This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : 88

Unique Paper Code : 32351101 I

Name of the Paper : Calculus

Name of the Course : B.Sc. (H) Mathematics

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

(Attempt any *four* questions from Section I)

1. If $y = \log(x + \sqrt{x^2 + 1})$, show that :

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0.$$

2. Sketch the graph of the function

$$f(x) = \frac{3x - 5}{x - 2}$$

by determining all critical points, intervals of increase and decrease, points of relative maxima and minima, concavity of the graph, inflection points and horizontal and vertical asymptotes.

P.T.O.

3. Evaluate : $\lim_{x \to 0} (e^x - 1 - x)^x$.

4. Given the cost $C(x) = \frac{1}{8}x^2 + 5x + 98$ of producing x units a particular commodity and the selling price $p(x) = \frac{1}{2}(75 -$ when x units are produced. Determine the level of producti that maximizes profit.

5. Sketch the graph of $r = \sin 2\theta$ in polar coordinates.

Section II

(Attempt any *four* questions from Section II)

6. Obtain the reduction formula for

$$\int \sec^n x \, dx.$$

Use it to evaluate $\int \sec^6 x \, dx$.

7. Find the volume of the solid generated by revolving the regi enclosed by $y = x$; $y = 2 - x^2$ and $x = 0$ is revolved about t x-axis.

8. Use cylindrical shells method to find the volume of the so generated when the region enclosed by $y = 2x - x^2$ a $y = 0$ is resolved about y-axis.

9. Show that the arc length of the curve $y = \cosh x$ betwe $x = 0$ and $x = \log 2$ is 3/4.

10. Find the area of the surface generated by revolving the cu $y = \sqrt{9 - x^2}$, $-1 \le x \le 1$, about x-axis.

Section III

(Attempt any *three* questions from Section III)

11. Find the equation of parabola having axis $y = 0$ and passing through the points (3, 2) and (2, –3).

12. Find the equation of ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.

13. Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

14. Rotate the coordinate axes to remove the xy-term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Identify the resultant conic.

Section IV

(Attempt any *four* questions from Section IV)

15. Given the vector functions

$$\vec{\mathbf{F}}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and

$$\vec{\mathbf{G}}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t\to 1}[\vec{\mathbf{F}}(t)\times\vec{\mathbf{G}}(t)] = [\lim_{t\to 1}\vec{\mathbf{F}}(t)] \times [\lim_{t\to 1}\vec{\mathbf{G}}(t)].$$

Section III

(Attempt any *three* questions from Section III)

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12. Find the equation of ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.

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and

$$\vec{\mathbf{G}}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t\to 1}[\vec{\mathbf{F}}(t)\times\vec{\mathbf{G}}(t)] = [\lim_{t\to 1}\vec{\mathbf{F}}(t)] \times [\lim_{t\to 1}\vec{\mathbf{G}}(t)].$$

16. A velocity of particle moving in space is

$$\vec{V}(t) = t^2\hat{i} - e^{2t}\hat{j} + \sqrt{t}\hat{k}.$$

Find the particle's position as a function of t if the position at time $t = 0$ is $\vec{R}(0) = \hat{i} + 4\hat{j} - \hat{k}$.

17. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from ground level :

(*i*) Find the maximum height attained by the shell.

(*ii*) Find the time of flight and the range of the shell.

18. Find the tangential and normal components of the acceleration of an object that moves with position vector

$$\vec{R}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}.$$

19. Find the curvature $\kappa(t)$ for the curve given by the vector equation

$$\vec{R}(t) = 4\cos t\hat{i} + 4\sin t\hat{j} + t\hat{k} \ (0 \le t \le 2\pi).$$

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This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **89**

Unique Paper Code : **32351102** **I**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Find polar representation of the complex number : 6

$$z = \sin a + i(1 + \cos a),\ a \in [0, 2\pi).$$

(*b*) Find $|z|$ and $\arg z$, $\arg(-z)$ for : 6

(*i*) $z = (1 - i)(6 + 6i)$

(*ii*) $z = (7 - 7\sqrt{3}i)(-1 - i)$.

(*c*) Solve the equation : 6

$$z^4 = 5(z - 1)(z^2 - z + 1).$$

2. (*a*) For $a, b \in \mathbf{Z}$, define $a \sim b$ iff $a^2 - b^2$ is divisible by 3 : 6

(*i*) Prove that $\sim$ is an equivalence relation on Z.

(*ii*) Find the equivalence classes of 0 and 1.

(*b*) Define : 6

$$f : \mathbf{Z} \to \mathbf{Z} \text{ by } f(x) = x^2 - 5x + 5$$

(*i*) Is f one-to-one ?

(*ii*) Is f onto ?

Justify each answer.

(c) Show that the open intervals (0, 1) and (4, 6) have the same cardinality. 6

3. (a) Suppose a, b and c are three non-zero integers with a and c relatively prime. Show that : 6

$$\gcd(a, bc) = \gcd(a, b).$$

(b) (i) Solve the following congruence if possible. If no solution exists, explain why not :

$$4x \equiv 2 \pmod{6}.$$

(ii) Find three positive and three negative integers in $\overline{5}$ w.r.t. congruence mod 7. 6

(c) Use mathematical induction to establish the following inequality : 6

$$n! > n^3, \text{ for all } n \geq 6.$$

4. (a) Find the general solution to the following linear system : 6½

$$\begin{aligned} 3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15. \end{aligned}$$

(b) Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$.

Is u in the subspace of $\mathbf{R}^3$ spanned by the columns of A. Why or why not ? 6½ (*c*) Let :

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

(*i*) For what values of h is v_3 in span $\{v_1, v_2\}$?

(*ii*) For what values of h is $\{v_1, v_2, v_3\}$ linearly dependent ? Justify each answer. 6½

5\. (*a*) Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$, and define by $T : R^3 \to R^3$ by $T(x) = Ax$. Find all x in R^3 such that $T(x) = 0$. Does $b = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ belong to range of T ? 6½

(*b*) A linear transformation $T : R^2 \to R^2$ first reflects points through the x_1-axis and then reflects points through the x_2-axis. Show that T can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation ? 6½

(*c*) Let

$$A = \begin{bmatrix} 2 & -3 & -4 \\ -8 & 8 & 6 \\ 6 & -7 & -7 \end{bmatrix} \text{ and } u = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}.$$

Is u in Nul A ? Is u in Col A ? Justify each answer. 6½

6. (*a*) Given $b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and $B = \{b_1, b_2\}$ is basis of subspace H of R^2.

(*i*) Determine if $x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ belongs to H.

(*ii*) Find $[x]_B$, the B-coordinate vector of x. 6½

(*b*) Determine the basis of the null space of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix}.$$ 6½

(*c*) Is $\lambda = -2$ an eigenvalue of $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

If so, find one corresponding eigenvector. 6½

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 90

Unique Paper Code : 32351301 I

Name of the Paper : Theory of Real Functions

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts from each question.

All questions are compulsory.

(a) Let $A \subseteq R$, $f : A \rightarrow R$ and $c \in R$ be a cluster point of A. Then prove that f can have only one limit at c. 5

(b) Use the $\in$-δ definition of the limit to prove that $\lim_{x \to c} x^3 = c^3$ for any $c \in R$. 5

(c) State divergence criterion for limit of a function. Show that $\lim_{x \to 0} (x + \text{sgn}(x))$ does not exist. 5

P.T.O.

(*d*) Prove that :

(*i*) $\lim_{x \to 0+} \frac{1}{x} = \infty$

(*ii*) $\lim_{x \to \infty} \frac{1}{x^2} = 0.$

2. (*a*) Let $A \subseteq R$, $f, g, h : A \to R$ and $c \in R$ be a clust point of A. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A$, $x \neq$ and if $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$, then prove th $\lim_{x \to c} g(x) = L.$

(*b*) State and prove sequential criterion for continuity of real valued function.

(*c*) Let the function $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x & : \text{if } x \text{ is rational} \\ x+3 & : \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous.

(*d*) Let $x \to [x]$ denote the greatest integer function. Determi the points of continuity of the function $f(x) = x - [$ $x \in R$.

3.

3. (*a*) Let f be a continuous real valued function defined on $[a, b]$. By assuming that f is a bounded function show that f attains its bounds on $[a, b]$. 5

(*b*) State Bolzano's Intermediate value theorem and show that the function $f(x) = xe^x - 2$ has a root c in the interval $[0, 1]$. 5

(c) Let $f : R \rightarrow R$ is continuous on R and suppose that $f(r) = 0$ for every rational numbers r. Show that $f(x) = 0$ for all $x \in R$. 5

(*d*) Define uniform continuity of a function. Prove that if a function is continuous on a closed and bounded interval I, then it is uniformly continuous on I. 5

4. (*a*) Show that the function $f(x) = 1/x^2$ is uniformly continuous on $A = [0, \infty[$ but it is not uniformly continuous on $B =]0, \infty[$. 5

(*b*) Determine where the following function $f : R \rightarrow R$ is differentiable, $f(x) = |x - 1| + |x + 1|$. 5

(c) Let f be defined on an interval I containing the point
Then prove that f is differentiable at c if and only if ther
exists a function ϕ on I that is continuous at c and satisfies
$f(x) - f(c) = \phi(c)\,(x - c)$ for all $x \in$ I. In this case, w
have $\phi(c) = f\,'(c)$. Using the above result find the functio
ϕ for $f(x) = x^3$, $x \in$ R.

(d) State and prove Mean Value Theorem.

5\. (a) State Darboux's theorem. Suppose that $f : [0, 2] \to$ R i
continuous on [0, 2] and differentiable on]0, 2[and tha
$f(0) = 0$, $f(1) = 1$, $f(2) = 1$. (i) Show that there exist
$c_1 \in (0, 1)$ such that $f'(c_1) = 1$. (ii) Show that there exist
$c_2 \in (1, 2)$ such that $f'(c_2) = 0$. (iii) Show that there exist
$c \in (0, 2)$ such that $f'(c) = 1/10$.

(b) Let $f : \text{I} \to$ R be differentiable on the interval I. Then
prove that f is increasing on I if and only if $f'(x) \geq 0$ fo
all $x \in$ I.

(c) State Taylor's theorem. Use it to prove tha
$1 - x^2/2 \leq \cos x$ for all $x \in$ R.

(d) Find the Taylor series for e^x and state why it converge
to e^x for all $x \in$ R.

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: 91	I
Unique Paper Code	: 32351302	
Name of the Paper	: Group Theory - I	
Name of the Course	: B.Sc. (Hons.) Mathematics	
Semester	: III	
Duration	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **two** parts from each question. All questions are compulsory.*

1. (a) Define a group. Give an example of:
   (i) an abelian group consisting of eight elements,
   (ii) a non-abelian group consising of six elements,
   (iii) an infinite abelian group, and
   (iv) an infinite non-abelian group.

(b) Show that the set {5, 15, 25, 35} is a group under multiplication modulo 40. What is the identity element of this group? Find the inverse of each element.

(c) Prove that the intersection of an arbitrary family of subgroups of a group G is again a subgroup of G. What can you say about the union of two subgroups? Justify your answer.

$2 \times 6 = 12$

P. T. O.

2. (a) (i) Prove that in (Z, +), the group of integers unde addition, every non-zero element is of infinit order.

(ii) Let G be a group and $a \in G$. If $|a| = n$ and k is positive divisor of n, then prove that $|a^{n/k}| = k$.

(b) Prove that the order of a cyclic group is equal to th order of its generator.

(c) Define a cyclic group. If $G = (a)$ is a finite cycli group of order n, then prove that the order of an subgroup of G is a divisor of n, and for each positive divisor k of n, G has exactly one subgroup of order k namely, $(a^{n/k})$.

2×6.5=13.

3. (a) Prove that if the identity permutation $\varepsilon = \beta_1 \cdots \beta_r$ where the β's are 2-cycles then r is even.

(b) Show that for $n \geq 3$, $Z(S_n) = \{I\}$.

(c) Prove that:

(i) a group of prime order has no proper, non-trivial subgroup. State its converse. Is it true?

(ii) a group of prime order is cyclic and any non-identity element can be taken as its generator.

2×6=12

4. (a) Let G be a finite group of permutations of a set S. Then prove that for any i from S:

$$|G| = |orb_G(i)|\,|stab_G(i)|.$$

(b) (i) Prove that the center Z(G) of a group G is a subgroup of G and is normal in G.

(ii) If H is a subgroup of G such that H is contained in the center Z(G), then prove that H is a normal subgroup of G. Is the converse true? Justify your answer.

