has an element of order n. Also show, by an example, that the assumption that G is finite is necessary.

(ii) If G is a non abelian group then show that $Aut(G)$ is not cyclic.

(c) Define the characteristic subgroup of a group G. Prove that every subgroup of a cyclic group is characteristic.

(4) (a) If p is a prime and G is a group of prime power order p^α for some positive integer $\alpha \geq 1$, then show that G has a non trivial centre.

(b) Find all conjugacy classes of the dihedral group D_8 of order 8 and the quaternion group Q_8 of order 8 and hence verify the class equation.

(c) Prove that if H has a finite index n in G then there is a normal subgroup K of G where K is subgroup of H and the index of K in G ($|G : K|$) is less than or equal to $n!$.

(5) (a) Prove that if p is a prime and G is a group of order p^α for some positive integer α then every subgroup of index p is normal in G. Deduce that every group of order p^2 has a normal subgroup of order p.

(b) Prove that a group of order 56 has a normal Sylow p-subgroup for some prime p dividing its order.

(c) Prove that two elements of the symmetric group on n letters S_n are conjugate in S_n if and only if they have same cycle type.Also show that the number of conjugacy classes equals the number of partitions of n.

(6) (a) Define a simple group. Prove that if G is an abelian simple group then G is isomorphic to the cyclic group $\mathbb{Z}_p$ for some prime p.

(b) (i) Prove that group of order 280 is not simple.

(ii) Show that the alternating group A_5 of degree 5 can not contain a subgroup of order 30 or 20 or 15.

(c) (i) If the centre of G is of index n, then prove that every conjugacy class has at most n elements.

(ii) Prove that the centre $Z(S_n)$ of symmetric group S_n contains only the identity of S_n for all n greater than or equal to three.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6771 **HC**

Unique Paper Code : 42341102

Name of the Paper : Problem Solving with Computers.

Name of the Course : **B.Sc. (Prog.)/ Math Sc.**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any five questions from **Section B.**
4. Parts of a question must be answered together.
5. Write program statements in Python.

Section A

1. (a) What do you mean by unary and binary operators? Give an example of each. (2)

(b) Determine the output of the following statements: (2)

(i) math.ceil(7.4)

(ii) math.floor(7.4)

(iii) max("hello", "how", "are", "you")

(iv) ((not (9==8)) and ((7+1) ! = 8)) or (6 < 4.5)

(c) Rewrite the following program segment using for loop (2)

```
total=0
count=1
while count < 5:
    total += count
    count += 1
print total
```

(d) Identify the syntactical errors in the following code and rewrite the code after removing them: (2)

```
def add:
   a = b = 7
   result = a + b
```

(e) For 1st = [2, 5, 6, 8, 12, 22] (2)

write the output of the following:

(i) del lst [2:4]

(ii) Ist.pop(2)

(f) For colors = 'Red, Green, Blue, Orange, Yellow, Cyan' (2)

Write the output of the following:

(i) colors, split()

(ii) colors.partition(',")

(g) Write the steps of searching the number 31 in the given list using Binary Search: (2)

17,20,26,31,44,54,55,65,77,93

(h) Write a function to return front element of a queue. (3)

(i) What are data scanning devices? How are these better as compared to keyboard devices? (2)

(j) What is an algorithm? Name any two commonly used ways to represent an algorithm. (2)

(k) Name the Hardware technology used in Second and Fourth generation computers? (2)

(l) Write full form of the following abbreviations used in computer technology? (2)

MBR, VLSI, ALU, GIGO.

Section B

2. (a) Write a function that prints Fibonacci series for first n terms. Fibonacci series takes 0 and 1 as the first two values. Third and subsequent values in the series are computed as the sum of previous two terms. Take n as input from the user. (5)

(b) Evaluate the following expressions involving bitwise operators (show calculations):

(i) 40 >> 3

(ii) -15 & 22

(iii) ~11

(iv) 10 ^ 6

(v) 15 | 22 (5)

3. (a) Write a program to find maximum of three numbers using nested function approach. Define a function max3 that takes three numbers as input and computes maximum of three using another function max2 that finds maximum of two numbers. (5)

(b) Consider the following function:

```
def nMultiple(a=0, num=1):
    return a* num
```

Determine the output obtained when the following calls are made:

(i) nMultiple(5)

(ii) nMultiple(5, 6)

(iii) nMultiple(num=7)

(iv) nMultiple(num=1, a=5)

(v) nMultiple(5, num=6) (5)

. (a) Write statements in Python to count the number of occurrences of all vowels in the string "Encyclopedia". (5)

(b) (1) Determine the output of the following statement: (2+2+1=5)

result= [x+y for x in range(1,5) for y in range (1,5)]

(2) Determine the output of the following statements for the given input:
names= ['Ram', 'Sita', 'Gita', 'Sita']

(i) names.insert(2,'Shyam')

(ii) names.sort()

(iii) names.reverse()

(iv) names.sort(reverse=True)

(3) Determine the output of the following statemen[t] for the given input:

digitsl = set([0, 1, 2, 3])

digits2= set([2, 4, 5, 6])

digits3 = set([0, 7, 8, 9, 2])

(i) set.difference(digits 1, digits2, digits3)

(ii) digitsl | digits2

5. (a) Define a class **Employee** that stores information abou[t] employees in the company. The class should contain th[e] following data members: (5)

Name - Employee Name

Department- Department in which Employee is workin[g]

Basic, DA, HRA- Components of Salary

Salary- Salary of the Employee

The Class should support the following methods:

(i) **__init__()** method for initializing data memebers

(ii) **findSalary() method** for determining salary as sum of Basic, DA and HRA

(iii) **empDisplay()** for displaying information about the employee.

(b) Consider the following string: (5)

message= 'Hello!! How are you?'

Determine the output for the following functions:

(i) len(message)

(ii) message[-10:-5]

(iii) message.find('o')

(iv) message.rfind('o')

(v) message.capitalize()

(a) What is an exception? (5)

Name the exceptions that can occur on executing the following statements:

(i) colors=['red', 'green','blue']

colors[4]

(ii) result = 'sum of 4 and 2 is' + 6

(iii) 45/0

(iv) for i in range (0, 10)

print i

(b) Explain how exceptions are handled in Python? (5)

7. (a) Sort the list P = [56, 48, 12, 75, 88,9). Show the modified list after each iteration of the Insertion Sort method. (5)

(b) Under what conditions is binary search used? Write a recursive function to implement binary search algorithm. (5)

8. (a) What is a stack? Write a function to remove an element from the top of stack. Explain the overflow and underflow conditions? (5)

(b) Consider a queue of three elements which are integers and perform the following operations in sequence on the queue and show the modified queue in each case : (5)

enqueue 5

enqueue 10

dequeue

enqueue 20

enqueue 25

dequeue

dequeue

dequeue

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Roll No. | | | | | | | | | | | |

. No. of Question Paper : **7948**

Inique Paper Code : **62351101** HC

lame of the Paper : **Calculus**

lame of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Ouration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Discuss the existence of the limit of the function : 6

$$f(x) = \frac{e^{\frac{1}{x^2}}}{1 - e^{\frac{1}{x^2}}}$$

at $x = 0$.

P.T.O.

(*b*) Discuss the continuity of : 6

$$f(x) = |x - 1| + |x - 2|$$

at $x = 1$ and $x = 2$. Also state the kind of discontinuity, if any.

(*c*) Examine the following function for differentiability at $x = 0$: 6

$$f(x) = \begin{cases} x\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} \; ; & x \neq 0 \\ 0 \; ; & x = 0 \end{cases}$$

2. (*a*) Find the *n*th derivative of $\cos(x + 5)$. 6

(*b*) If 6

$$y = \left[x + \sqrt{1 + x^2}\right]^m,$$

prove that :

$$(1 + x^2)\, y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 - m^2)\, y_n = 0.$$

(c) If 6

$$u = \log\left(\frac{x^2 + y^2}{x + y}\right),$$

then using Euler's theorem, prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1.$$

3. (a) If the tangent to the curve : 6½

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$$

cuts off intercepts p and q from the axis of x and y respectively, show that :

$$\frac{p}{a} + \frac{q}{b} = 1.$$

(b) Find the equation of the tangent to the curve $y^2 = 4x$ which makes an angle 45° with the x-axis. 6½

(c) Show that radius of curvature is $4a\cos\frac{\theta}{2}$ for the cycloid : 6½

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta).$$

4. (a) Find the asymptotes of the curve : 6½

$$x^3 - 4x^2y + 5xy^2 - 2y^3 + 3x^2 - 4xy + 2y^2 - 3x + 2y - 1 = 0.$$

(b) Find the equation of the tangent to the curve : 6½

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$

at (–1, –2), and show that it is a cusp.

(c) Trace the curve : 6½

$$x^3 + y^3 = 3axy,\ a > 0.$$

5. (a) State Lagrange's mean value theorem. Can we drop some condition of Lagrange's mean value theorem ? Justify your answer. 6

(b) Let $f(x) = \tan x$ for all x in **R**. Using Lagrange's mean value theorem, for the function f, show that : 6

$$|\tan^{-1} x - \tan^{-1} y| < |x - y| \quad \forall\ x, y \in \mathbf{R}.$$

(c) Let f be a function defined by : 6

$$f(x) = x^3 - 6x^2 + 9x + 1 \quad \forall\, x \in \mathbf{R}.$$

Find the interval in which the function f is increasing or decreasing.

6. (a) Find the maximum and minimum values of the function : 6½

$$f(x) = 2x^3 - 15x^2 + 36x + 10 \quad \forall\, x \in \mathbf{R}.$$

(b) Define extremum of a function. Give an example of a function with no extremum. Justify your answer. 6½

(c) Evaluate : 6½

$$\lim_{x \to 0^+} (\cot x)^{\sin x}.$$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **5190** **H**

Unique Paper Code : 235351

Name of the Course : **B.A. (Programme)**

Name of the Paper : Integration and Differential Equations

Semester : III

Time : Three Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each questions.