(c) Let N be a normal subgroup of a group G and let H be a subgroup of G. If N is a subgroup of H, prove that H/N is a normal subgroup of G/N if and only if H is a normal subgroup of G. 2×6.5=13

5. (a) Let **C** be the complex numbers and:

$$\mathbf{M} = \left\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in R\right\}.$$

Prove that **C** and **M** are isomorphic under addition and $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$ and $\mathbf{M}^* = \mathbf{M} \setminus \{0\}$ are isomorphic under multiplication.

(b) Prove that an infinite cyclic group is isomorphic to (Z, +). Hence show that every subgroup of an infinite cyclic group is isomorphic to the group itself.

(c) Let G be a group of permutations. For each σ in G, define

$$\text{sgn}(\sigma) = \begin{cases} 1, & \text{if } \sigma \text{ is an even permutation,} \\ -1, & \text{if } \sigma \text{ is an odd permutation.} \end{cases}$$

Prove that sgn is a homomorphism from G to $\{1, -1\}$. What is the kernel? 2×6 =12

6. (a) Let ϕ be a homomorphism from a group G to a group $\tilde{G}$. Let g be an element of G. Then:

(i) $\phi(g^n) = \phi(g)^n$ for all $n \in \mathbf{Z}$.

(ii) ϕ is one-one if and only if $\ker(\phi) = \{e\}$, where e is the identity of G.

(b) State and prove the First Isomorphism Theorem.

(c) (i) Suppose ϕ is a homomorphism from $U(30)$ to $U(30)$ and $\text{Ker}(\phi) = \{1, 11\}$.

If $\phi(7) = 7$, find all elements of $U(30)$ that map to 7.

(ii) Let G be a group. Prove that the mapping $\phi(g) = g^{-1}$, for all $g \in G$, is an isomorphism from G onto G if and only if G is Abelian. $2\times6.5=13$

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Roll No. | | | | | | | | | | | |

. No. of Question Paper : **92**

Inique Paper Code : **32351303** **I**

lame of the Paper : **C-7 Multivariate Calculus**

lame of the Course : **B.Sc. (Hons.) Mathematics**

emester : **III**

Duration : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Section I

Attempt any *six* questions from this section.

1. Let f be the function defined by $f(x, y) = \dfrac{x^2 + 2y^2}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

   (*a*) Find $\displaystyle\lim_{(x, y) \to (2, 1)} f(x, y)$.

   (*b*) Prove that f has no limit at $(0, 0)$.

P.T.O.

2. The temperature at the point (x, y) on a given metal pl

in the xy-plane is determined according to the form

$T(x, y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at wh

the temperature changes with distance if we start at (2,

and move :

(a) parallel to the vector **j**. 5

(b) parallel to the vector **i**. (

3. The Company sells two brands X and Y of a commerc

soap, in thousand-pound units. If x units of brand X a

y units of brand Y are sold, the unit price for brand X

$p(x) = 4{,}000 - 500x$ and for brand Y is $q(y) = 3{,}000 - 450$

(a) Find the total revenue R in terms of p and q. 8

(b) Suppose the brand X sells for \$ 500 per unit and bra

Y sells for \$ 750 per unit. Estimate the change in to

revenue if the unit prices are increased by \$ 20 for bra

X and \$ 18 for brand Y.

f

$$w = f\left(\frac{r-s}{s}\right),$$

show that

$$r\frac{\partial w}{\partial r} + s\frac{\partial w}{\partial s} = 0.$$

Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at P(1, –1) in the direction toward Q(2, 3).

Find the absolute extrema of $f(x, y) = 2 \sin x + 5 \cos y$ in the rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5).

Let $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = \|\mathbf{R}\|$, evaluate $\text{div}\left(\frac{1}{r^3}\mathbf{R}\right)$.

Section II

Attempt any *five* questions from this section.

By using iterated integral, compute

$$\iint_R x\sqrt{1-x^2}\, e^{3y}\, dA,$$

where R is the rectangle $0 \le x \le 1$, $0 \le y \le 2$.

9. Evaluate the double integral :

$$\iint_D \frac{dA}{y^2+1},$$

where D is the triangular region bounded by $y = -x$ an 14.

$y = 2.$

10. Evaluate the double integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$$

by converting to polar co-ordinates.

11. Find the volume of the tetrahedron T bounded by the pla

$2x + y + 3z = 6$ and the co-ordinates plane $x = 0$, $y = 0$ ar

$z = 0.$

12. Find the volume of the solid D bounded by the parabolo

$z = 1 - 4\left(x^2 + y^2\right)$ and the xy-plane.

13. Evaluate

$$\iint_D (x+y)^5 (x-y)^2\, dy\, dx$$

by using change of variable $u = x + y$ and $v = x -$

where D is the region in the xy-plane which is bounded

the co-ordinate axes and the line $x + y = 1.$

Section III

Attempt any *four* questions from this section.

14. Evaluate the line integral

$$\int_C \mathbf{F} \, . \, d\mathbf{R},$$

where

$$\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2}} i - \frac{y}{\sqrt{x^2 + y^2}} j$$

and C is the quarter circle path $x^2 + y^2 = a^2$, traversed from $(a, 0)$ to $(0, a)$.

15. Show that the vector field

$$\mathbf{F}(x, y, z) = \langle \sin z, -z \sin y, x \cos z + \cos y \rangle$$

is conservative and evaluate

$$\int_C \mathbf{F} \, . \, d\mathbf{R}$$

for any piecewise smooth path joining A(1, 0, −1) to B(0, −1, 1).

16. Use Green's theorem, to find the work done by the force field

$$F(x, y) = (3y - 4x)\boldsymbol{i} + (4x - y)\boldsymbol{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

17. Use Stokes' theorem, to evaluate the line integral

$$\oint_C (3y\, dx + 2z\, dy - 5x\, dz)$$

where C is the intersection of the xy-plane and the hemisphere

$$z = \sqrt{1 - x^2 - y^2},$$

traversed counterclockwise as viewed from above.

18. Evaluate

$$\iint_S (\mathbf{F} \cdot \mathbf{N})\, dS,$$

where $F = x^2\boldsymbol{i} + xy\boldsymbol{j} + x^3y^3\boldsymbol{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes, with outward unit normal vector **N**.

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: 93	I
Unique Paper Code	: 32351501	
Name of the Paper	: **Metric Spaces**	
Name of the Course	: **B.Sc. (Hons.) Mathematics**	
Semester	: V	
Duration	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.
All questions are compulsory

1. (*a*) (i) Let $X = \mathbf{R} \cup \{\infty\} \cup \{-\infty\}$. Define the metric d on X by :

$$d(x, y) = |\tan^{-1} x - \tan^{-1} y|, x, y \in X,$$

where $\tan^{-1}(\infty) = \pi/2$ and $\tan^{-1}(-\infty) = -\pi/2$. Show that (X, d) is a metric space.

(ii) Let X denote the set of all Riemann integrable functions on $[a, b]$. For f, g in X, define:

$$d(f, g) = \int_a^b |f(x) - g(x)| \, dx.$$

Show that d is not a metric on X. 3+3=6

(*b*) Prove that a sequence in $\mathbf{R}^n$ is Cauchy in the Euclidean metric d_2 if and only if it is Cauchy in the maximum metric d_∞. 6

P. T. O.

(c) (i) Show that the metric space (X, d) of ratio numbers is an incomplete metric space.

(ii) Let X be any nonempty set and d be the discr metric defined on X. Prove that the metric spa (X, d) is a complete metric space. 3+3=6

2. (a) Let (X, d) be a metric space. Prove that the int section of any finite family of open sets in X is an op set in X. Is it true for the intersection of an arbitra family of open sets? Justify your answer.

(b) Prove that if A is a subset of the metric space (X, then $d(A) = d(\overline{A})$.

(c) Let F be a subset of a metric space (X, d). Prove tl the following are equivalent:

(i) $x \in \overline{F}$

(ii) $S(x, \epsilon) \cap F \neq \emptyset$ for every open ball $S(x, \epsilon)$ center at x;

(iii) There exists an infinite sequence $\{x_n\}$, $n \geq 1$ points (not necessarily distinct) of F su that $x_n \to x$.

3. (a) Let (X, d) be a metric space and $Z \subseteq Y \subseteq X$. If cl_X and $cl_Y(Z)$ denote, respectively, the closures of Z in metric spaces X and Y, then show that:

$$cl_Y(Z) = Y \cap cl_X(Z).$$

(b) (i) Let Y be a nonempty subset of a metric space (X, d_X), and (Y, d_Y) is complete. Show that Y is closed in X.

(ii) Is the converse of part (i) true? Justify your answer.

4+2=6

(c) Let d_p (p ≥ 1) on the set $\mathbf{R}^n$ be given by:

$$d_p(x, Iy) = \left(\sum_{j=1}^{n}|x_j - y_j|^p\right)^{1/p},$$

for all $x=(x_1, x_2, \ldots, x_n)$, $y=(y_1, y_2, \ldots, y_n)$ in $\mathbf{R}^n$. Show that ($\mathbf{R}^n$, d_p) is a separable metric space. 6

4. (a) Prove that a mapping f: (X, d_X) → (Y, d_Y) is continuous on X if and only if f^{-1}(F) is closed in X for all closed subsets F of Y.

6 ½

(b) (i) Define an isometry between the metric spaces (X, d_X) and (Y, d_Y), and show that it is a homeomorphism.

(ii) Is the completeness of a metric space preserved under homeomorphism? Justify your answer.

4+2½=6½

(c) State and prove the Contraction Mapping Principle.

1½+5=6½

5. (a) Let f be a mapping of (X, d_X) into (Y, d_Y). Prove that f is continuous on X if and only if for every subset F of Y:

$$f^{-1}(F^o) \subseteq (f^{-1}(F))^o$$ 6½

P. T. O.

(b) Prove that the metrics d_1, d_2 and d_∞ defined on by:

$$d_1(x, y) = \sum_{j=1}^{n} |x_j - y_j|;$$

$$d_2(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^2\right)^{1/2}; \text{ and}$$

$$d_\infty(x, y) = \max\{|x_j - y_j| : j = 1,2,\ldots,n\}$$

for $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n) \in \mathbf{R}^n$ are equivalent. 6

(c) Prove that a metric space (X, d) is disconnected if and only if there exists a continuous mapping of (X, d) onto the discrete two element space (X_0, d_0). 6½

6. (a) If every two points in a metric space X are contained in some connected subset of X, prove that X is connected. 6½

(b) Let (X, d) be a metric space and Y a subset of X. Prove that if Y is compact subset of (X, d), then Y is bounded. Is the converse true? Justify your answer. 6½

(c) If f is a one-to-one continuous mapping of a compact metric space (X, d_X) onto a metric space (Y, d_Y), then prove that f is a homeomorphism. 6½ question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | |

o. of Question Paper : **94**

que Paper Code : **32351502** **I**

ne of the Paper : **Group Theory-II**

ne of the Course : **B.Sc. (H) Mathematics**

nester : **V**

ration : **3 Hours** Maximum Marks : **75**

ite your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Question No. **1** has been divided in 10 parts

and each part is of **1½** marks.

Each question from **2** to **6** has 3 parts and each part is of

6 marks. Attempt any *two* parts from each question.

State true (T) or false (F). Justify your answer in brief :

(*a*) $\mathbf{Z}_2 \oplus \mathbf{Z}_3$ is isomorphic to $\mathbf{Z}_6$ where $\mathbf{Z}_n$ is used for group $\{0, 1, 2, \ldots\ldots\ldots\ldots, n-1\}$ under addition modulo n.

(*b*) The largest possible order of any element of external direct product $\mathbf{Z}_3 \oplus \mathbf{Z}_6 \oplus \mathbf{Z}_2$ is 36.

P.T.O.

(*c*) If H, K and L are normal subgroups of a group G. Th
G is internal direct product of H, K and L if G = HKL a
$H \cap K \cap L = \{e\}$ where e is identity of G.

(*d*) The order of the group of inner automorphisms
additive group of integers is greater than 1.

(*e*) The dihedral group D_8 of order 8 is a subgroup of t
symmetric group S_4.

(*f*) For any two groups G_1 and G_2, $G_1 \oplus G_2$ is isomorphic
$G_2 \oplus G_1$.

(*g*) Let G be a non-abelian group. A map $G \times G \rightarrow G$ is giv
by $(g, a) \mapsto g.\, a = ag$ for all g and a in G. This is an acti
of G on itself.

(*h*) Every subgroup H of a group G of index 2 is norm
in G.

(*i*) If order of a group G is greater than 1, then the conjuga
action of G on itself is transitive.

(*j*) In S_3 the all conjugacy classes are {(1 2), (1 3), (2 3)} a
{(1 2 3), (1 3 2)}.

(*a*) Prove that for any positive integer n, $Aut(\mathbf{Z}_n)$ is isomorphic to $U(n)$, where $\mathbf{Z}_n$ is the group {0, 1, 2 ......, $n - 1$} under addition modulo n and $U(n)$ the group of units under multiplication modulo n and $Aut(\mathbf{Z}_n)$ denotes the group of automorphisms of $\mathbf{Z}_n$.

(*b*) Define the commutator subgroup G′ of a group G. Prove that G/G′ is abelian and if G/N is abelian then G′ is subgroup of N.

(*c*) Prove that the order of an element of a direct product of fnite number of finite groups is the least common multiple of the orders of the components of the element.

(*a*) Prove that if a group G is the internal direct product of a fnite number of subgroups H_1, H_2, ..., H_n, then G is isomorphic to the external direct product of H_1, H_2, ..., H_n.

(*b*) Find all subgroups of order 4 in $\mathbf{Z}_4 \oplus \mathbf{Z}_4$.

(*c*) Let G = {1, 7, 17, 23, 49, 55, 65, 71} be the group under multiplication modulo 96. Express G as an internal direct product of cyclic groups.

94

4. (*a*) Let G be an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G.

(*b*) (*i*) Let G be a group acting on a non-empty set A. Define kernel of action of G on A and explain when this action will be called faithful.

(*ii*) Consider the action of the dihedral group D_8 of order 8 on the set A = {{1, 3}, {2, 4}} of the unordered pair of opposite vertices of a square. Show that this action is not faithful. Further, show that for either $a \in A$ (a = {1, 3} or {2, 4}), the stabilizer of a in D_8 equals the kernel of the action.

(*c*) Let G be a group and A be any subset of G. Define centralizer $C_G(A)$ and normalizer $N_G(A)$ of A in G. Further, for the symmetric group S_3 and a subgroup A = {I, (1, 2)} of S_3, find centralizer and normalizer of A in S_3 where I denotes identity of S_3.

(a) Let G be a group, H be a subgroup of G and let G act by left multiplication on the set A of left cosets of H in G. Let π_H be the associated permutation representation afforded by this action. Then, show that the following hold :

(i) G acts transitively on A.

(ii) The stabilizer in G of $1H \in A$ is a subgroup of H where 1 is identity of G.

(iii) Kernel of π_H is equal to $\bigcap_{x \in G} xHx^{-1}$ and the kernel of π_H is the largest normal subgroup of G contained in H.

(b) Let G be a group acting on a non-empty set A given by $g.a$ for all $g \in G$ and for all $a \in A$. If $a, b \in A$ and $b = g.a$, for $g \in G$, then show that $G_b = gG_ag^{-1}$. Deduce that, if G acts transitively on A, then kernel of the action is $\bigcap_{g \in G} gG_ag^{-1}$ where G_x denotes stabilizer of x in G.

(c) (i) State the class equation for a finite group G. Find all conjugacy classes and their sizes in the alternating group A_4.

(ii) Let G be a group of order p^2 for some prime p. Show that it is isomorphic to either $\mathbf{Z}_{p^2}$ or $\mathbf{Z}_p \times \mathbf{Z}_p$.

6. (*a*) Show that for any positive integer n greater than or eq to 5, the alternating group A_n of degree n does not h a proper subgroup of index less than n.

(*b*) Prove that if order of a group G is 105, then it has nor Sylow 5-subgroup and normal Sylow 7-subgroup.

(*c*) State and prove the Index theorem. Hence or otherw show that there is no simple group of order 216.

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[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **136** **I**

Unique Paper Code : 42341102

Name of the Course : **B.Sc.(Prog.) / B.Sc. Mathematics Sciences**

Name of the Paper : Problem Solving Using Computers

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **Section - A** is compulsory.

(c) Answer any **five** questions from **Section- B.**

(d) Answer all parts of a question together.

Section - A

1. (a) Write full form of RAM, EPROM. 2

(b) Draw a block diagram to illustrate the basic organization of a computer system. 2

P.T.O.

(c) Write at least two characteristics , each Second , Third and Forth generati computers.

(d) Draw a flowchart to find the maximum of three numbers.

(e) If X = 32 and Y = 16, find the value of X a Y after the following operations :

(i) X << 2

(ii) X ^ Y

(iii) X & Y

(f) Evaluate the following expression :

7% 10 + 15 – 25 * 8//5

(g) Write a function to search an element i list using Binary search.

(h) Identify the syntax errors in the follow code :

```
if (X =Y):
    print(" Equal")
else if(x<y):
    print("Smaller")
else;
    print("Larger")
```

(i) When does need for exception handling routine arise ? 2

(j) What will be the contents of D, if the following statement is executed ?

D = [x+y for x in range (1, 3) for y in range (1, 4)] 2

Section - B

. (a) Find the output of the following program codes : 2

```
total =0
for i in range (10,0, -1);
        total + = i
 print total
```

(b) 2

```
a = b= 40
x=y=50
if a<100;
 if b>50:
      x+ = 1
else ;
  y+ = 1
print x
print y
```


P.T.O.

(c)
```
List1 = List( )
List2 = List( )
for i in range ( 0, 10):
    if (i % 2 ==0):
        List1 . append (i)
    else:
        List2 . append (i)
print List1, List2
```

(d) Given a string S, what will be output executing the following statements ?

S = "University of Delhi"

(i) S [: 10] + S [10 :]

(ii) S [0 : len (S)]

(iii) S [–5 :]

(iv) S [0 : 10 : 2]

3. (a) Write a function that takes an intege parameter n and prints the following patter with n number of lines using for loop. F example if n = 4 then the following patter should be printed :

1

12

123

1234

4

(b) Given the following lists : 4

List1 = [1, 2, 3, 4, 5]

List2 = ['A', 'E', 'I']

What will be the output if the following statements are executed :

(i) List2.extend(['O', '10'])

print List2

(ii) List2.append(['O', '10'])

print List2

(iii) List[2]==2

(iv) List3 = List1[1 : –1]

print List3

4. (a) Differentiate between : 5

(i) Testing and Debugging

(ii) PROM and EPROM

(b) Write a function that accepts an integer as input and return the reverse integer. e.g. if the input is 2896, the output should be 6982. 5

5. (a) What will be the output of executing the following statements on Python command prompt : 5

A = { 1, 2, 3, 5, 7, 9 }

B = { 2, 4, 6, 8, 9, 10 }

(i) A . intersection (B)

(ii) A . symmetric_difference (B)

(iii) A – B

(iv) B – A

(v) 10 in B

(b) Write a function that takes length of three sides of triangle as parameter and return the area of the triangle. Also assert that sum of length of any two sides is greater than third side. 5

6. (a) Define a class Bank that keeps track of bank customers. The class should contain the following data members : 8

Name - Name of the customer

Account Num - Account Numbeer

Type - Account Type (Savings or Current)

Amount - Total amount deposited in the bank

The class Bank should support the following methods :

(i) __init__ method for initialising the data members

(ii) Deposit method for depositing money in the account

(iii) Withdraw method for withdrawing money from the account

(iv) __str__ method that displays the information about bank the customer

(b) When do you need multiple except clauses in a try ...... except block ? 2

7. (a) Write a function that takes a list of numbers as parameter and sort it using Bubble Sort. 5

(b) Consider the following list of numbers
10, 23, 45, 67, 89, 99, 105, 150

Show step by step iterations for searching the number 105 in the above list using Binary Search. Also write the number of iterations required to find the number. 5

8. (a) Write a program to reverse a string using stacks. 5

(b) Evaluate the following postfix expression. Show the stack status after execution of each expression. 5

5, 20, 15, –, *, 25, 2, *, +

This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **138** **I**

Unique Paper Code : 42351101

Name of the Course : **B.Sc.(Mathematical Sciences)/B.Sc. (Prog.)**

Name of the Paper : Calculus and Matrices

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **Two** questions from each section.

Section - I

1. (a) Prove that the set $\{X_1, X_2\}$ of vectors in R^n is linearly independent iff X_1 and X_2 are collinear. 6

(b) Define a subspace of a vector space. Examine whether the subset 6

$W = \{(a,b,2); a.b \in R\}$ of R^3 is a subspace or not.

2. (a) Define Linear Transformation. Find and sketch the image of unit square with vertices (0,0), (1,0), (0,1) and (1,1) under the dilation of factor 3. 6

P.T.O.

(b) Define eighen value of a matrix. Find eighen values and corresponding eighen vectors of matrix 6

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}.$$

3. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \end{bmatrix}$ using Elementary Transformations.

(b) Solve, if consistent, the system of equations :

$$x + y + 3z = 1$$
$$2x + 3y - z = 3$$
$$5x + 7y + z = 7$$

6, 6

Section - II

4. (a) Discuss the convergence of the following sequences : 6

(i) $\left((-1)^n \cdot \frac{1}{n}\right)$ (ii) (x^n)

where $-1 < x < 1$

(b) Sketch the graph of the function $f(x)=\frac{1}{2}x^2-3x+\frac{11}{2}$.

Mention the transformations used at each step. 6

(c) Radium is known to decay at the rate proportional to the amount present. If half life of radium is 1600 years, what percentage of radium will remain in a given sample after 800 years ? 6

(a) If $y=\sin(m\sin^{-1}x)$, prove that

$(1-x^2)y_{n+2}=(2n+1)xy_{n+1}+(n^2-m^2)y_n$.

(b) Find the Taylor's series generated by $f(x)=\frac{1}{x}$

at $x=2$. When does this series converges to $\frac{1}{x}$. 6

(c) Verify that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$, where z is given by $z=(x^2+xy+y^2)^{-1}$. 6

(a) Find the nth order derivative of the function given by $y=\sin(ax+b)$, where a,b are fixed constants. 6

P.T.O.

(b) Define heat equation and hence verify th[at] $\varphi(x,t) = e^{-c^2 x^2 t} \sin \pi x$ is a solution of h[eat] equation.

(c) Find the limit of the following sequences

(i) $\frac{1^2 + 2 + 3^2 + ...n^2}{6n^3 - n^2 + 3n + 4}$ (ii) $\left(5^{\frac{1}{n}}\right)$

Section - III

7. (a) Show that modulus of sum of two compl[ex] numbers is always less than or equal to t[he] sum of their moduli.

(b) Form an equation in lowest degree with re[al] coefficients which has 2-3i and 3+2i as t[wo] of its roots.

8. (a) Solve the equation $z^7 + z = 0$.

(b) Simplify $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$.

9. (a) Find the equation of circle whose radius [is] 3 and whose centre has affix 1-i .

(b) Find the equation of the right bisectors [of] the line joining the points z_1 and z_2.

his question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

No. of Question Paper : **1278**

Unique Paper Code : **62351101** **I**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

(*a*) Find the value of a if the function f given by :

$$f(x) = \begin{cases} 2x - 1, & x < 2, \\ a, & x = 2, \\ x + 1, & x > 2 \end{cases}$$

is continuous at $x = 2$. 6

(*b*) Examine the continuity of the function f defined by :

$$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, \quad x \neq 0$$

$$f(0) = 0,$$

at $x = 0$. Also discuss the kind of discontinuity if any. 6

P.T.O.

(c) Discuss the derivability of $f(x) = |x-1| + |x+1$

$x = -1, 1$.

2. (a) If

$$u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), x \neq 0, y \neq 0$$

then prove that :

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(b) If $z = \tan^{-1}\left(\frac{y}{x}\right)$, verify that :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(c) Find the nth derivative of $\sin^3 x$.

3. (a) Find the condition for the curves :

$$ax^2 + by^2 = 1, \qquad a_1 x^2 + b_1 y^2 = 1$$

to intersect orthogonally.

(b) Find the equations of the tangent and normal at any point (x, y) of the curve : 6½

$$\frac{x^m}{a^m}+\frac{y^m}{b^m}=1.$$

(c) Find the radius of curvature at any point to the curve $x=a(\cos t+t\sin t), y=a(\sin t-t\cos t)$. 6½

(a) Find the asymptotes of the curve :

$$x^3+y^3-3ax=0.$$ 6½

(b) Find the multiple points on the curve :

$$x^4-2ay^3-3a^2y^2-2a^2x^2+a^4=0.$$ 6½

(c) Trace the curve :

$$y^2x^2=x^2-a^2.$$ 6½

(a) Verify the Rolle's theorem for the function $f(x)=(x-a)^m(x-b)^n$; m, n being positive integers; $x\in[a, b]$.