1. (a) Find the area of the region bounded above by y = x + 6 bounded below by $y = x^2$ and bounded on the sides by the lines x = 0 and x = 2. 6

(b) Evaluate : 6

(i) $\int \frac{2x+3}{\sqrt{3+4x-4x^2}} dx$

(ii) $\int \frac{dx}{4+5\sin x}$

(c) Find the value of $\int_0^{\pi/4} \frac{\cos x - \sin x}{5 + \sin 2x} dx$ 6

2. (a) Find the reduction formula for $I_{m,n} = \int \sin^m x \cos^n x \, dx$ where m and n are positive integers & hence evaluate

$\int \sin^4 x \cos^3 x dx$ $6\frac{1}{2}$

(b) Find the volume of the solid that results when the region enclosed by the given curve is revolved about the x-axis

$y = 9 - x^2$, $y = 0$. $6\frac{1}{2}$

(c) Find arc length of the curve $y = x^{2/3}$ from $x = 1$ to $x = 8$.

3. (a) Find the area of surface generated by revolving the given curve about the x-axis $y = \sqrt{4 - x^2}$ 6

(b) Solve :

$ydx - xdy + \log x \, dx = 0$

(c) Find the orthogonal trajectories of the family of curves $y = cx^2$ where c is a parameter. 6

4. (a) Solve :

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$ $6\frac{1}{2}$

(b) Solve :

$$\frac{d^2y}{dx^2} - \frac{6}{x^2}y^2 = x\log x$$ $6\frac{1}{2}$

(c) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. What is the general solution ? Find the solution y(x) satisfying y(0) = 2, y(0) = -3. $6\frac{1}{2}$

5. (a) A certain culture of bacteria grows at a rate that is proportional to the number present. It is found that the number doubles in 4 hours. How many may be expected at the end of 12 hours ? 6

(b) Solve 6

(yz + 2x) dx + (zx + 2y) dy + (xy + 2z) dz = 0

(c) Solve the following differential equation by method of variation of parameter

$$\frac{d^2y}{dx^2} + y = \tan x.$$ 6

P.T.O.

6. (a) Form the partial differential equation by eliminating the constants a, b from the equation.

(i) $z = ax + by + a^4 + b^4$

(ii) $z = (x + a)(y + b)$

$6\frac{1}{2}$

(b) Find the general solution of the Lagrange's equation $6\frac{1}{2}$

$x(y - z)p + y(z - x)q = z(x - y)$

(c) Solve the partial differential equation by Charpit's method

$(p^2 + q^2)y = qz$

$6\frac{1}{2}$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **5191** **H**

Unique Paper Code : 235351

Name of the Course : **B.A. (Programme)**

Name of the Paper : Integration and Differential Equations

Semester : III

Time : Three Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **any two** parts from each questions.

1. (a) Find the area of the region enclosed by curve $y^2 = 4x$ and $y = 2x - 4$ by integrating with respect to x. 6

(b) Evaluate 6

$$\int \frac{dx}{\sqrt{(x^2+2x+5)}}$$

$$\int \frac{dx}{5+4\cos x}$$

P.T.O.

(c) Find the value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ 6

2. (a) Find the reduction formula $I_{m,n} = \int \cos^m x \sin nx \, dx$ where m and n are positive integers. $6\frac{1}{2}$

(b) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x -axis. $6\frac{1}{2}$

(c) Find the arc length of the curve $x = \frac{1}{3}(y^2+2)^{3/2}$ from y = 0 to y = 1. $6\frac{1}{2}$

3. (a) Find the area of surface generated by revolving the given curve about x-axis

$y = \sqrt{x} \qquad 1 \leq x \leq 4$ 6

(b) Solve 6

$$y(x^2y^2+2)dx + x(2-2x^2y^2)dy = 0$$

(c) Solve 6

$$y = 2px + y^2p^3, p = \frac{dy}{dx}$$

4. (a) Solve $6\frac{1}{2}$

$$\left(e^{x}+1\right)y\ dy=\left(y+1\right)e^{x}dx$$

(b) Find orthogonal trajectories of the family of curves $cx^2 + y^2 = 1$ where c is a parameter.

$6\frac{1}{2}$

(c) Evaluate Wronskian of the functions $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ and hence concluded whether or not they are linearly independent. Also form the differential equation.

$6\frac{1}{2}$

5. (a) A bacteria culture is known to grow at a rate proportional to the amount present. If the initial number is 300 and if it is observed that the population has increased by 20 percent after 12 hours determine the number of bacteria present in the culture after 2 days. 6

P.T.O.

(b) Solve the system of equations 6

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dx}{dt} + 2y + y = 3e^t$$

(c) Solve the differential equation by the method of variation of parameter :

$$\frac{d^2y}{dx^2} + y = \tan x$$ 6

6. (a) (i) Form a partial differential equations by eliminating the functions from $z = (x + y) + f(xy)$.

(ii) Eliminate the constants from $z^2 = ax^2 + by^2 + 1$ to form a partial differential equation. $6\frac{1}{2}$

(b) Find the general solution of following

$y^2 - x^2 = z\,(xp - yq)$ $6\frac{1}{2}$

(c) Solve the equations by Charpit's method

$(p^2 + q^2)\,y = qz$ $6\frac{1}{2}$

[This question paper contains 4 printed pages.]

Your Roll No..............

r. No. of Question Paper : 6778 HC

nique Paper Code : 42354302

ame of the Paper : Algebra

ame of the Course : B.Sc. Physical Sciences / Mathematical Sciences/ Analytical Chemistry (Part-II)

mester : III

ration : 3 Hours Maximum Marks : 75

structions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Attempt **any two** parts from each questions.

All questions are compulsory.

Marks are indicated.

Unit-I

(a) Define Group. Show that in a group G, the right and left cancellation laws hold. (6)

(b) Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G. (6)

P.T.O.

(c) Let G be a group and H a nonempty subset of G. T[illegible] show that H is a subgroup of G if ab^{-1} is in H whe[illegible] a and b are in H.

2. (a) Let $G = GL(2,R)$ and $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a \text{ and } b \text{ ar[illegible] zero integers} \right\}$. Prove or disprove that H is a subg[illegible] of G.

(b) Let $G = \langle a \rangle$ be a cyclic group of order n. Then s[illegible] that $G = \langle a^k \rangle$ if and only if $\gcd(k,n) = 1$.

(c) Let

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix} \text{ and}$$

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

Compute each of the following:

(a) Write α and β as product of disjoint cycles.

(b) Compute $\alpha\beta$ and α^{-1}. (6)

3. (a) State Lagrange's theorem for groups. Show that group of prime order is cyclic. (6)

(b) Suppose that a has order 15. Compute all the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$. (6)

(c) Show that every permutation on a finite set can be written as a cycle or a product of disjoint cycles. (6)

Unit-II

4. (a) Let $S = \{a + bi : a, b \in \mathbb{Z}, b \text{ is even}\}$. Show that S is a subring of the ring $\mathbb{Z}[i]$ of Gaussian integers, but not an ideal of $\mathbb{Z}[i]$. (6½)

(b) Define a ring and an integral domain. Give an example of a ring which is not an integral domain. (6½)

(c) Prove that every finite integral domain is a field. (6½)

Unit-III

5. (a) Prove that the intersection of two subspaces of a vector space V(F) is a subspace of V(F). Is the result true for the union of two subspaces? If not, give example. (6½)

(b) Show that $S = \{(1,0,-1,0), (2,1,3,0), (-1,0,0,0), (1,0,1,0)\}$ is a linearly dependent set in $\mathbb{R}^4$.

(c) Let $\{a,b,c\}$ be a basis for the vector space $\mathbb{R}^3$. Prove that the set $\{a+b,\ b+c,\ c+a\}$ is also a basis of $\mathbb{R}^3$. (6½)

6. (a) (i) Show that the mapping $T:\mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1,x_2) = (x_1, x_1 + x_2, x_2)$ is a linear transformation.

(ii) Let $T : V \to W$ be a linear transformation. If v_1, v_2, v_3 are linearly dependent vectors in V, prove that $T(v_1), T(v_2), T(v_3)$ are linearly dependent in W. (6½)

(b) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1,1) = (1,3)$, $T(-1,1) = (3,1)$. Find $T(a,b)$ for any $(a,b) \in \mathbb{R}^2$. (6½)

(c) Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,y) = (x, x + y, y)$, then find the range, rank, kernel and nullity of T. (6½)

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7987**

Unique Paper Code : **62354343** **HC**

Name of the Paper : **Analytical Geometry and Applied Algebra**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Identify and sketch the curve :

$$(x + 2)^2 = -(y + 2)$$

and also label the focus, vertex and directrix. 6

(*b*) Sketch the ellipse :

$$9(x - 3)^2 + 25(y + 1)^2 = 225$$

also label foci, vertices and ends of major and minor axes. 6

(*c*) Describe the graph of the equation :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$ 6

2. (*a*) Find the equation of the parabola that has its vertex at (1, 1) and directrix $y = -2$. Also state the reflection property of parabola. 6

(*b*) Find an equation for the ellipse with length of major axis 10 and with vertices (3, 2) and (3, – 4) and also sketch it. 6

(*c*) Find and sketch the curve of the hyperbola whose asymptotes are $y = 2x + 1$ and $y = -2x + 3$ and the hyperbola passes through the origin. 6

3. (*a*) Consider the equation $x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$. Rotate the coordinate axes to remove the xy term and then identify the type of the conic represented by the above equation. 6

(*b*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 60°$.

(*i*) Find the $x'y'$-coordinate of the point whose xy-coordinates are (2,6).