6

(b) If $f(x) = (x - 1)(x - 2)(x - 3)$; $x \in [0, 4]$, then using Lagrange's Mean Value Theorem find the value of such that $c \in (0, 4)$.

(c) Prove that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, for $0 \le x \le 1$.

6. (a) State and prove Lagrange's Mean Value Theorem. 6

(b) Evaluate :

$$\lim_{x \to \pi/2} \frac{\tan 3x}{\tan x}$$

(c) Investigate the maximum and minimum values of the function f defined by :

$$f(x) = 2x^3 - 15x^2 + 36x + 10, \forall x \in \Re .$$

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **140** **I**

Unique Paper Code : 42344304

Name of the Course : **B.Sc.(Prog.)/ B.Sc. Math. Science**

Name of the Paper : Operating Systems

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **Section - A** is compulsory.

(c) Attempt any **five** questions from **Section-B**.

(d) **All** parts of a question must be attempted together.

Section - A

(Compulsory)

1. (a) What is a fault tolerant system ? 2

(b) What system calls have to be executed by command interpreter or shell in order start a new process ?

(c) Explain the convoy effect in CPU schedulin

(d) What is memory compaction ?

(e) Give difference between primitive and no primitive scheduling. State why strict no preemptive scheduling is unlikely to be use

2.

(f) Name **three** criteria based on which we c compare various CPU scheduling algorithm

(g) What is dynamic loading ?

(h) Explain how locality of reference helps getting reasonable performance in dema paging ?

(i) Why threads are called light weight processes ? 2

(j) What is absolute pathname ? Explain with the help of an example. 2

(k) What is the difference between "cp" and "mv" command of Unix ? 2

Section - B

(Attempt any five)

2. (a) Explain **three** benefits of multi-threaded programming. 3

(b) How does cache help to improve system performance ? What problems do they cause? 4

(c) What are the **three** advantages of multiprocessor systems ? 3

P.T.O.

3. (a) What is the purpose of the comman
interpreter ? Why is it usually separate fror
the kernel ?

(b) Consider a paging system with the page tabl
stored in memory.

(i) If a memory reference takes 5
nanoseconds, how long does a page
memory reference take ?

(ii) If we add TLBs, and 75 percent of a
page-table references are found in th
TLBs, what is the effective memor
reference time ? (Assume that finding
page-table entry in the TLBs takes 5
nanoseconds, if the entry is present.)

(c) Explain the difference between internal a
external fragmentation.

4. (a) Draw a process state diagram and expla
the state transitions.

(b) Write the shell script to perform the following : 1×5=5

(i) List the details of directories in the current working directory.

(ii) Remove a file interactively.

(iii) Compare two files while listing the unique lines of both the files

(iv) Count the number of users currently logged in the system

(v) Give permission to a file such that only the owner has execute permission

5. (a) Explain the layered approach of the OS structure. What are the advantages and disadvantages of layered approach to system design ? 5

(b) What is a page fault ? How is it handled ? 5

6. (a) What is the role of a dispatcher ?

(b) Explain how the following schedulin algorithms favor short processes :

(i) FCFS

(ii) RR

(iii) Multilevel feedback Queue

(c) What is the hardware support required fo demand paging ?

(d) Give three cases where the entire progra need not be in memory for execution.

7. Suppose the following processes arrive fo execution at the time indicated :

Process	Burst Time	Arrival Tim
P0	7	0
P1	4	1
P2	2	1
P3	3	3
P4	4	4

8.

(i) Draw Gantt charts illustrating the execution of these processes using FCFS, SJF, RR (time quantum = 3). 3

(ii) What is the turnaround time for process P0, P3 in each of the scheduling algorithms ? 3

(iii) What is the average waiting time for the processes in each of the scheduling algorithms ? 3

(iv) Which algorithm gives minimum average waiting time ? 1

8. (a) Consider a logical address space of 64 pages of 1,024 bytes each, mapped onto a physical memory of 32 frames. 4

(i) How many bits are there in the logical address ?

(ii) How many bits are there in the physical address ?

(b) What is degree of multiprogramming ? Whic scheduler controls the degree multiprogramming ? Why ?

(c) What is a privileged instruction ? Expla its use with the help of an example.

[This question paper contains 4 printed pages]

Your Roll No.	:	
S. No. of Q. Paper	: 143	I
Unique Paper Code	: 42354302	
Name of the Course	: **B.Sc.(Prog.)/ B.Sc. Mathematical Sciences**	
Name of the Paper	: Algebra	
Semester	: III	

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **Two** parts from each question.

(c) **All** questions are compulsory.

(d) **Marks** are indicated.

Unit- I

1. (a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R}; a \neq 0 \right\}$

Show that G is a group under matrix multiplication. 6

P.T.O.

(b) (i) Let G be a group such that if a, b, c and ab = ca ⇒ b = c, then prove tha is abelian.

(ii) Let $H=\{x \in U(20) : x \equiv 1 \bmod 3\}$.

List all elements of H.

Prove or disprove that H is a subgrou U(20).

(c) Prove that the intersection of two subgroup a group is a subgroup but their union is not.

2. (a) Define cyclic group. Prove that every cy group is Abelian. Is the converse tru Justify.

(b) Give an example of a non cyclic group al whose proper subgroups are cyclic.

(c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix}$

and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

(i) Write α and β as product of disj cycles.

(ii) Find $o(\alpha\beta)$ and $o(\alpha^{-1})$

3. (a) Let 'a' be an element of a finite grou Prove that $a^{o(G)} = e$.

(b) Consider the subgroup H = {1, 9} of group G = U(20) under multiplication modulo 20. Find the number of cosets of H in G and determine all the distinct cosets of H in G. 6

(c) Prove that the center Z (G) of a group G is a normal subgroup of G. 6

Unit- II

(a) Prove that a non empty subset S of a ring R is a subring of R if and only if

$a-b \in s$ and $ab \in S \;\forall\; a, b \in S$. 6.5

(b) Prove that $\mathbb{Q}\left[\sqrt{2}\right]=\left\{a+b\sqrt{2} : a,b \in \mathbb{Q}\right\}$ is an integral domain. 6.5

(c) (i) Let $\mathbb{Z}$ be the ring of integers and n be a fixed integer.

Show that I = <n> $\{nx : x \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z}$. 3.5

(ii) Give an example of a finite , non commutative ring . 3

Unit- III

(a) Determine whether or not the set

$$\left\{\begin{pmatrix}2 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 2\end{pmatrix}, \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\right\}$$

is linearly independent over $\mathbb{Z}_5$. 6.5

 P.T.O.

(b) Define the liner span of a subset of a vecto space V (F) and prove that the linear span a set S is a subspace of V(F) containing S.

6.

(c) Determine whether or not { (1, 3, 2), (2, 0, 1), (1, 1, 1) } from a basis of R^3.

6.

6. (a) Matrix of a linear transformation T with respect to basis {(1,2), (0,1)} of R^2 is given by $\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$.

Determine the linear transformation T.

6.

(b) Let U and V be two finite dimensional vecto spaces over F. Let T from U to V be a linea transformation. If $\{u_1, u_2, u_3, \ldots, u$ generates U then show that Range space T is generated by $\{T(u_1), T(u_2), T(u_3), \ldots, T(u_n)\}$.

6.

(c) Find the range, rank, kernel (Null space) ar nullity of T where linear transformatic $T: R^2 \rightarrow R^3$ is defined by

$T(x, y) = (y, x + 2y, x + y)$.

6

This question paper contains 5 printed pages.

***Your Roll No.* ....**

No. of Ques. Paper	**: 1319**	**I**
que Paper Code	**: 62354343**	
ne of Paper	**: Analytic Geometry and Applied Algebra**	
ne of Course	**: B.A. (Prog.) Mathematics**	
ester	**: III**	
ation	**: 3 hours**	
imum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has six questions in all.

Attempt any **two** *parts from each question.*

All *questions are compulsory.*

1 (a) Describe and draw the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$ 6½

(b) Writing the basic steps, describe and draw the graph of the equation :

$$(x+2)^2 = -(y+2).$$ 6½

(c) Identify and sketch the curve :

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$ 6½

(a) Find the equation of the ellipse with foci $(\pm 1, 0)$ and $b = \sqrt{2}$. Also state the reflection property of the ellipse. 6

P.T.O.

(b) Find the equation of the parabola whose vertex i
(5, −3); axis is parallel to y-axis and passes thro
(9, 5).

(c) Find the equation of the hyperbola with the verti
$(0, \pm 2)$ and asymptotes $y = \pm\frac{2}{3}x$. Also sketch
graph.

3. (a) Let any $x'y'$-coordinate system be obtained by rotat
an xy-coordinate system through an angle $\theta = 60^\circ$. F
the $x'y'$-coordinates of the point whose xy-coordina
are (−2, 6). Also find the equation of the cur
$\sqrt{3}\,xy + y^2 = 6$ in $x'y'$-coordinates.

(b) Rotate the coordinate axes to remove the xy-term
the conic :

$$6x^2 + 24xy - y^2 - 12x + 26y + 11 = 0.$$

Then name the conic.

(c) (i) Find the angle that the vector $-\sqrt{3}\mathbf{i} + \mathbf{j}$ makes w
positive x-axis.

(ii) Find the orthogonal projection of vector $v = 6\mathbf{i} + 3\mathbf{j} +$
on the vector $\mathbf{b} = \mathbf{i} - 2\mathbf{y} - 2\mathbf{k}$.

4. (a) Find the equation of two spheres that are centered
the origin and are tangent to the sphere of radius
centered at (3, −2, 4).

(b) (i) Find the direction cosines of the vector v = 2i + 3y – 6k, if it makes angles α, β and γ with x-axis, y-axis and z-axis, respectively. Then show that :

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$ 3

(ii) For any two vectors **u** and **v**, prove that :

$$\mathbf{u}.\mathbf{v} = \frac{1}{4}\|\mathbf{u}+\mathbf{v}\|^2 - \frac{1}{4}\|\mathbf{u}-\mathbf{v}\|^2.$$ 3½

(c) Let $\mathbf{u} = \mathbf{i} - 3\mathbf{i} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$, and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find the length of $3\mathbf{u} - 5\mathbf{v} + 2\mathbf{w}$. Also find the volume of the parallelopiped with adjacent edges **u**, **v** and **w**. 6½

(a) (i) Find the parametric equation of line passing through the point (1, 2, –3) and parallel to the vector **u** = 4i + 5j + 7k. 3

(ii) Find the equation of plane through the point (–1, 2, –5) and perpendicular to the planes $2x - y - z = 1$ and $x + y - 2z = 8$.

(b) Show that the lines :

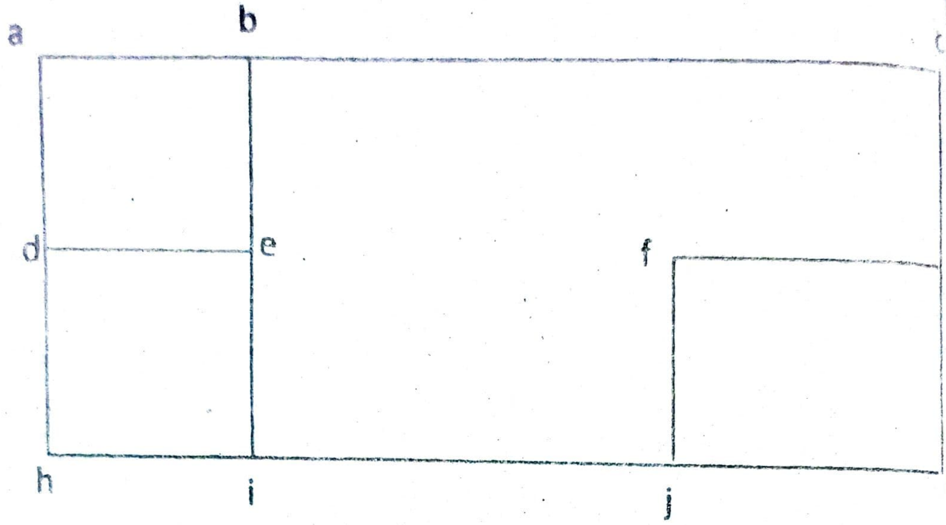
$$L_1 : x = -2 + t, \quad y = 3 + 2t, \quad z = 4 - t$$

$$L_2 : x = 3 - t, \quad y = 4 - 2t, \quad z = t$$

are parallel. Also find the equation of the plane they determine. 6

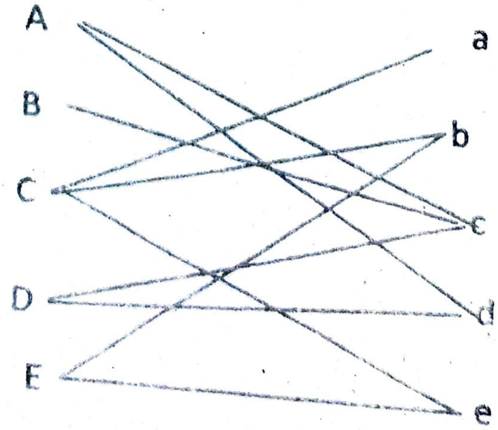
(c) Let the graph represent a section of a city's street map. What is the smallest number of policemen that should be positioned at corners (vertices) so that they can keep

every block (edge) under surveillance? Give a deta[illegible]
logical analysis.

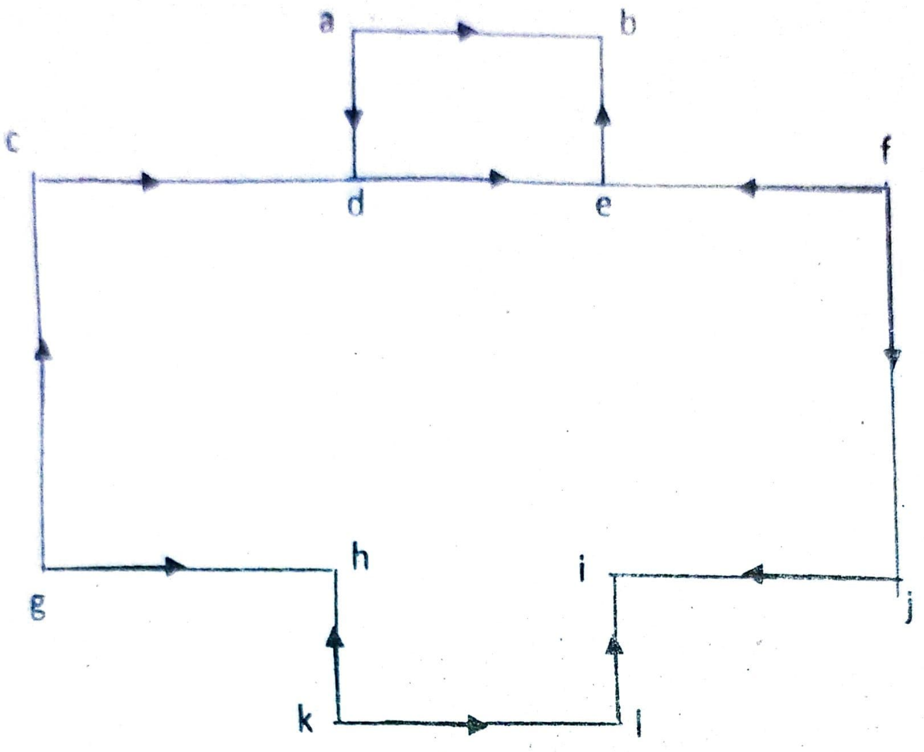


6. (a) Three pitchers of sizes 10 litres, 4 litres and 7 litres [illegible] given. If initially 10 litres pitcher is full and the ot[illegible] two are empty, find a minimal sequence of pouring [illegible] as to have exactly 3 litres of water in two pitchers. [illegible]

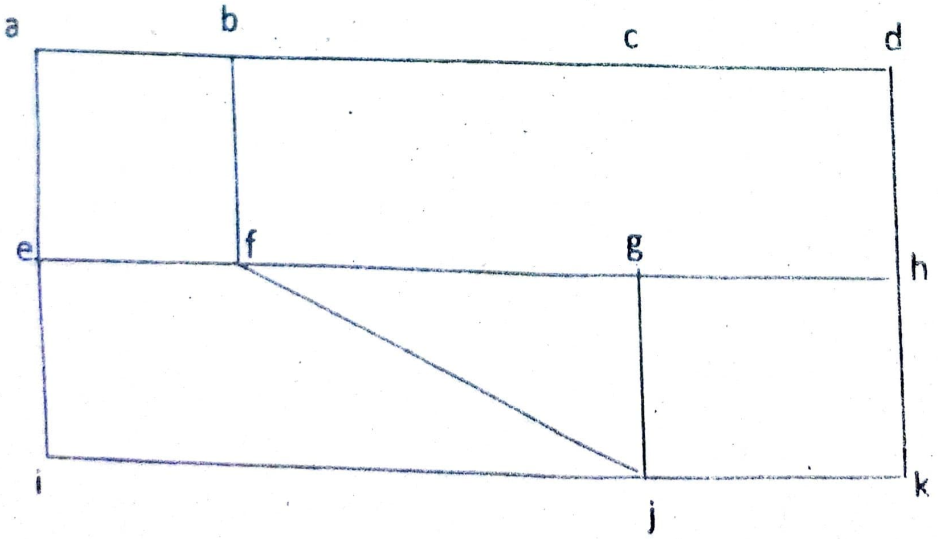
(b) (i) Find a matching or explain why none exists for [illegible] following graph :



(ii) Find a vertex basis for the following graph : 3½



(c) Find a maximum independent set in the following graph. Justify your answer. 6½



s question paper contains 4 printed pages] (300)

Roll No. | | | | | | | | | | | |

No. of Question Paper : 1347

ique Paper Code : 62353326 I

me of the Paper : **Mathematical Typesetting System : LaTeX—NC**

ame of the Course : **B.A. (Programme) Mathematics : Skill Enhancement**

emester : III

Duration : 2 Hours Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks; any *four* parts from the following : 4×0.5=2

(*i*) In LaTeX, ... command is used to get text in italics.

(*ii*) .... command is used to start new line in TeX document.

P.T.O.

(*iii*) In LaTeX, ... command is u[...]

a section.

(*iv*) .... command produce () in [...]

(*v*) .... command puts a horizon[...]

its argument.

2. Answer any *eight* parts from the following:

(*i*) Explain *four* ellipsis command in LaTeX, wh[...]

in any mode.

(*ii*) Write the code in LaTeX to obtain the [...]

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

(*iii*) Illustrate the difference between enume[...]

environments by giving an example.

(*iv*) Write the LaTeX code for the followi[...]

$$\frac{x+y}{1+\sqrt{\frac{y}{z+1}}}.$$

(*v*) Give the command in LaTeX to type

$$\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}.$$

(vi) Write a code in LaTeX to produce :

$$\int_0^\pi x \sin x \, dx = \int_0^\pi (\pi - x) \sin x \, dx$$

$$\therefore \int_0^\pi x \sin x \, dx = \pi.$$

(vii) Give the command in LaTeX to produce an output :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots.$$

(viii) Write a code in LaTeX to produce an output :

$$z = \begin{cases} y & \text{if } y > 0 \\ x + y & \text{otherwise.} \end{cases}$$

(ix) Write the following postfix expression in standard form :

x sqrt x 2 exp add 1 x sub div.

(x) Give a command to draw sector of a circle of radius 1.5 centered at (2, 2), going from reference angle 0 to 45 degrees.

3. Answer any *three* parts from the following : 4+4+4=12

(a) Write the code in LaTeX to plot the curves $y = \sqrt[3]{|x|}$ as dotted curve and $y = x^3$ as dashed curve in the same coordinate system.

P.T.O.

(*b*) Explain the command \qbezier(10, 20) (20, 30)

and draw its figure.

(*c*) Write the code in LaTeX to produce the matri

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}.$$

(*d*) Draw an output of the following commands :

(*i*) \put(20, 0){\circle{20}}

(*ii*) \put(50. 0){\circle*{5}}

4. Write a presentation containing in beamer with the content :

Title of the presentation with author and date, the l chapters :

Getting started with LaTeX, PStricks and Beamer, on different slides, including one definition chapter.

'his question paper contains 8 printed pages]

our Roll No. : ....

l. No. of Q. Paper : 167 I

nique Paper Code : 42347902

ame of the Course : **B.Sc.(Prog.) /B.Sc. Math. Sciences : DSE - 2A**

ame of the Paper : Analysis of Algorithms and Data structures

emester : V

Time : 3 Hours *Maximum Marks : 75*

ıstructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Question **NO.1** is compulsory.

(c) Attempt any **five** of question nos. **2** to **8**.

(d) **Parts** of a question must be answered together.

(a) Consider an array of numbers {4, 6, 3, 7, 8}: 2

(i) Can linear search be applied to find 5 ?

(ii) Can binary search be applied to find 8 ?

P.T.O.

(b) Arrange the following running times
increasing order.

$O(n^2)$, $O(n\log n)$, $O(2^n)$, $O(\log n^2)$

(c) Consider an integer array A of dimens
$m \times n$, at what memory location will
element A[i][j] be located, consider
column major address mapping ?

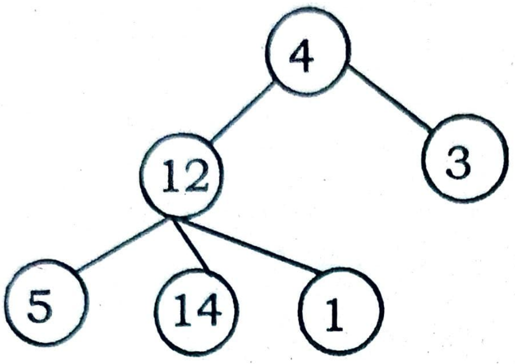
(d) Consider the 0-1 knapsack problem;
greedy strategy always give the optin
solution ? If yes, prove; if no, give a coun
example.

(e) Perform selection sort on the arr
{3, 5, 1, 8, 7}, show the steps after ea
iteration. Report the number of compariso

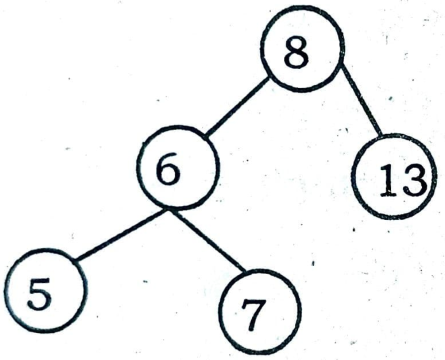
(f) Write a recursive algorithm to compute
product of two integers *a* and *b*.

(g) For each of the following trees, specify whether it is a binary search tree or not. Give reasons for your answers. 6

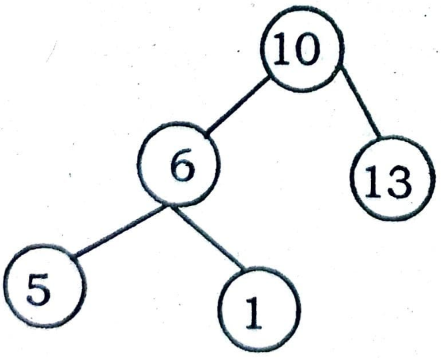
(i)



(ii)



(iii)



2. (a) Write an algorithm for push operation a pop operation for a Stack implemented us linked lists.

(b) Write an algorithm for finding an eleme in an array using Binary Search.

3. (a) Consider the following sequence operations performed on an initially em doubly linked list :

InsertBeginning(5),

InsertBeginning(8),

InsertEnd(3),

InsertEnd(10),

DeleteBeginning(),

Deletenode(3)

Show the contents of the list, links betwe nodes, head and tail after each operatio

(b) Consider a function f() to computer Fibona numbers as defined below :

	0 if n=0
Fib (n)	1 if n=1
	Fib(n-1) + Fib(n-2) if > =2

How many times will f( ) be called to f the value of Fib(6) ?

(a) Write a recursive algorithm to compute the sum of n natural numbers. 3

(b) Do the following transformations : 4

(i) Postfix to Infix

A B C D E - + $ * E F * –

(ii) Infix to Prefix

A $ B * C – D + E / F / (G + H)

(Note : $ is the exponent operation)

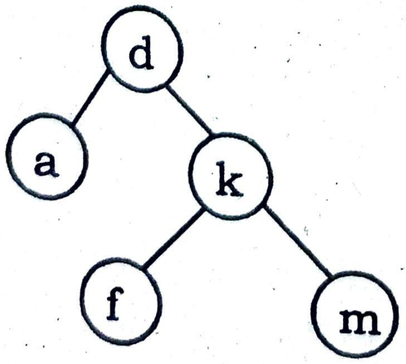
(c) Perform Merge sort on the given array of numbers {6, 5, 4, 3, 2, 1}. Show each step. 3

(a) For the given binary search tree, give the following : 6

(i) Pre-order traversal

(ii) In-order traversal

(iii) Post-order traversal



(b) Consider the following applications a specify which data structure may be us to implement them and why ?