(*ii*) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinates. 6

(*c*) Find the equation of the sphere with center at (2,–1,–3) and is tangent to the zx-plane. 6

4. (*a*) (*i*) Find a vector **v** having opposite direction as the vector from the point P (1, 0, –6) to Q (–3, 1, 1) with $\|\mathbf{v}\| = 5$.

(*ii*) Sketch the surface $z^2 + y^2 = 4$ in 3-space. 3+3½

(*b*) (*i*) Using vector, find the area of triangle with vertices A(2, 2, 0), B(–1, 0, 2) and C(0, 4, 3).

(*ii*) Let $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find the volume of the parallelopiped with adjacent edges **u**, **v** and **w**. 3+3½

(*c*) Prove that

$$\mathbf{u}.\mathbf{v} = \tfrac{1}{4}\left(\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2\right).$$ 6½

5. (*a*) Find the distance between the skew lines : 6½

$$L_1 : x = 1 + 7t \quad y = 3 + t \quad z = 5 - 3t, \quad -\infty < t < \infty$$

$$L_2 : x = 4 - t \quad y = 6 \quad z = 7 + 2t, \quad -\infty < t < \infty$$

(*b*) (*i*) Determine whether the points P_1 (–6, 4, 8), P_2(9, – 2, 0) and P_3 (1, –5, 3) lie on the same line.

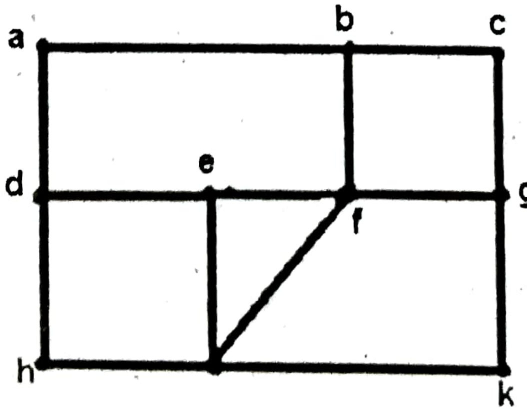
(*ii*) Where does the line

$$x = 2 - t,\ y = 3t,\ z = 1 + 2t$$

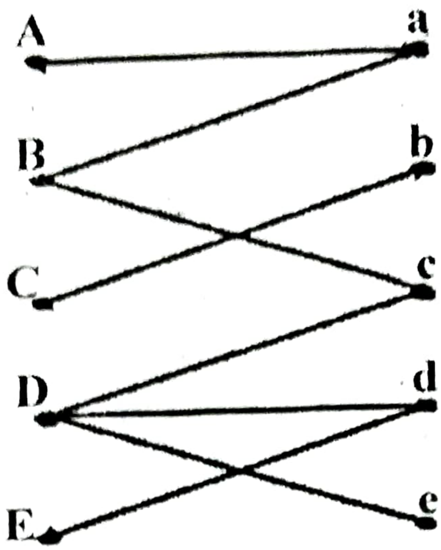
intersect the plane $2x - 7y + 3z = 6$. 3+3½

(*c*) Find the equation of the plane through the points $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$. 6½

6 (*a*) Find a maximum independent set of vertices for the following graph. What is the minimum number of independent set needed to cover all the vertices ? 6½



(*b*) (*i*) Find a matching or explain why none exists for the following graph :



(*ii*) Given three pitchers: 8, 5 and 3 liters capacity. Only 8 liter pitcher is full. Make at least one of them contain exactly 4 liter of water with the minimum number of water transfers. 3+3½

(*c*) Defing Latin square. Construct a Latin square of order 5 on $\{e, e^2, e^3, e^4, e^5\}$. 6½

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	: **5230**	**H**
Unique Paper Code	: **235551**	
Name of Paper	: **Analysis**	
Name of Course	: **B.A. Programme**	
Semester	: **V**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are three Sections. Each Section consists of 25 marks.

Attempt any ***two*** *parts from each question in each Section.*

Marks are indicated againt each question.

SECTION I

1. (a) Define a bounded set, its supremum and infimum. Find the supremum and infimum of the following sets:

(i) $\left\{\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3} \ldots\ldots\ldots\right\}$

(ii) $\left\{\frac{n}{n+1}; n=1, 2, 3, \ldots\ldots\ldots\right\}$

(iii) Z, the set of integers. 6

(b) Define open set and prove that the union of an arbitrary family of open sets is an open set. 6

Turn over

(c) Give an example of a set which has:

(i) No limit point

(ii) Unique limit point

(iii) Infinite number of limit points. 6

2. (a) Show that the function f defined as:

$$f(x)=\begin{cases}(1+2x)^{\frac{1}{x}}, & \text{when } x\neq 0\\ e^2, & \text{when } x=0\end{cases}$$

is continuous at $x = 0$. 6½

(b) Show that $f(x)=\frac{1}{x}$ is not uniformly continuous on [0, 1]. 6½

(c) (i) Define neighbourhood.

(ii) Define closed set.

(iii) Give an example of a set whose derived set is void. 6½

SECTION II

3. (a) Show that $\lim_{n\to\infty} r^n = 0$, if $|r|<1$. 6½

(b) If $\langle a_n\rangle$ and $\langle b_n\rangle$ be two sequences such that:

$$\lim_{n\to\infty} a_n = a,\ \lim_{n\to\infty} b_n = b,\ b_n \neq 0 \text{ and } b\neq 0$$

then show that:

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)=\frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}=\frac{a}{b}.$$ 6½

(c) Prove that a monotone sequence is convergent iff it is bounded. 6½

(a) If $\sum_{1}^{\infty} u_n$ is a convergent series then show that $\lim_{n \to \infty} u_n = 0$. Does the converse of this result hold? Justify your answer. 6

(b) State Raabe's test for convergence of the series $\sum_{1}^{\infty} u_n$ and hence test the convergence of the series:

$$\sum_{1}^{\infty} \frac{1.3.5 \ldots\ldots\ldots (2n-1)}{2.4.6 \ldots\ldots\ldots 2n} \cdot \frac{1}{n}. \quad 6$$

(c) Test the absolute convergence of the following series:

(i) $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$

(ii) $\sum_{1}^{\infty} \frac{\sin nx + \cos nx}{n^{3/2}}$

(iii) $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n}$. 6

SECTION III

(a) Show that continuous function f defined on a closed and bounded interval $[a, b]$ is integrable. 6

(b) Test the convergence of the improper integral:

$$\int_{0}^{\infty} x^{n-1} e^{-x} \, dx. \quad 6$$

Turn over

(c) Define Gamma function and show that $\int_{-\infty}^{\infty} e^{-x^2}$ ...

6. (a) Find the Fourier series of the function f defi... follows:

$$f(x)=\begin{cases}1, & \text{for } -\pi<x\leq 0\\ -2, & \text{for } 0<x\leq \pi\end{cases}.$$

(b) Show that $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{n^p\left(1+x^{2n}\right)}$ converges absolutel... uniformly for all real values of x if $p > 1$.

(c) (i) Find the radius of convergence of the power ...

$$\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n.$$

(ii) Discuss the Riemann integrability of the fu... $f(x) = |x|$ on $[-1, 1]$.

5230

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : 6780 HC

Unique Paper Code : 42357501

Name of Paper : Differential Equations

Name of Course : DSE for Mathematical Science / Prog.

Semester : V

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

*Attempt any **two** parts from each question.*

Marks of each part are indicated.

Use of non-programmable scientific calculator is allowed.

1. (a) Solve the initial value problem:

$(2y \sin x \cos x + y^2 \sin x)\, dx + (\sin^2 x - 2y \cos x)\, dy = 0,$

$y(0) = 3.$ 6½

(b) Solve: $\frac{dy}{dx} + 3x^2 y = x^2,\ y(0) = 2.$ 6½

(c) Solve: $y = 2px + yp^2.$ 6½

2. (a) Solve the initial value problem:

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2xe^{2x} + 6e^x,\ y(0) = 1,\ y(0) = 0.$ 6

(b) Solve: $$x^3\frac{d^3y}{dx^3}-x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=x^3.$$ 6

(c) Consider the differential equation:

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$$

(i) Show that e^{2x} and e^{3x} are linearly independent solutions of the given differential equation over IR.

(ii) Write the general solution of the given equation.

(ii) Find the solution that satisfies the conditions $y(0)=2$, $y'(0)=3$. Explain why this solution is unique. 6

3. (a) Find the general solution of:

$$(x+1)^2\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+2y=1,$$

given that $y=x+1$ and $y=(x+1)^2$ are linearly independent solution of the corresponding homogeneous equation.

6½

(b) Given that $y = e^{2x}$ is a solution of the given differential equation:

$$(2x+1)\frac{d^2y}{dx^2}-4(x+1)\frac{dy}{dx}+4y=0.$$

Find a linearly independent solution by reducing the order. Also, write the general solution. 6½

(c) Check the condition of integrability and then solve the given differential equation:

$$yz(y+z)\,dx+xz(z+x)\,dy+xy(x+y)\,dz=0.$$ 6½

4. (a) Solve:

$$\frac{dx}{dt} = 5x - 2y, \frac{dy}{dt} = 4x - y.$$ 6

(b) Solve:

$$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{nl(z-x)} = \frac{n\,dz}{lm(x-y)}.$$ 6

(c) Solve : $(y^2 + yz)\,dx + (zx + z^2)\,dy + (y^2 - xy)\,dz = 0$. 6

5. (a) Form a partial differential equation corresponding to the complete integral given by $z = xy + f(x^2 + y^2)$, where f is an arbitrary function. 6½

(b) Define a linear partial differential equation. Form a partial differential equation corresponding to the complete integral given by $z = (x + a)(y + b)$, where a and b are arbitrary constants. 6½

(c) Explain the criteria to classify a partial differential equation into parabolic, elliptic or hyperbolic equation. Illustrate by telling the nature of the following partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}.$$ 6½

6. (a) Find the complete integral of the partial differential equation $p^2 x + q^2 y = z$, by using Charpit's method. 6

P.T.O.

(b) Use Lagrange's method to find the general solution of the partial differential equation $p \tan x + q \tan y = \tan z$. 6

(c) Find the complete integral of the partial differential equation $p = (z + qy)^2$, by using Charpit's method. 6

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 6780** **HC**
Unique Paper Code	**: 42357501**
Name of Paper	**: Differential Equations**
Name of Course	**: DSE for Mathematical Science / Prog.**
Semester	**: V**
Duration	**: 3 hours**
Maximum Marks	**: 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any ***two*** *parts from each question.*

Marks of each part are indicated.

Use of non-programmable scientific calculator is allowed.

1. (a) Solve the initial value problem:

$(2y \sin x \cos x + y^2 \sin x)\, dx + (\sin^2 x - 2y \cos x)\, dy = 0,$

$y(0) = 3.$ 6½

(b) Solve: $\frac{dy}{dx} + 3x^2 y = x^2,\ y(0) = 2.$ 6½

(c) Solve: $y = 2px + yp^2.$ 6½

2. (a) Solve the initial value problem:

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2xe^{2x} + 6e^x,\ y(0) = 1,\ y(0) = 0.$ 6

P.T.O.

(b) Solve: $x^3\frac{d^3y}{dx^3}-x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=x^3.$ 6

(c) Consider the differential equation:

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$$

(i) Show that e^{2x} and e^{3x} are linearly independent solutions of the given differential equation over IR.

(ii) Write the general solution of the given equation.

(ii) Find the solution that satisfies the conditions $y(0)=2$, $y'(0)=3$. Explain why this solution is unique. 6

3. (a) Find the general solution of:

$$(x+1)^2\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+2y=1,$$

given that $y=x+1$ and $y=(x+1)^2$ are linearly independent solution of the corresponding homogeneous equation.

6½

(b) Given that $y = e^{2x}$ is a solution of the given differential equation:

$$(2x+1)\frac{d^2y}{dx^2}-4(x+1)\frac{dy}{dx}+4y=0.$$

Find a linearly independent solution by reducing the order. Also, write the general solution. 6½

(c) Check the condition of integrability and then solve the given differential equation:

$$yz(y+z)\,dx+xz(z+x)\,dy+xy(x+y)\,dz=0.$$ 6½

4. (a) Solve:

$$\frac{dx}{dt} = 5x - 2y, \frac{dy}{dt} = 4x - y.$$ 6

(b) Solve:

$$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{nl(z-x)} = \frac{n\,dz}{lm(x-y)}.$$ 6

(c) Solve : $(y^2 + yz)\,dx + (zx + z^2)\,dy + (y^2 - xy)\,dz = 0$. 6

5. (a) Form a partial differential equation corresponding to the complete integral given by $z = xy + f(x^2 + y^2)$, where f is an arbitrary function. 6½

(b) Define a linear partial differential equation. Form a partial differential equation corresponding to the complete integral given by $z = (x + a)(y + b)$, where a and b are arbitrary constants. 6½

(c) Explain the criteria to classify a partial differential equation into parabolic, elliptic or hyperbolic equation. Illustrate by telling the nature of the following partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}.$$ 6½

6. (a) Find the complete integral of the partial differential equation $p^2x + q^2y = z$, by using Charpit's method. 6

(b) Use Lagrange's method to find the general solution of the partial differential equation $p \tan x + q \tan y = \tan z$. 6

(c) Find the complete integral of the partial differential equation $p = (z + qy)^2$, by using Charpit's method. 6

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6828 HC

Unique Paper Code : 42347902

Name of the Paper : Analysis of Algorithms and Data Structures

Name of the Course : B.Sc. (P) Discipline Specific Elective

Semester : V

Duration : 3 hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. **1** is compulsory.

3. Attempt any **five** of Question nos. **2** to **8.**

4. Parts of a Question must be answered together.

1. (a) Give two differences between arrays and linked list. (2)

   (b) Why is linear implementation of queues using arrays an inefficient method of implementation? (2)

P.T.O.

(c) What is the role of stacks in the implementation of recursion? (2)

(d) Develop the representation of the following sparse matrix in row major ordering

$$\begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 6 & 0 & 0 & 7 & 0 & 0 & 3 \\ 0 & 0 & 0 & 9 & 0 & 8 & 0 & 0 \\ 0 & 4 & 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(2)

(e) Can you perform binary search on the following list <2, 4, 1, 9, 3, 7> ? Why? (2)

(f) Let T(n) be the time taken by an algorithm A giving a solution to a problem of size n in $3n^2$ amount of time in the worst case. Let T(n) = 3n. For each of the following specify whether it is true or false :

(i) $T(n) = \Omega(n^2)$

(ii) $T(n) = \Omega(n^3)$

(iii) $T(n) = O(n^2)$

(iv) $T(n) = O(n^3)$ (2)

(g) Convert the following infix expression to

(i) Prefix expression

(ii) Postfix expression

(A / B) – (C * D) + (F – G) (4)

(h) Match the following algorithms to their worst case running times :

Selection sort	n lg n
Binary search	n
Merge sort	lg n
Linear search	n^3
Matrix multiplication	n^2

(5)

(i) State true or false

(i) In the worst case linear search is slower than binary search.

(ii) Stacks use the FIFO method of access.

(iii) A doubly linked list uses more space than a singly linked list.

(iv) The nodes in a tree that have no children are called root nodes. (4)

2. (a) Showing changes after each step, sort the following array using bubble sort. How many exchanges will occur during the first pass?

43, 52, 76, 22, 87, 33, 92 (4)

(b) Write an algorithm for insertion sort. (3)

(c) Sort the following list [10, 1234, 9, 7234, 67, 9181] using Radix sort and show the values in the list after each step. (3)

3. (a) Do the following transformations

(i) Postfix to Infix

AB-C+DEF-+$

(ii) Infix to Postfix

A-B/(C*D$E)

(iii) Infix to Prefix

(A+B)*(C-D) (6)

(b) Write an algorithm to calculate factorial of a number using :

(i) Iteration

(ii) Recursion (4)

4. (a) Consider the following sequence of operations performed on an initially empty doubly linked list :

InsertBeginning(15),

InsertBeginning(18),

InsertEnd(13),

InsertEnd(10),

DeleteBeginning(),

Deletenode(13)

Show the head, tail, contents of the list and links between nodes after each operation. (6)

(b) Write a program to implement a circular queue using arrays. Include the following functions-

(i) Insert an element n into the queue

(ii) Delete an element n from the queue (4)

5. (a) Create a class stack. Declare appropriate data members. Declare and define the functions *push*() for inserting a value, *pop*() for removing a value. (6)

(b) Consider an initially empty stack of size 4 implemented using arrays. Perform the given sequence of operations and show the contents of the stack after each operation.

push (14),

pop(),

push (13),

push (18),

push (12),

push (16),

push (30),

pop( ) (4)

6. (a) Write member functions to perform the following operations on Singly Linked Lists :

(i) Insert an element after *n*th element of the list.

(ii) Delete an element present at the end of the list. (6)

(b) Consider the 0-1 knapsack problem; will greedy strategy always give the optimal solution? If yes, prove; if no, give counter example. (4)

7. (a) Perform binary search to find 2 in the array <1, 2, 3, 6, 7, 10, 12, 14, 15>. Show each step. (4)

(b) Write a recursive function to perform binary search. What are the conditions that are to be taken care while writing a recursive program? (4)

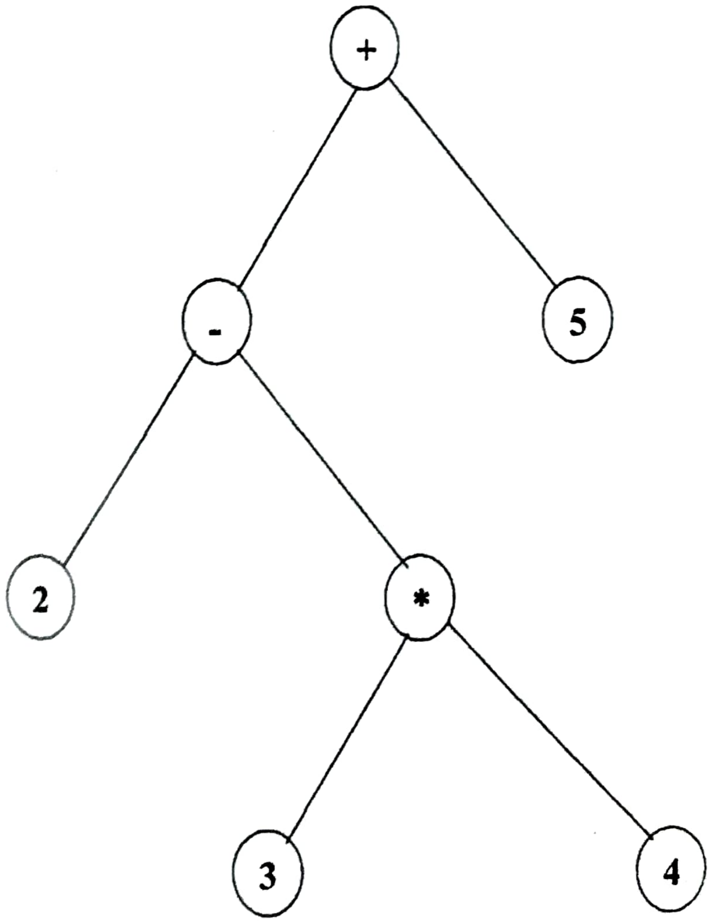
(c) Considering root of the binary tree at level 0, what is the maximum and minimum number of nodes at level i. (2)

3\. (a) Consider the following expression tree and perform

(i) Pre-order traversal

(ii) In-order traversal

(iii) Post-order traversal (6)



(b) Create a binary search tree using the following value

20, 5, 16, 12, 30, 14, 23 (4

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8078**

Unique Paper Code : **62357502** **HC**

Name of the Paper : **Differential Equations**

Name of the Course : **B.A. (Prog.) DSE : Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$(ye^x + 2e^x + y^2)\, dx + (e^x + 2xy)\, dy = 0;\ y(0) = 6.$$

(*b*) Solve : $(x^2 + y^2 + x)dx + xydy = 0.$ 6

(*c*) Solve : $(x - 2y + 5)dx - (2x + y - 1)dy = 0.$ 6

2. (*a*) Solve : $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$ 6.5

(*b*) Solve : $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin \ln x.$ 6.5

(*c*) Consider the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0.$$ 6.5

P.T.O.