(i) Scheduling processes on the CPU.

(ii) Converting an infix expression to postfix expression.

6. (a) Consider an initially empty circular que of size five implemented using arra Perform the given sequence of operatio and show the position of front and rear af each operation.

Enqueue(4),

Dequeue,

Enqueue(3),

Enqueue(8),

Enqueue(2),

Enqueue(6),

Enqueue(13),

Dequeue,

Enqueue(1)

(Note : Enqueue is inserting a values int queue, Dequeue is removing a value fr the queue)

(b) Sort the following array using radix sort, show the array contents after each iteration. {245, 12, 5673, 78, 43567, 33, 25, 46, 678}

4

(a) Write an algorithm to search for an element and delete it if found, in a doubly linked list.

4

(b) Give worst case and best case running times for the following algorithms : 4

(i) Linear Search

(ii) Insertion Sort

(c) Which of the following uses divide and conquer technique for solving problems ?

2

(a) Linear search

(b) Binary Search

(c) Quick Sort

(d) Count Sort

(a) If k integer elements are to be stored :

5

(i) Determine the amount of memory used when these elements are stored using an array of size n=50 (assume $k \leq n$) and when they are stored in a singly linked list. Assume pointers require as much memory as an integer.

P.T.O.

(ii) How large can the ratio of two mem
requirement get ?

(b) Write an algorithm to sort an array usi
count sort.

question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

of Question Paper : **1454**

e Paper Code : **62353505** **I**

of the Paper : **Statistical Software-R**

of the Course : **B.A. (Prog.) Mathematics : SEC**

ter : **V**

on : **2 Hours** Maximum Marks : **38**

your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

All commands should be written in software R.

Do any *five* of the following : 5×1

State whether the following statements are true *or* false.

The commands for the following mathematical expressions are :

(*i*) $\sqrt{2}+3$ is sq(2) + 3

(*ii*) 4 ! is fact(4)

(*iii*) $\tan^{-1} x$ is atan(x)

(*iv*) $|x| + 3$ is abs(x) + 3

(*v*) Is R language key sensitive

(*vi*) If datap is a ten item vector then datap[1 : 3] command show only one and third items.

5

2. Do any *five* of the following :

Fill in the blanks :

(*i*) .... command is used to plot histogram. (h
histo( ))

(*ii*) The command to produce five basic qua
is .... (quartile( )/quantile( ))

(*iii*) If you have an xtabs object "Y", then write the com
to ressemble it into a data frame ....
(as.data.frame(as.matrix(Y))/as.data.frame(Y).

(*iv*) Data frames are .... dimensional. (one/two)

(*v*) To generate ten random numbers uniformly, we
command .... guinf(10)/ruinf(10)).

(*vi*) The Kolmogorov-Smirnov test is applied for com
.... distributions (within one/two)

3. (*a*) Write the commands for the following :

(*i*) sin (30°)

(*ii*) last 150 commands executed.

(*b*) (*i*) Using scan command create simple data
containing the text stating the following da
the week :

Mon Tue Wed Thu Fri Sat.

(*ii*) Write a command to remove all the ele
containing '*r*'.

(c) Why should you use R language for statistical work ?

(d) Generate a 4 × 4 matrix and name it as MAT. Then find the mean of the second row of the matrix MAT. Also, find the row sums of the same matrix.

(e) Write syntax to generate 'n' random values of :

(i) normal distribution

(ii) uniform distribution.

(f) Describe density function with 3 × 4 matrix example.

(g) A data file is given with name bird :

	A	B	C	D	E
X	12	14	15	40	10
Y	08	04	07	09	11
Z	30	20	25	10	35

(i) Extract third columns

(ii) Transpose bird data

(iii) Find max and min items

(iv) Make histogram of X

(*h*) Make a score data file

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Draw a stem leaf plot.

4. Do any *four* of the following :

(*a*) Consider the following course grades of ran selected students :

40	38	20	31
26	35	38	21
50	33	29	40
42	46	20	48
43	48	41	27

Write commands for :

(*i*) Putting data into a variable x

(*ii*) Creating a scatter plot of x

(*iii*) Creating a box plot of x

(*iv*) Creating a stem and leaf plot of x

(*v*) Creating a normal probability plot of x.

(*b*) The following data gives, for each amount by which an elastic band is stretched over the end of a ruler, the distance that the band moved when released :

Stretch	Distance
46	148
54	182
48	173
50	166
44	109
42	141
52	166

(*i*) Create data frame of the above data.

(*ii*) Convert the data frame into matrix.

(*iii*) Convert the data frame into table.

(*iv*) Draw box plot of the given data.

(*v*) Label the axis of the plot.

(c) (i) Create a sample of 50 numbers which incremented by 1.

(ii) Create the binomial distribution of 50 numbers probability 0.5.

(iii) Find the probability of getting 26 or less heads a toss of a coin. (using binomial distribution

(iv) How many heads will have a probability of 0.25 come out when a coin is tossed 51 times ?

(v) Find 8 random values from a sample of 150 probability of 0.4. (using binomial distribution

(d) Generate 50 random variable using Poisson distribut binomial distribution and plot one distributio another.

(e) If a data2 file is given :

data2 = 3, 5, 8, 7, 9, 6, 8, 6, 3, 5, 4, 7, 3, 6, 2

Which test apply to compare this sample to no distribution also write command.

ıuestion paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

. of Question Paper : **155**

e Paper Code : **42357501** **IC**

of the Paper : **Differential Equations**

of the Course : **B.Sc. (Math. Sci.)/B.Sc. (Prog.) :**

DSE-1

ster : **V**

tion : **3 Hours** Maximum Marks : **75**

e your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any *two* parts from each question.

(*a*) Solve : 6.5

$$\left(2xy^2 + y\right)dx + \left(2y^3 - x\right)dy = 0.$$

P.T.O.

(b) Solve :

$$\frac{dy}{dx} - \frac{y}{x} = -\frac{y^2}{x}.$$

(c) Solve :

$$p^2 + 2py \cot x = y^2.$$

2. (a) Solve the initial value problem :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2xe^{2x} + 6e^x, \ y(0) = 1, \ y'(0$$

(b) Find the general solution of the differential equat

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3.$$

(c) For the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0,$$

show that e^x and xe^x are solutions on the

$-\infty < x < \infty$. Are these linearly independent

Find the solution that satisfies the conditions

$y'(0) = 4$.

(a) Using the method of variation of parameters, solve the differential equation : 6

$$\frac{d^2y}{dx^2} + y = \tan^2 x.$$

(b) Given that $y = x$ is a solution of : 6

$$\left(x^2 - 1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

(c) Find the general solution of : 6

$$\left(x^2 + 2x\right)\frac{d^2y}{dx^2} - 2(x + 1)\frac{dy}{dx} + 2y = (x + 2)^2,$$

given that $y = x + 1$ and $y = x^2$ are linearly independent solutions of the corresponding homogeneous equation.

(a) Solve : 6

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}.$$

(*b*) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} - x + 5y = t^2,$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 4y = 2t + 1.$$

(*c*) Check condition of integrability and solve :

$$\left(y^2 + yz\right)dx + \left(xz + z^2\right)dy + \left(y^2 - xy\right)dz = 0.$$

5. (*a*) Eliminate the arbitrary function f from the equati

$$z = f\left(\frac{xy}{z}\right)$$

to form the corresponding partial differential eq

(*b*) Find the general integral of the partial diff

equation :

$$px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$$

(*c*) Show that the equations :

$$xp - yq = x, \quad x^2p + q = xz$$

are compatible and find their solution.

(a) Find the complete integral of the equation : 6.5

$$p = (z + qy)^2.$$

(b) Find the complete integral of the equation : 6.5

$$zpq = p + q.$$

(c) Reduce the following differential equation to canonical form : 6.5

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0.$$

s question paper contains 7 printed pages]

r Roll No. : ....

o. of Q. Paper : 607 I

ue Paper Code : 32357501

e of the Course : B.Sc.(Hons.)

Mathematics : DSE - I

of the Paper : Numerical Methods

ester : V

e : 3 Hours *Maximum Marks* : 75

uctions :

a) Write your Roll No. on the top immediately on receipt of this question paper.

b) Use of non-programmable scientific calculator is allowed.

c) Attempt **all** questions selecting **two parts** from each question.

P.T.O.

1. (a) A scheme for approximating the square of a positive real number a is based on

recursive formula $x_{n+1} = \frac{x_n^3 + 3 ...}{3x_n^2 + ...}$

Construct an algorithm for approximatin square root of a positive real number a u this formula.

(b) Show that when Newton's method is ap to the equation $\frac{1}{x} - a = 0$ the resul iteration function is $g(x) = x(2-ax)$. He or otherwise, find the order of converg of the method.

(c) Use the bisection method to determine smallest positive root of the equa $\ln(1+x) - \cos x = 0$. Further show that theoretical error bound at each iterati satisfied.

(a) Consider the function $g(x) = 1 + x - \frac{1}{8}x^3$. Verify analytically that this function has a unique fixed point on the real line. Perform six iterations using the fixed point iteration scheme to approximate the fixed point of g(x) starting with $p_0 = 0.5$.

(b) Let g be a continuous function on the closed interval $[a, b]$ with $g : [a, b] \to [a, b]$. Show that g has a fixed point p in $[a, b]$. Furthermore, if g is differentiable on the open interval (a, b) and there exists a positive constant $k < 1$ such that $g'(x) \leq k < 1$ for all x belongs to (a, b), then the fixed point in $[a, b]$ is unique.

(c) Find the approximated root of $f(x) = e$ (
by the method of False Position, ta
$p_0 = 0$ and $p_1 = 1$ until $|p_n - p_{n-1}| < 5 \times 1$

3. (a) Using scaled partial pivoting during the
step, find matrices L, U and P such tha

$$= PA \text{ where } A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}. \text{ Hence, (a}$$

$$\text{the system } Ax = b \text{ where } b = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

(b) Use Jacobi method to solve the follo
system of linear equations. Use the i
approximation $x^{(0)} = 0$ and perform (
iterations.

$$\begin{aligned} 4x_1 - x_2 \quad &= 0 \\ 2x_1 + 4x_2 - x_3 \quad &= 2 \\ -2x_2 + 4x_3 - x_4 &= -3 \\ -2x_3 + x_4 &= 1 \end{aligned}$$

Use the SOR method with $\omega = 0.7$ to solve the system of equations $Ax = b$, where

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & -6 & 3 \\ -9 & 7 & -20 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ -13 \\ 7 \end{bmatrix}.$$

Use $x^{(0)} = 0$ and perform three iterations.

13

) Suppose that f is continuous and has continuous first and second order derivatives on the interval $[x_0, x_1]$. Derive the following bound on the error due to linear interpolation of f : $|f(x) - P_1(x)| \le \frac{1}{8} h^2 \max_{x \in [x_0, x_1]} |f''(x)|$, where $h = x_1 - x_0$.

) (i) Construct the difference table for the sequence of the values

$f(x) = (0, 0, 0, \varepsilon\ 0, 0, 0)$.

(ii) Prove that :

$$\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1} \Delta f_i$$

P.T.O.

(c) Obtain the Newton's form of interp
polynomial for the data set :

X	–1	0	1	2
Y	3	–1	–3	1

5. (a) Use the formula

$$f''(x_0) \approx \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

approximate the second order derivat
the function $f(x) = 1 + x + x^3$ at $x_0 = 1$, t
$h = 1, 0.1, 0.01$ and 0.001.

(b) Find the highest degree of the polyn
for which the for

$$f'(x_0) \approx \frac{f(x_0+h) - f(x_0-h)}{2h},$$ for the

derivative provides the exact value
derivative regardless of h.

(c) Derive second-order backward diffe
approximation to the first order derivat
a function.

) Using Simpson's rule determine the approximate value of the integral $\int_0^{\pi} \sin x \, dx$. Further verify the theoretical error bound.

) Apply Euler's method to find the approximate solution of the given initial value problem

$x' = (\sin x - e^t) / \cos x$, $(0 \le t \le 1)$, $x(0) = 0$, $N = 4$.

) Consider the initial value problem (IVP) $x' = t - x, (0 \le t \le 4), x(0) = 1, N = 4$ whose exact solution is given by $x(t) = 2e^{-t} + t - 1$. Obtain the solution of the IVP and compare the absolute error with theoretical error bound, assuming the Lipschitz constant L's equal to 1.