(i) Show that each of the functions e^x, e^{4x} and $2e^x - 3e^{4x}$ is a solution. Also show that e^x and $2e^x - 3e^{4x}$ are linearly independent.

(ii) Write the general solution.

3. (a) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} + y = \sec^2 x.$$ 6.5

(b) Using the method of undetermined coefficients to find the general solution of the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}.$$ 6.5

(c) Given that $y = e^x$ is a solution of the differential equation :

$$x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$ 6.5

Find a linearly independent solution by reducing the order and write the general solution.

4. (a) Solve : $\dfrac{dx}{x(y^2 - z^2)} = \dfrac{dy}{y(z^2 - x^2)} = \dfrac{dz}{z(x^2 - y^2)}$. 6

(*b*) Solve : $yz(y + z)dx + xz(x + z)dy + xy(x + y)dz = 0.$ 6

(*c*) Solve : 6

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dx}{dt} + 2x + 5y = e^t.$$

5. (*a*) Find the general solution of the differential equation

$$(y + z)p + (z + x)q = x + y.$$ 6.5

(*b*) Find the complete integral of the differential equation

$$(p^2 + q^2)x = pz.$$ 6.5

(*c*) (*i*) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$u_{xx} + (1 + x^2)^2 u_{yy} = x^2.$$ 2.5

(*ii*) Eliminate the parameters *a* and *b* from the following equation to find the corresponding partial differential equation :

$$ax^2 + by^2 + z^2 = 1.$$ 4

6. (*a*) Find the complete integral of the equation :

$$p^2z^2 + q^2 = 1.$$ 6

(*b*) Eliminate the arbitrary function f from the equation $z = f\left(\frac{x}{y}\right)$ to find the corresponding partial differential equation. 6

(*c*) Find the general solution of the partial differential equation :

$$yzp + xzq = x + y.$$ 6

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: **8372**

HC

Unique Paper Code : **32357505**

Name of Paper : **Discrete Mathematics**

Name of Course : **Mathematics : DSE for Hons.**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each questions.

Section 1

1 (a) Define 'covering relation' in an ordered set. Prove that if P and Q are two ordered sets, then (a_2, b_2) covers (a_1, b_1) in $P \times Q$ if and only if either ($a_1 = a_2$ and b_2 covers b_1) or (a_2 covers a_1 and $b_1 = b_2$).

(6)

(b) Let N_0 be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n. Draw a Hasse diagram and find out maximal and minimal elements, if they exist, for the subset $\{2,3,4,6,10,12,0\}$ of $(N_0, \leq)$. Does it have the smallest and the greatest elements? Justify your answer.

(6)

(c) Define an order isomorphism for ordered sets. Show that every order isomorphism is bijective but the converse is not true.

(6)

2 (a) Let $(L, \leq)$ be a lattice as an ordered set. Define two binary operations + and $\cdot$ on L by $x + y = x \vee y = \sup\{x, y\}$ and $x \cdot y = x \wedge y = \inf\{x, y\}$. Prove that $(L, +, \cdot)$ is an algebraic lattice.

(6.5)

P. T. O.

(b) Let L be a lattice and let $x, y, z \in L$. Prove that

(i) $y \le z \Rightarrow x \wedge y \le x \wedge z$ and $x \vee y \le x \vee z$

(ii) $((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$,

(c) Let $f : L \to K$ be a lattice homomorphism. Show that

(i) If S is a sublattice of L, then $f(S)$ is a sublattice of K.

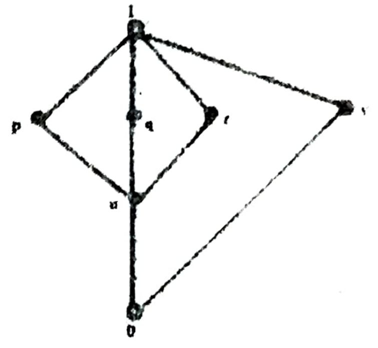
(ii) If T is a sublattice of K and $f^{-1}(T)$ is non-empty, then $f^{-1}(T)$ is a sublatt

Section II

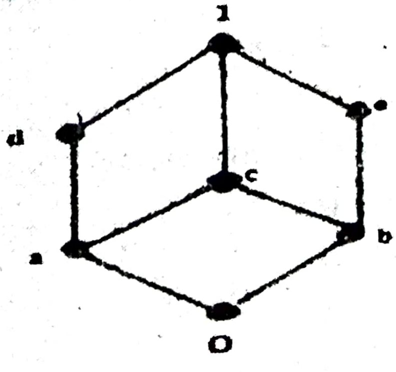
3 **(a)** Prove that a lattice L is distributive if and only if $\forall a, b, c \in L$ we have

$$(a \vee b = c \vee b \text{ and } a \wedge b = c \wedge b) \Rightarrow a = c.$$

(b) Use M_3-N_5 Theorem to find if the lattices L_1 and L_2 given below are modular



L_1



L_2

(c) Find the Conjunctive Normal form of

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'.$$

(6)

4 (a) Define sectionally complemented lattice. Show that every Boolean Algebra is sectionally complemented.

(6.5)

(b) Find all the prime implicants of $xy'z + x'yz' + xyz' + xyz$ and form the corresponding prime implicant table.

(6.5)

(c) Draw the contact diagram and give the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1x_3 + x_1'x_2)(x_2' + x_3)$$

(6.5)

Section III

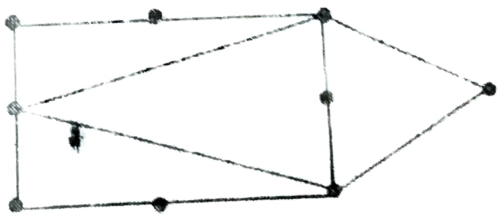
5 (a) (i) Answer the Königsberg bridge problem and explain your answer with graph.

(ii) Draw $K_{3,6}$ and $K_{4,4}$

(3, 3)

(b) (i) Draw a graph with 5 vertices and as many edges as possible. How many edges does your graph contain. What is the name of this graph and how is it denoted?

(ii) What is bipartite graph? Determine whether the graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite.



(3, 3)

(c) (i) Draw a graph whose degree sequence is 1,1,1,1,1,1.

(ii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4. Justify your answer.

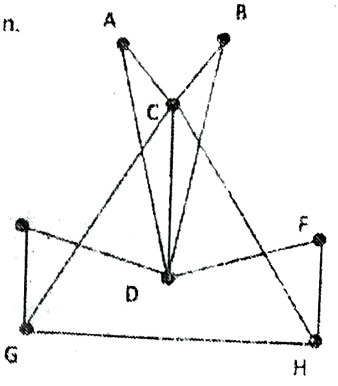
(iii) Draw pictures of the subgraphs G \{e} and G \{u} of the following graph G:

G

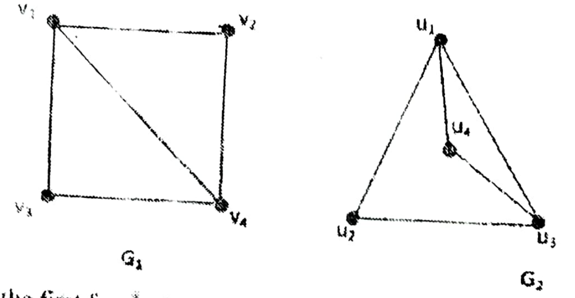
(2, 2, 2)

6 (a) (i) Consider the graph G given below. Is it Hamiltonian? If no, explain your an[swer, if yes]
find a Hamiltonian cycle.

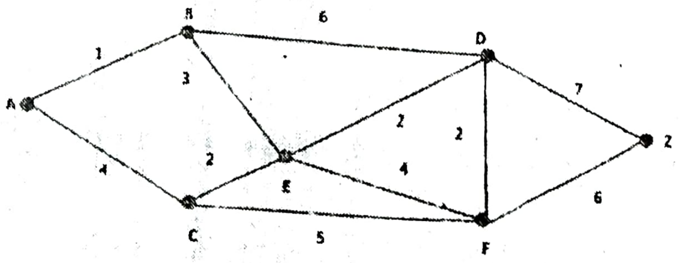
(ii) Is it Eulerian? Explain.



(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown bel[ow, and a]
permutation matrix P such that $A_2 = PA_1P^T$.



(c) Apply the first form of Dijkstra's Algorithm to find a shortest path from A to Z i[n the]
graph shown. Label all vertices.



This question paper contains 6 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	: **8508**	**HC**
Unique Paper Code	: **32357501**	
Name of Paper	: **Numerical Methods**	
Name of Course	: **Mathematics : DSE for Honours**	
Semester	: **V**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use of non-programmable scientific calculator is allowed.

*Attempt **all** questions, selecting **two** parts from each question.*

1. (a) Give the geometrical construction of the Newton's method to approximate a zero of a function. Write an algorithm to find a root of $f(x) = 0$ by Newton's method.