12

is question paper contains 4 printed pages]

ır Roll No. : ....

No. of Q. Paper : 608 I

que Paper Code : 32357502

ne of the Course : **B.Sc.(Hons.) Mathematics : DSE - I**

ne of the Paper : Mathematical Modelling & Graph Theory

ıester : V

ıe : 3 Hours *Maximum Marks : 75*

ructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **three** parts of each question.

(c) **All** questions are compulsory.

(a) Solve the initial value problem using the Laplace transform : 6

$$x^{(3)} + x'' - 6x' = 0;\ x(0) = x''(0) = 1$$

b) (i) Find the inverse Laplace transform of :

$$F(s) = \frac{1}{s^2 - 4}.$$

2

(ii) Show that :

$$L\{t\sin kt\} = \frac{2sk}{\left(s^2+k^2\right)^2}.$$

(iii) Find the inverse Laplace transform

$$F(s) = \frac{1}{\left(s^2+s-6\right)^2}.$$

(c) Find two linearly independent Frober series solutions of :

$$6x^2y''+7xy'-\left(x^2+2\right)y=0.$$

(d) Use power series to solve the initial va problem :

$$\left(4x^2+16x+17\right)y''-8y=0; y(-2)=1, y'(-2)$$

2. (a) Explain Linear Congruence Method and it to generate 10 random numbers us $a = 5$, $b = 1$ and $c = 8$. Was there cycling so, when did it occur ?

(b) Using Monte Carlo Simulation, write algorithm to approximate the area under curve $f(x) = \sqrt{x}$, over the interval $\frac{1}{2} \le x \le$

(c) Using algebraic analysis : 6

Maximize $5x + 3y$

subject to $x + y \leq 6,$

$3x - y \leq 9,$

$x, y \leq 0.$

(d) Using graphical analysis : 6

Minimize $5x + 7y$

subject to $2x + 3y \geq 6$

$3x - y \leq 15,$

$-x + y \leq 4,$

$2x + 5y \leq 27,$

$x, y \geq 0.$

(a) (i) Is it possible to draw a 3-regular graph with 3 vertices ? 3

(ii) Draw the eleven unlabelled simple graphs with four vertices. 3

(b) (i) Define an Eulerian trail and semi-Eulerian trail. Give **one** example for each. 4

(ii) Draw a simple connected graph with degree sequence (1, 1, 2, 3, 3, 4, 4, 6). 2

(c) Prove that there is no knight's tour on a 3 x 6 chessboard. 6

(d) Prove that a bipartite graph with odd number of vertices is not Hamiltonian. 6

4. (a) Use the factorization :

$$s^4 + 4a^4 = \left(s^2 - 2as + 2a^2\right)\left(s^2 + 2as + 2a^2\right)$$

and apply inverse Laplace transform to sh
that :

$$L^{-1}\left\{\frac{s^2}{s^4 + 4a^4}\right\} = \frac{1}{2a}(\cosh at \sin at + \sinh$$

cos at).

(b) Find the general solutions in power of
the following differential equation :

$$y'' + xy' + y = 0.$$

(c) Solve the problem :

Maximize $25x + 30y$

subject to $20x + 30y \leq 690,$

$5x + 4y \leq 120,$

$x, y \geq 0.$

Determine the sensitivity of the optim
solution to change in C_1 using the obje
tive function $C_1x + 30y$.

(d) Write down a Gray code of 4 – digi
binary words.

This question paper contains 4 printed pages.

Your Roll No. ....

No. of Paper	: 753	I
nique Paper Code	: 32357502	
ame of the Paper	: **Mathematical Modelling and Graph Theory**	
ame of the Course	: **B.Sc. (H) Mathematics : DSE-2**	
mester	: **V**	
uration	: **3 hours**	
aximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions, selecting **three** parts from each question.*

(a) Solve the initial value problem using the Laplace transform: 6

$$x''-6x'+8x=2;\ x(0)=0, x'(0)=0.$$

(b) (i) Find the inverse Laplace transform of: 2

$$F(s)=\frac{5s-6}{s^2-3s}.$$

(ii) Show that: 2

$$L=\{t\sinh kt\}=\frac{2ks}{(s^2-k^2)^2}$$

(iii) Find the inverse Laplace transform of: 2

$$F(s)=\frac{1}{s^4-16}.$$

(c) Find two linearly independent Frobenius series solutions of: 6

P. T. O.

$$2x^2y''+xy'-(3-2x^2)y=0.$$

(d) Use power series to solve the initial val
problem:

$$(x^2-6x+10)y''-4(x-3)y'+6y=0;\ y(3)=2,\ y'(3)=$$

2. (a) Explain Middle-Square Method and use it
generate 10 random numbers taking $x_0=10$
Comment about the results. Was there cycli
Illustrate.

(b) Using Monte Carlo Simulation, write an algori
to calculate the area trapped between the
curves $y=x^2$ and $y=6-x$ and the x- and
axes.

(c) Using the simplex method:

Maximize $3x+y$

subject to $2x+y\le 6,$
$x+3y\le 9,$
$x,\ y\ge 0.$

(d) Using algebraic analysis:

Maximize $10x+35y$

subject to $4x+3y\le 24,$
$4x+y\le 20,$
$x,\ y\ge 0.$

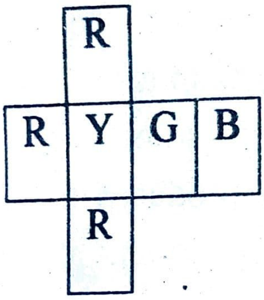
3. (a) (i) Determine the number of edges of $K_{9,10}$, Q_5
C_{10}.

(ii) Define complete bipartite graph. How many vertices and edges does a complete graph $K_{m,n}$ have? 3

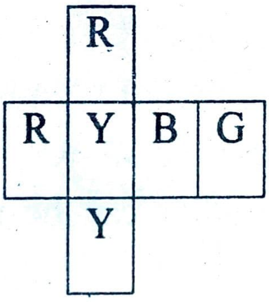
) (i) Prove that in any balanced signed graph every cycle has an even number of edges. 4

(ii) Draw a simple connected graph with degree sequence (3, 3, 3, 3, 3, 5, 5, 5). 2

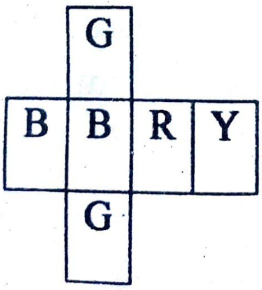
c) Determine whether the given four cubes having four colors, can be stacked in a manner so that each side of the stack formed will have all the four colors exactly once. 6



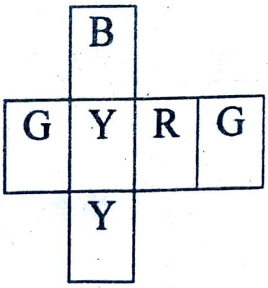
Cube 1



Cube 2



Cube 3



Cube 4

(d) Define a r-regular graph. Prove that, a r-regular graph with n vertices has $nr/2$ edges. 6

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show

$$L^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cosh at \cos at .$$

(b) Find the general solutions in power of x of following differential equation:

$$y'' + xy' + y = 0 .$$

(c) A carpenter realizes a net unit profit of $ 2 table and $ 30 per bookcase. He has up to board-feet of lumber and up to 120 hours of to devote weekly to the project. The lumber the labor can be used productively elsewh not used in the production of tables bookcases. He estimates that it requires 20 feet of lumber and 5 hours of labor to comp table and 30 board-feet of lumber and 4 ho labor to complete a bookcase. For mathematical model and use graphical anal determine how many of each piece of furni should make each week to maximize his pro

(d) Prove that there is no knight's tour on chessboard.

his question paper contains 8 printed pages]

ur Roll No. : ....

No. of Q. Paper : 610 I

ique Paper Code : 32357504

me of the Course : **B.Sc.(Hons.) Mathematics : DSE -I**

me of the Paper : Mathematical Finance

mester : V

me : 3 Hours *Maximum Marks : 75*

tructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) Following values may be used if needed $e^{0.025} = 1.0253$, $e^{-0.025} = 0.975$, $e^{0.0125} = 1.0125$ and $e^{-0.0125} = 0.9875$.

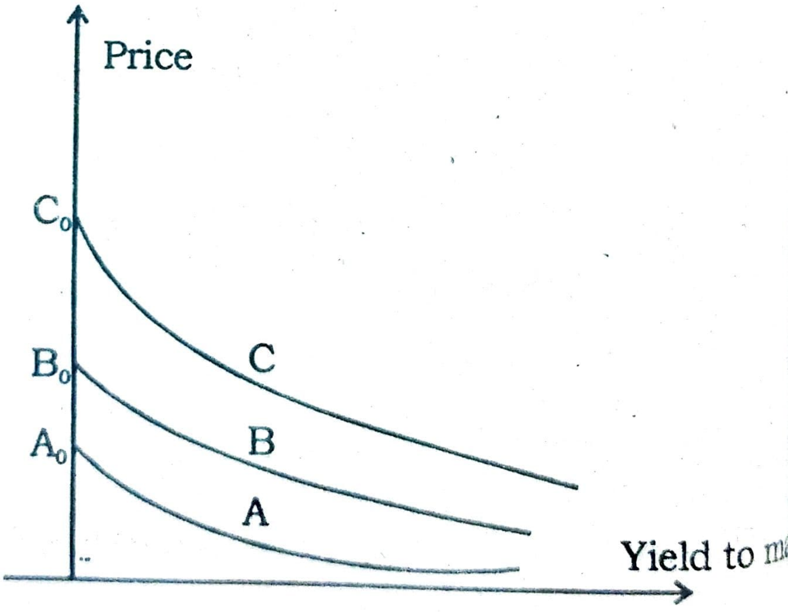
(a) State and prove annuity formula. 6

(b) A young couple has made a non-refundable deposit of the first month's rent (equal to $1,000) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent

P.T.O.

is only $900. They plan to be in the apart[...]
only 6 months. Should they switch to [...]
new apartment ? What if they plan to st[...]
year ? Assume an interest rate of 12%.

(c) Consider three bonds, each with mat[...]
30 years having respective coupon rat[...]
10%, 5%, 0%.



Match price-yield curves A, B, C wit[...]
three given bonds. Find A_0, B_0 and C_0; [...]
of intersection of price-yield curves A, [...]
with the price axis. Justify your answe[...]

(a) Find future value and present value of the cash flow stream (–1, 2, 2), having each period as one year when the prevailing interest rate is 10% per annum. Also find IRR for the given cash flow stream. 6

(b) Suppose that you have the opportunity to plant trees that later can be sold for lumber. This project requires an initial outlay of money in order to purchase and plant the seedlings. No other cash flow occurs until the trees are harvested. However, you have a choice as to when to harvest: after 1 year or after 2 years. If you harvest after 1 year, you get your return quickly; but if you wait an additional year, the trees will have additional growth and the revenue generated from the sale of trees will be greater. Assume that the cash flow streams associated with these two alternatives are

(i) (-1, 2) cut early

(ii) (-1, 0, 3) cut later.

Also assume that the prevailing interest rate is 10%. Find out when is it best to cut the trees under NPV criteria. 3+3

(c) Define portfolio return and derive expression for variance of portfolio retu[rn]

3. (a) What do the various symbols stand fo[r] the following bond price formula ?

$$P = \frac{F}{[1+(\lambda/m)]^n} + \frac{C}{\lambda}\left\{1 - \frac{1}{[1+(\lambda/m)]^n}\right\}$$

Should the price be higher or lower i[f] yield is higher ?

(b) The correlation ρ between assets A a[nd] is 0.1, and other data are given in the t[able] below where $\rho = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$.

Asset	$\bar{r}$	σ
A	10%	15%
B	18%	30%

(i) Find the proportions α of A and (1 of B that define a portfolio of A a[nd] having minimum standard deviatio[n]

(ii) What is the value of this minimum standard deviation ?

(iii) What is the expected return of this portfolio ? 6.5

(c) (i) Define spot rate s_t for t years. How is s_t determined under yearly, m periods per year and continuous compounding conventions ? 4.5

(ii) Describe security market line. 2

4. (a) (i) Define and describe minimum-variance set, efficient frontier of feasible set for any given n assets. 4

(ii) Define total return of an asset. What is beta of a portfolio ? 2.5

(b) Consider two 5-year bonds: one has a 9% coupon and sells for 101.00; the other has a 7% coupon and sells for 93.20. Find the price of a 5-year zero-coupon bond. Both bonds have the same face value normalized to 100. 6.5

(c) (i) Let the risk-free rate be $r_f = 8\%$. Suppose the rate of return of the mark[et] has an expected value of 12% and standard deviation of 15%. Consider a asset having covariance 0.045 with t[he] market. Find β and the expected rate [of] return of asset.