   (b) Define order of convergence of an iterative scheme $\{x_n\}$. Determine the order of convergence for the recursive scheme:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

Turn over

(c) Define the rate of convergence of an iterative scheme $\{x_n\}$. Use the bisection method to determine the root of the equation $x^5 + 2x - 1 = 0$ on $(0, 1)$. Further, compute the theoretical error bound at the end of fifth iteration and the next enclosing (bracketing) interval. 13

2. (a) Differentiate between the method of false position and the secant method. Apply the method of false position to $\cos x - x = 0$ to determine an approximation to the root lying in the interval $(0, 1)$ until the absolute error is less than 10^{-3} $(p = 0{\cdot}739085)$.

(b) Let g be a continuous function on the closed interval $[a, b]$ with $g : [a, b] \to [a, b]$. Furthermore, suppose that g is differentiable on the open interval (a, b) and there exists a positive constant $k < 1$ such that $|g'(x)| \leq k < 1$ for all x belongs to (a, b). Then:

(i) The sequence $\{p_n\}$ generated by $p_n = g(p_n - 1)$ converges to the fixed point p for any p_0 belonging to $[a, b]$;

(ii) $|p_n - p_{n-1}| \leq k^n \max(p_0 - a, b - p_0)$.

(c) Find the approximated root of $f(x) = x^3 + 2x^2 - 3x - 1$ by secant method, taking $p_0 = 2$ and $p_1 = 1$ until $|p_n - p_{n-1}| < 5 \times 10^{-3}$. 13

(a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that $LU = PA$ where

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}.$$

Hence, solve the system $Ax = b$, given

$$b = \begin{bmatrix} 14 \\ 36 \\ 7 \end{bmatrix}.$$

(b) Use Jacobi method to solve the following system of linear equations. Use the initial approximation $x^{(0)} = 0$ and perform three iterations.

$$\begin{aligned} 4x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + 4x_2 + x_3 &= -1, \\ -x_1 + x_2 + 4x_3 &= 1. \end{aligned}$$

(c) (i) Consider the matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular matrix U with ones along its diagonal such that $A = LU$.

(ii) Determine the spectral radius of the matrix:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}.$$ 13

4. (a) (i) If $x_0, x_1, x_2, \ldots\ldots x_{n+1}$ are $n + 1$ distinct points and f is defined at $x_0, x_1, x_2, \ldots\ldots x_n$, then prove that interpolating polynomial P, of degree at most n, is unique.

(ii) Define the shift operator E and central difference operator δ. Prove that:

$$E = 1 + \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}.$$

(b) For the function $f(x) = e^x$, construct the Lagrange form of interpolating polynomial of f passing through the points $(-1, e^{-1})$, $(0, 1)$ and $(1, e)$. Estimate $\sqrt{e}$ using the polynomial. What is the error in the approximation? Verify that theoretical error bound is satisfied.

(c) (i) Write the following data in the usual divided difference tabular form and determine the missing values:

$$x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 3,$$

$$f[x_0] = 2,\ f[x_1] = 6,\ f[x_2] = 6,$$

$$f[x_0, x_1] = 4,\ f[x_2, x_3] = 0,\ f[x_1, x_2, x_3] = 0.$$

(ii) Prove that:

$$\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0.$$

12

5. (a) Use the formula:

$$f'(x) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

to approximate the derivative of the function $f(x) = 1 + x + x^3$ at $x_0 = 1$, taking $h = 1, 0.1, 0.01$, and 0.001. What is the order of approximation?

(b) Verify:

$$f'(x) \approx \frac{-3f(x_0)+4f(\pm h)-f(x_0+2h)}{2h},$$

the difference approximation for the first derivative provides the exact value of the derivative regardless of h, for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

(c) Derive second-order forward difference approximation to the first order derivative of a function. 12

6. (a) Approximate the value of the integral $\int_1^2 \frac{1}{2} dx$ using Simpson rule. Further verify the theoretical error bound.

(b) Apply Euler's method to approximate the solution of the given initial value problem $x' + \frac{4}{t} = t^4$, $(1 \le t \le 3)$, $x(1) = 1$, $N = 5$. Further it is given that the exact solution is $x(t) = \frac{1}{9}(t^5 + 8t^{-4})$. Compute the absolute error at each step.

(c) Consider the initial value problem

$$x' = 1 + \frac{x}{t}, (1 \le t \le 3), x(1) = 1$$

whose exact solution is given by $x(t) = t(1 + \ln t)$. Using the step-size of 0.5, obtain the solution of the IVP and compare the absolute error with theoretical error bound, assuming the Lipschitz constant L equals 1. 12

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: **8509** **HC**

Unique Paper Code : **32357502**

Name of Paper : **Mathematical Modelling & Graph Theory**

Name of Course : **Mathematics : DSE for Hons.**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any three parts from each question.

SET- A

1. (a) Solve the initial value problem using Laplace transform: (6)

$$x''+6x'+25x=0;\ x(0)=2, x'(0)=3.$$

(b) (i) Find the inverse Laplace transform of (2)

$$F(s)=\frac{s-1}{(s+1)^3}.$$

(ii) Show that (2)

$$L\{t\, Cosh\, kt\}=\frac{s^2+k^2}{(s^2-k^2)^2}.$$

(iii) Find the inverse Laplace transform of (2)

$$F(s)=\frac{s^3}{(s-1)^4}.$$

(c) Find two linearly independent Frobenius series solutions of (6)

P. T. O.

$$2xy''+3y'-y=0.$$

(d) Use power series to solve the initial value problem: (6)

$$y''+xy'-2y=0;\ y(0)=1, y'(0)=0.$$

2. (a) Explain Middle-Square Method and use it to generate random numbers taking $x_0 = 2041$. Does this method has any drawbacks? Illustrate. (6)

(b) Using Monte Carlo Simulation, write an algorithm to calculate the volume of the sphere

$$x^2+y^2+z^2 \le 1$$

that lies in the first octant, $x>0, y>0, z>0$. (6)

(c) Using graphical analysis (6)

Minimize $x-y$
subject to

$$x+y \ge 6,$$
$$2x+y \ge 9,$$
$$x, y \ge 0.$$

(d) Using simplex method (6)

Maximize $6x+4y$
subject to

$$-x+y \le 12,$$
$$x+y \le 24,$$
$$2x+5y \le 80,$$
$$x, y \ge 0.$$

3. (a) (i) Draw two non-isomorphic regular graphs with 8 vertices and 12 edges. (3)

(ii) Prove that if G is a simple graph with at least 2 vertices then G has two or more vertices of same degree. (3)

(b) (i) Determine for what values of n, r and s the graphs given below are Eulerian and Semi-Eulerian.

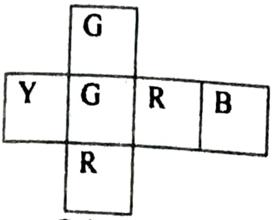
A) the complete graph K_n

B) the complete bipartite graph $K_{r,s}$

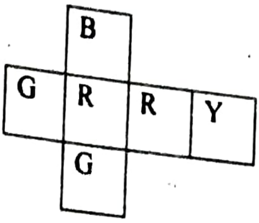
C) the $n-$cube Q_n. (4)

(ii) State Handshaking Lemma. (2)

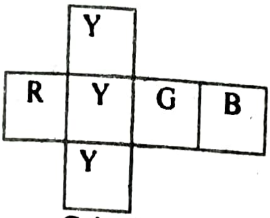
(c) Show that there will be no solution to the four cubes problem for the following set of cubes. (6)



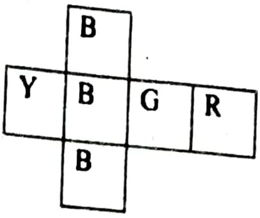
Cube 1



Cube 2



Cube 3



Cube 4

(d) Prove that there is no knight's tour on a 3 x 3 chessboard. (6)

4. (a) Use the factorization: (7)

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that:

$$L^{-1}\left\{\frac{s}{s^4 + 4a^4}\right\} = \frac{1}{2a^2} Sinh\,at\ Sin\,at\ .$$

(b) Solve the initial value problem: (7)

$$y'' + (x-1)\,y' + y = 0;\ y(1) = 2,\ y'(1) = 0.$$

(c) Fit the model $y = cx$ to the data using Chebyshev's criterion to minimize the largest deviation (7)

x	1	2	3
y	2	5	8

(d) Prove that if G be a graph in which every vertex has an even degree, then G can be split into cycles, such that no two cycles have an edge in common. (7)

is question paper contains 3 printed pages.]

Your Roll No..............

No. of Question Paper : **6824** **HC**

que Paper Code : 4234501

e of the Paper : System Administration and Maintenance

e of the Course : **B.Sc. Program/Mathematical Science : SEC**

ester : V

tion : 2 Hours Maximum Marks : 25

ructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Question No. **1** is compulsory.

Attempt any **3** questions from **Q. 2** to **Q. 6.**

(a) The ____________ interacts with the hardware and interacts with the user. (2)

P.T.O.

(b) The two modes of operation in Dual Mode of a operating system are ________ and ________ respectively. (2

(c) File permission rwx r - x r-- in Linux represen ________ in octal method. (1

(d) How does the operating system provide security to i users ? (

(e) Name any two administrative tools of Control Panel Windows OS. (

(f) Explain any three variations of date command. (

2. (a) Linux is a multiprogramming and multitasking operati system. Explain. (

(b) What is the difference between ps and who comman in Linux ? Explain. (

3. (a) Explain the difference between Absolute path name an Relative path name giving suitable examples. (

(b) Compare the features of Windows server 2003 an 2008. (2

4. List the function of each of the following commands :

(a) cal

(b) man

(c) ipconfig

(d) hostname

(e) rm (5)

5. (a) Describe basic Windows architecture components. Also draw the diagram. (3)

(b) Differentiate between Workgroup and Domain network types. (2)

6. (a) Explain the terms NTFS and FAT. (3)

(b) What is a firewall and what are its applications ? (2)

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : 6820A HC

Unique Paper Code : 42353327

Name of the Course : Mathematics Skill Enhancement Course

Name of the Paper : Mathematical Typesetting System : LaTeX

Semester : III

Time : 2 Hours *Maximum Marks : 38*

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

1. Fill in the blanks any **four** parts from the following : 4 × 0.5 = 2

(i) The command .... in the LaTeX document produces a medium space.

(ii) The .... command tells TeX to print its entire argument on the same line.

(iii) Indentation can be prevented in a paragraph of a LaTeX document with the .... command.

P.T.O.

(iv) In pspicture environment, the command .... produces an ellipse centered at (0,0) with major axis 6 units and minor axis 4 units.

(v) A mathematical formula appearing in the display mode is enclosed by .... and .... commands.

2. Answer any **eight** parts from the following : 8 × 2 = 16

(i) Write the difference between \hspace and \hspace* commands.

(ii) Typeset the following in a displayed formula :

$$\underbrace{a+\overbrace{b+\ldots+y}^{24}+z}_{26}.$$

(iii) Explain the \qbezier command in the LaTeX picture environment.

(iv) Draw a square of side 4 units with reference point(1,–2) and rounded corners.

(v) Write the command to draw an arrow at (4,4) of length 10 units in the direction of positive x-axis.

(vi) In PS Tricks picture environment, write a command to change unit-length of x-axis and y-axis by 2 centimeter and 3 centimeter, respectively.

(vii) Give the command in LaTeX to produce an expression :

$$\frac{1}{b-a}\int_a^b f'(x)\,dx = \frac{f(b)-f(a)}{b-a}.$$

(viii) Write the code in LaTeX in display math mode to produce an output.

If $x \not> y$ then $x \not\geq y+1$.

(ix) Write the following postfix expression in standard from :

x sin 1 x cos 2 exp add div 3 exp.

(x) Give a command to draw sector of a circle of radius 2 units centered at (3,3), going from reference angle 0 to 60 degrees.

3. Answer any **three** parts from the following : 4+4+4=12

(a) Plot step function $f(x) = [x]$, $0 \leq x < 5$ in the picture environment.

(b) Write the code in LaTeX to obtain an expression :

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

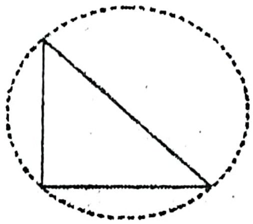
$$e^x = \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \ldots\ldots$$

$$e^{-1} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots$$

(c) Make the following equation in LaTeX u delimiters :

$$\begin{vmatrix} \hat{i} & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

(d) Write a code in LaTeX using PSTric draw the following :



4. Write a presentation containing in beamer the following content.

Slide-1 : Title of the presentation with au and date.

Slide-2: Fermat's Last Theorem. Let n>2 any interger, then the equation $x^n + y^n = z^n$ has no solutions in positive inte for any x, y and z .

Slide-3 : This result is called his last theor because it was the last of his cla in the margins to be either prove disproved. Andre Wiles found the accepted proof in 1995, some years later, Wiles proof exceptionally long and difficult !

Slide-4: Thank you

is question paper contains 6 printed pages.

Your Roll No. ....

No. of Ques. Paper : 6825 A HC

ique Paper Code : 42353503

me of Paper : Statistical Software R

me of Course : Mathematics : Skill Enhancement Course

mester : V

ration : 2 hours

ximum Marks : 38

Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions.

All commands should be written in software R.

Do any *four* of the following: 1x4

State whether the following statements are true or false:

(a) savehistory(file=' .Rhistory') is same as history( ) command.

(b) ls.str( ) is used to find the structure of all the defined objects.

(c) c(3 5 7 9) gives a vector.

(d) The commands mean( ) and colMeans( ) for a data frame give the same output.

(e) Pie chart cannot be formed of the data given in matrix form.

2. Do any *six* of the following. 1x6

Fill in the blanks:

(a) .... command to find the variance of data.

(var( )/ vara( ))

(b) .... command is used to make scatter plot.

(splot() / plot())

(c) $ command is used for ....

(copy a data, extract from a data).

(d) hist() command is used for .... (history, histogram).

(e) sample() command selects .... elements from data. (random, beginning).

(f) rep() command is used for repeat .... items.

(one, multiple).

(g) .... command to rearrange the items in a vector to be in a order. (sort, order)

3. Answer the following questions: 2x8=16

(a) (i) Write a command to list all the variables defined ending with 'm'.

(ii) Write "Jan", "Feb", "Mar", "Apr", "May" as a factor.

(b) (i) Can we use scan( ) command for the text Ajay, Anil, Raju, Ravi, Sanjay? Justify your answer.

(ii) What are the differences between save( ) and load( ) commands for files?

(c) Differentiate between seq(5) and seq_along(5) commands.

(d) Create a pie chart of any data with labels with one example.

(e) Rearrange the data in increasing order and draw a stem and leaf plot, where data is:

X = 3,5,7,5,3,2,6,8,5,6,9

(f) A data file is given with name bird.

	A	*B*	*C*	*D*	*E*
X	12	14	15	40	10
Y	08	04	07	09	11
Z	30	20	25	10	35

(i) Extract third columns.

(ii) Transpose bird data.

(iii) Find max and min items.

(iv) Make histogram of X.

(g) Make a score data file:

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Draw a stem leaf plot.

(h) Consider the data2 = 6,7,3,5,6,6,7,9,3,6. Write the sequence of first four positioned items of data2 and also write a sequence having the last four positioned items of data2.

Do any *four* of the following: 3×4

(a) Write commands for the following statements:

(i) Create a sequence of 25 numbers which are incremented by 1.

(ii) Create the binomial distribution of 25 numbers with probability 0.5.

(iii) Find the probability of getting 15 or less heads from a toss of a coin. (using binomial distribution)

(iv) How many heads will have a probability of 0.2 will come out when a coin is tossed 51 times.

(v) Find 8 random values from a sample of 150 with probability of 0.4. (using binomial distribution).

(b) Consider the following data frame object "x":

	C1	C2
R1	4	7
R2	3	4
R3	2	3
R4	4	4
R4	4	2
R5	6	3

Write commands for the following statements:

(i) Find the minimum and maximum value of the data frame x.

(ii) Find the column means and column sums of x.

(iii) Find the row mean of x.

(iv) Create a scatter chart of x.

(v) Create a line chart plot of vector C1.

(c) Make a dataframe file:

81	81	96
95	98	73
92	79	82
80	86	89
79	62	NA

Then convert this into a matrix

(d) Generate 50 random variables using normal distribution, negative-binomial distribution.

(e) Consider the following course grades of randomly selected students:

32	40	20	31
26	35	38	21
12	44	22	45
42	46	20	48
45	48	41	27

Write commands for:

P.T.O.

(i) Putting data into a variable x.

(ii) Creating a box plot of x

(iii) Creating a scatter plot of x

(iv) Creating a stem and leaf plot of x

(v) Creating a normal probability plot of x.

[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8119A** **HC**

Unique Paper Code : 62353505

Name of the Course : **B.A. (Prog.) Mathematics: SEC**

Name of the Paper : Statistical Software- R

Semester : V

Time : 2 Hours ***Maximum Marks : 38***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

(c) **All** commands should be written in software R.

1. Do any **five** of the following : 1×5=5

State whether the following statement are ***true or false*** :

(i) Command to evaluate sin (60^0) is

$$\sin\left(\frac{\pi}{60}*180\right)$$

P.T.O.

(ii) setwd () command is used to find the default path of the files saved

(iii) If a = 2 + 5 and b = e^5, then a + b = c gives 7 + e^5.

(iv) savehistory (file =' Rhisotry') is same as loadhistory (file =' Rhistory') command.

(v) If datag is a ten item vector then datag [-3] command show only third items.

(vi) The commands seq_along (dataname) and se(along = dataname) gives the same output

2. Do any **five** of the following. 1×5=5

Fill in the blanks.

(i) .... command is used to verify if a given object "X" is a matrix data object. (is. matrix (X)/class(X)).

(ii) In two digit number, ... digit represent the stem value in stem-and-leaf plot. (ones/tens) (iii)............... command is used to find the row sums of any data frame object "Z". (row Sums()/rowsums()).

(iv) hist() command is used for .... (history, histogram).

(v) For calling function, we use.............. bracket ((),[]).

(vi) To rearrange data, we use .... command (sort().order()).

3. (a) (i) Using scan command enter the following data : 2×8=16

Mon, Tue, Wed, Thus, Fri, Sat, Sun

(ii) Write command to read a csv file.

(b) Write a command for the following :

(i) To list all the elements starting with either 'n' or 'j'.

(ii) To remove all the variables containing 'I' as the last alphabet.

(c) Identify the errors in the command and correct them

Seq [from = 1, To = 10, by = 2]

(d) (i) What will be the class of the resulting vector if you concatenate a number and NA.

(ii) How will you convert a data frame into a table.

(e) Differentiate between seq(5) and seq along(5) commands.

(f) Consider a matrix X

	Q1	Q2	Q3	Q4
R1	Jan	Apr	Jul	Oct
R2	Feb	May	Aug	Nov
R3	Mar	Jun	Sep	Dec

(i) Write command to change the name of rows with a,b,c and name of columns with A,B,C,D respectively.

(ii)Print all items of 2nd columns.

(g) Rearrange the data in increasing order and draw a stem and leaf plot where data is :

X = 3,5,7,5,3,6,8,.5,4,5,9,7,4

(h) Make a score data file

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Draw a stem leaf plot

Do any **four** of the following :

3×4=12

(a) (i) How to make a comment in R ?

(ii) Create a vector

x : 12, 7, 3, 4.2, 18, -21, NA.

(iii) Find the mean and median of vector x.

(iv) Find mean of vector x by dropping NA values.

(v) Find the quantile of vector x.

(b) (i) Create data strings :

x : 3 7 9 5

labels : Landon New York Singapore Mumbai.

P.T.O.

[This question paper contains 2 printed pages.]

Sl. No. : **2625** *GC 3* ***Your Roll No..............***

Unique Paper Code : 32235908

Name of the Paper : Insect Vector and Diseases

Name of the Course : **Generic Elective for Hons courses - CBCS**

Semester : I

Duration : 03 Hours

Maximum Marks : 75 Marks

Instruction for Candidates

(Write your Roll No. on the top immediately on receipt of this question paper)
Attempt five questions in all.
Question No.01 is compulsory

1.(a) **Define the following:** 05
(i) VectorialCapacity
(ii) Epidemiology
(iii) OpisthognathousHead
(iv) HolometabolousInsects
(v) Haemotophagy

(b) **Match the following :** 05

(i) *Aedes*	(a) Chagas Disease
(ii) *Plasmodium falciparum*	(b) Plague
(iii) *Yersinia pestis*	(c) Brain Malaria
(iv) Rickettsia	(d) Chikungunya
(v) *Trypanosoma*	(e) Typhus Fever

(c) **Write the scientific name of the following:** 03
(i) Head louse
(ii) House Fly
(iii) Sand Fly

(d) **State the vector and pathogen for the following diseases :** 10
(i) Dengue
(ii) Relapsing Fever
(iii) Visceral Leishmaniasis
(iv) Trench Fever
(v) Encephalitis

P.T.O.

(e) **Fill in the blanks** 04

(i) Organ of Berlese is found in ....

(ii) Sand fly belongs to order....................

(iii) Aristate antennaeis found in.......................

(iv) Bed bugs are representatives of order.............................

2.(a) Describe the concept of vectors and host vector association. 06

(b) Define host specificity. List the diseases spread by flea and its control strategies. 06

3.(a) Illustrate the life cycle of Malarial parasite. 06

(b) Discuss various strategies applicable for Mosquito control. 06

4.(a) Summarise the key features of Diptera and Hemiptera. 06

(b) Describe the identifying features of sandfly,diseases spread and its control strategies. 06

5.(a) Describe the chronology in chagas disease. 06

(b) Mention bed bugs role as vectors including control and prevention measures. 06

6.(a) Write an essay on different kinds of Insect mouth parts. 06

(b) Identify key features of Siphonaptera and write about a vector from this order. 06

7. rite short notes on ANY THREE of the following : 4X3=12

(a) Filariasis

(b) Control of housefly

(c) Myiasis

(d) Triatome Bugs

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7336**

Unique Paper Code : **32355101** HC

Name of the Paper : **Calculus**

Name of the Course : **Generic Elective for Honours : Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *five* questions from each of the *three* Sections.

Each question is of *five* marks.

Section I

1. Use $\varepsilon - \delta$ definition to show that :

$$\lim_{x \to 4} (9 - x) = 5.$$

P.T.O.

2. Find the asymptotes for the curve :

$$f(x) = \frac{x^2 - 3}{2x - 4}.$$

3. Find the Linearization L(x) of $f(x)$ at $x = a$ where :

$$f(x) = x + \frac{1}{x} \text{ at } a = 1.$$

4. For $f(x) = (x - 2)^3 + 1$

(i) Find the intervals on which f is increasing and the intervals on which f is decreasing.

(ii) Find where the graph of f is concave up and where it is concave down.

5. Use L'Hôpital's rule to find :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right).$$

6. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

7. Find the length of the curve :

$$x = t^3, y = \frac{3t^2}{2}, 0 \le t \le \sqrt{3}.$$

Section II

8. State Limit comparison test. Using the limit comparison test, show that :

$$\int_1^\infty \frac{3dx}{e^x + 5} \text{ converges.}$$

9. Identify the symmetries of the curve and then sketch the graph of :

$$r^2 = \cos\theta.$$

16. Find the derivative of the function f at p_0 in the direction of $\vec{A}$ where $f(x, y, z) = 3e^x \cos yz$, $p_0(0, 0, 0)$, $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$.

17. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces : $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$, $x^2 + y^2 + z^2 = 11$

Point : $(1, 1, 3)$.

18. Find equations for the :

(a) Tangent plane and

(b) Normal line at the point p_0 on the given surface :

$f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $p_0(1, 2, 4)$.

19. Find the absolute maximum and minimum value of :

$$f(x, y) = 2 + 2x + 2y + x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

20. If

$$f(x, y) = x^2 - y^2, \quad g(x, y) = 3xy + y^2x,$$

show that :

(*i*) $\nabla(fg) = f\nabla g + g\nabla f$

(*ii*) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$.

21. If $w = x \sin y + y \sin x + xy$, show that $w_{xy} = w_{yx}$.

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7467** **HC**

Unique Paper Code : 32355301

Name of the Course : **Generic Elective for Honours : Mathematics**

Name of the Paper : Differential Equations

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Solve the differential equation by finding an integrating factor:

$$(e^{(x+y)}+ye^{y})dx+(xe^{y}-1)dy=0$$

6.5

(b) Solve the differential equation $y'=5.7y-6.5y^2$. 6.5

(c) Find the orthogonal trajectories of $x=c\sqrt{y}$.

P.T.O.

2. (a) Solve $((3x^2+2x+\sin(x+y))dx+\sin(x+y)dy=0$. 6

(b) Show that x^2 and x^{-2} form a basis of the following differential equation $x^2y''+xy'-4y=0$. Also find the solution that satisfies the conditions $y(1)=11$, $y'(1)=-6$. 6

(c) Find the radius of convergence of the series $\sum_{m=0}^{\infty}\frac{(-1)^m x^{3m}}{8^m}$ 6

3. (a) Find the general solution of the following differential equation using method of variation of parameters $y''+9y=\sec 3x$. 6.5

(b) Use the method of undetermined coefficients to find the solution of the differential equation: $y''+3y'+2.25y=-10\,e^{-1.5x}$, $y(0)=1, y'(0)=0$. 6.5

(c) Find a homogenous linear ordinary differential equation for which two functions x^{-3} and $x^{-3}\ln x$ $(x>0)$ are solutions. Also show the linear independence by considering their Wronskian. 6.5

4. (a) Find the general solution of the linear partial differential equation

$x(y^2-z^2)u_x+y(z^2-x^2)u_y+z(x^2-y^2)u_z=0.$ 6

(b) Find the general solution of the differential equation: $(x^2D^2+6xD+6I)y=0.$

Where $D=\frac{d}{dx}$ 6

(c) Find the particular solution of the linear system that satisfies the stated initial conditions:

$\frac{dy_1}{dt}=y_1+y_2, \quad y_1(0)=1$

$\frac{dy_2}{dt}=4y_1+y_2, \quad y_2(0)=6.$ 6

5. (a) Find the power series solution of the following differential equation, in powers of x

$y''-y'=0.$ 6.5

(b) Find the solution of the Cauchy problem: $xu_x+yu_y=xe^{-u}$, with the u=0 when $y=x^2$. 6.5

P.T.O.

(c) Reduce the equation: $u_x + xu_y = y$ to canonical form, and obtain the general solution. 6.5

6. (a) Solve the initial-value problem:

$au_x + bu_y = 0$, $u(x,0) = \alpha e^{\beta x}$ by the font is different. 6

(b) Reduce the: $u_{tt} - c^2 u_{xx} = 0, c \neq 0$ where c is a constant, into canonical form and hence find the general solution. 6

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx} + 2u_{xy} + u_{yy} = 0$$

into canonical form and hence find the general solution. 6

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8161A** **HC**

Unique Paper Code : 62355503

Name of the Course : **Mathematics : Generic Elective**

Name of the Paper : General Mathematics-I

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions as per directed question wise.

Section - I

1. Write a short note on the life and contributions of any **three** of the following mathematicians :

(a) Galois,

(b) Riemann,

(c) Weierstrass,

(d) Abel,

(e) Laplace

P.T.O.

Section - II

2. Attempt any **six** questions. Each question carries **five** marks.

(a) Define Perfect numbers and Amicable numbers. State the properties of Perfect numbers.

(b) Define the magic square and state properties of Benjamin Franklin's magic square.

(c) Define the Inversion and explain the Fifteen Puzzle.

(d) Find the remainder when

$12345 \times 123456 \times 1234567$ is divided by 11.

(e) Explain continued fraction and express $\frac{221}{41}$ as continued frction.

(f) Define unit fraction and express $\frac{2}{7}$ and $\frac{98}{100}$ as unit fraction .

(g) (i) In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together ?

(ii) Use the Egyptian method of duplation to find 58×93.

Section - III

3. Do any **three** questions. Each question carries **six** marks.

(a) If $A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ 3 & 5 \end{pmatrix}$, find a and b such that $AB = BA$.

(b) If $A = \begin{pmatrix} 6 & 2 & -1 \\ 4 & 3 & 1 \\ 1 & -2 & 0 \end{pmatrix}$, then calculate A^3 ?

(c) Express the matrix $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$ as sum of skew-symmetric and symmetric matrix.

(d) Find the inverse of the matrix $\begin{pmatrix} -4 & 7 & 6 \\ 5 & -5 & -4 \\ -2 & 4 & 3 \end{pmatrix}$, if it exist.

P.T.O.

4. Do any **two** questions. Each question is of **six** marks.

(a) $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 6 & 1 \\ 3 & 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 2 & -1 \\ 6 & 4 & 6 \end{pmatrix}$, then is AB = BA ? Verify.

(b) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -4 \\ 1 & 2 \end{pmatrix}$, then is determinant (AB) = determinant (BA) Verify.

(c) Use Cramer's rule to solve the system :

$5x_1 - 3x_2 - 10x_3 = -9$

$2x_1 + 2x_2 - 3x_3 = 4$

$-3x_1 - x_2 + 5x_3 -1$