(ii) Assume that the expected rate of retu[rn] on the market portfolio is 23% and th[e] rate of return on T-bills (the risk fr[ee] rate) is 7%. The standard deviation [of] the market is 32%. Assume that t[he] market portfolio is efficient. What is t[he] equation of the capital market line ?

2

5. (a) (i) Explain what is a short call position a[nd] a long put position in an American optio[n]. 6

(ii) Give differences between forward a[nd] futures contracts. Illustrate wi[th] examples.

(b) Consider a long forward contract to purchase a non-dividend paying stock in 3 months. Assume the current price is \$35, the 3-months risk-free interest rate r is 5% per annum, forward price is \$38. Is there a possibility of arbitrage ? Explain. 6

(c) An investor sells a European call option on a share for \$4. The stock price is \$47 and the strike price is \$50. Under what circumstances does the investor make a profit ? Under what circumstances will the option be exercised ? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option. 6

6. (a) Suppose price of a stock is \$31, exercise price is \$30, risk-free interest rate is 10% per annum, the price of a 3 month European call option is \$3 and the price of a 3 month European put option is \$1. Is there put-call parity ? Can an arbitrageur make profit at the end of 3 months ? Explain. 6.5

(b) (i) Draw and explain profit from buying European put option on one share of stock, given option price is $7 and stri price is $70.

(ii) List six factors that affect stock opti prices.

(c) (i) Give three reasons why the treasurer a company might not hedge t company's exposure to a particular ri

(ii) Explain the difference between hedg and arbitrage. Give an example for eac question paper contains 8 printed pages]

Roll No. : ....

. of Q. Paper : 611 I

e Paper Code : 32357505

of the Course : **B.Sc.(Hons.) Mathematics : DSE-I**

of the Paper : Discrete Mathematics

ster : V

: *3 Hours* *Maximum Marks : 75*

ctions for Candidates :

) Write your Roll No. on the top immediately on receipt of this question paper.

) Do any **two** parts from each question.

Section - I

) Define covering relation in an ordered set. Prove that if X is any set, then in the ordered set $\wp(X)$ equipped with the set inclusion relation given by $A \leq B$ if and only if $A \subseteq B$ for all $A, B \in \wp(X)$, a subset B of X covers a subset A of X if and only if $B = A \cup \{b\}$, for some $b \in X \sim A$. 6

P.T.O.

(b) Let N_0 be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n and let $\wp(N)$ be the power set of N equipped with the partial order given by $A \leq B$ if and only if $A \subseteq B$ for all $A, B \in \wp(N)$. In which of the following cases is the map $\varphi : P \to Q$ order-preserving?

(i) $P = Q = N_0$ and $\varphi(x) = nx \; \forall x \in P$, where $n \in N_0$ is fixed.

(ii) $P = Q = \wp(N)$ and φ defined by

$$\varphi(A) = \begin{cases} \{1\} \text{ if } 1 \in A \\ \{2\} \text{ if } 2 \in A \text{ but } 1 \notin A \\ \varnothing \text{ otherwise} \end{cases}$$

(c) Let P = {a, b, c, d, e, f, u, v}. Draw a diagram of the ordered set $(P, \leq)$ where

$v < a < c < d < e < u$, $a < f < u$,
$v < b < c$, $b < f$

Also, find out $a \vee b$, $a \wedge b$, $e \vee f$ and $e \wedge f$.

(a) Let V be a vector space and let M = Sub V, the set of all subspaces of V. Prove that $(M, \subseteq)$ is a lattice as an ordered set but is not a sublattice of the lattice $(L, \subseteq)$, where $L = \wp(V)$, the power set of V. 6.5

(b) Prove that in a lattice L, the following inequalities are satisfied :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c) \ \forall a, b, c \in L$ 3

(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) \quad \forall a, b, c \in L$ 3.5

(c) Let $(L, \leq)$ be a lattice as an ordered set. Define two binary operations + and. on L by x+y = $x \vee y = \sup\{x, y\}$ and $x \cdot y = x \wedge y = \inf\{x, y\}$. Prove that (L, +, .) is an algebraic lattice. 6.5

Section - II

(a) Define a distributive lattice. Prove that a homomorphic image of a distributive lattice is distributive . 6

P.T.O.

(b) Use the Quine-McCluskey method to find a minimal form of :

$xyz' + xy'z + xy'z' + x'yz + x'y'z$

(c) (i) Find the conjunctive normal form of :

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'.$$

(ii) Find the disjunctive normal form of :

$x_1'x_2 + x_3(x_1' + x_2).$

4. (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ for all x, y in a Boolean algebra B. 5. 3.5

(ii) Show that the lattice ({1, 2, 4, 5, 10, 20}, gcd, lcm) does not form a Boolean algebra for the set of positive divisors of 20.

(b) Using the Karnaugh Diagrams, find a minimum form for p and q where : 3.

$$p = (x_1 + x_2)(x_1 + x_3) + x_1x_2x_3$$

$$q = x_1x_2'x_3 + x_1x_2'x_3' + x_1'x_2x_3 + x_1'x_2'x_3 + x_1'x_2'x_3'$$

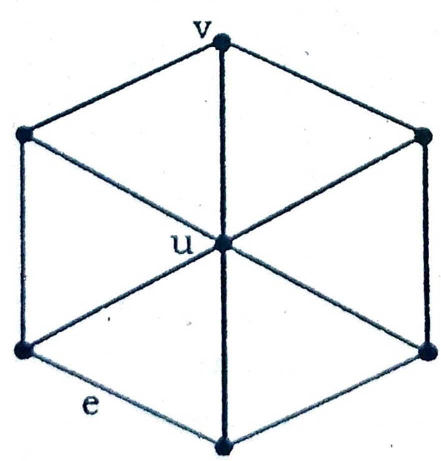
(c) Draw the contact diagram and give the symbolic representation (using seven gates) of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1x_3 + x_1'x_2)(x_2' + x_3)$$

6.5

Section - III

5. (a) (i) Draw pictures of the subgraphs G \{e}, G \{v} and G \{u} of the following graph G. 3



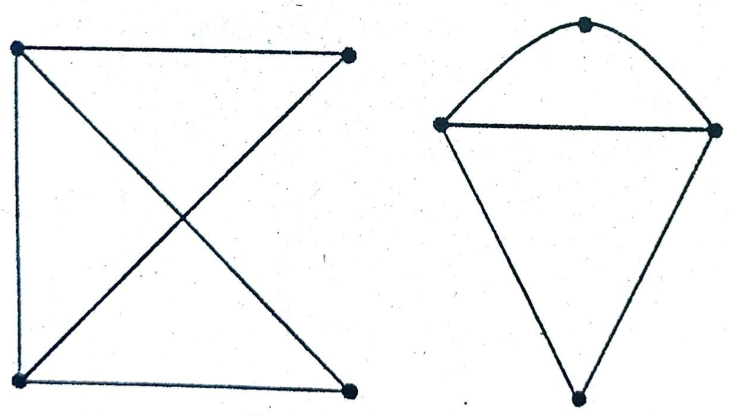
G

P.T.O.

(ii) Answer the Königsberg bridge problem and explain your answer with graph. 3

(b) (i) Draw K_4 and $K_{3,4}$. 3

(ii) For the below pair of graphs, either label the graphs so as to exihibit an isomorphism or explain why graphs are not isomorphic. 3

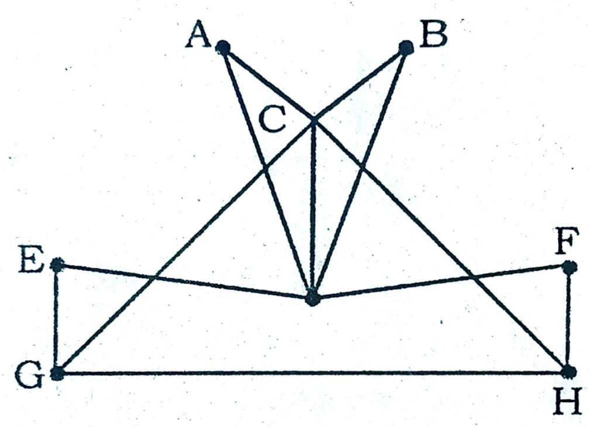


(c) (i) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4. Justify your answer. 2

(ii) A complete graph with more than two vertices is not bipartite. Justify this statement. 2

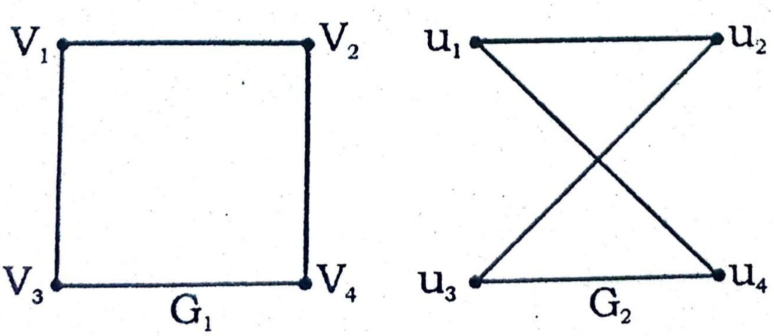
(iii) Draw a graph whose degree sequence is 1,1,1,1,1,1. 2

6. (a) Consider the Graph G given below. Is it Hamiltonian ? Is it Eulerian ? Explain your answers. 6.5



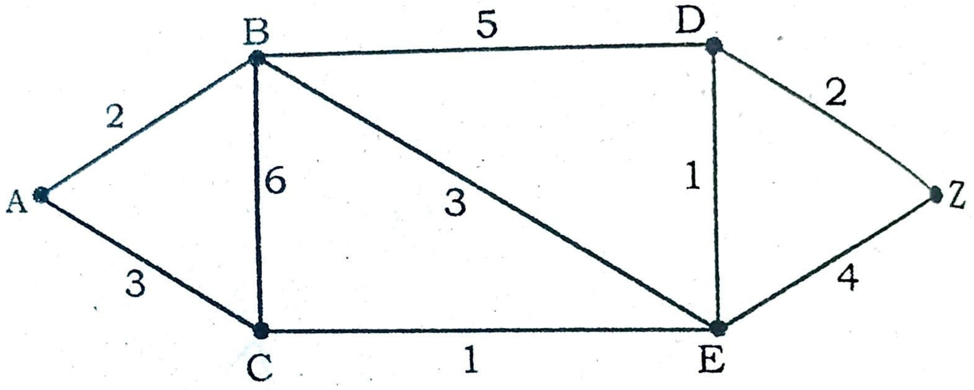
(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.

6.5



P.T.O.

(c) Apply the improved version of Dijkstra's Algorithm to find a shortest path from A to Z. Write steps. 6.5



is question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

No. of Question Paper : **1380**

ique Paper Code : **62357502** **I**

me of the Paper : **Differential Equations**

me of the Course : **B.A. (Prog.) Mathematics : DSE-1**

mester : **V**

ration : **3 Hours** Maximum Marks : **75**

rite your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

(*a*) Solve the initial value problem : 6

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0;\ y(0) = 2.$$

(*b*) Solve $x + py = p^3$. 6

(*c*) Solve $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$. 6

(*a*) Solve $\dfrac{d^4y}{dx^4} - 5\dfrac{d^3y}{dx^3} + 6\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 8y = e^x$. 6.5

(*b*) Solve $(x+1)^2 \dfrac{d^2y}{dx^2} - 3(x+1)\dfrac{dy}{dx} + 4y = x^2$. 6.5

P.T.O.

(c) $$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0.$$ 13

6

(i) Show that e^x, e^{-x} and e^{2x} are linearly independe solutions of the above equation.

(ii) Write the general solution.

3. (a) Using the method of variation of parameters, solve : 6

$$\frac{d^2y}{dx^2} + y = 2 - x.$$

(b) Using the method of undetermined coefficients to fi the general solution of the differential equation. 6

$$\frac{d^2y}{dx^2} - 9y = x + e^{2x}.$$

(c) Given that $y = x$ is a solution of the differential equati

$$x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right).$$

Find a linearly independent solution by reducing the ord and write the general solution.

4. (a) Solve $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

(b) Solve $(yz + z^2)dx - xzdy + xydz = 0$.

(c) Solve $\dfrac{d^2x}{dt^2} - 3x = 4y$, $\dfrac{d^2y}{dt^2} + x + y = 0$.

5. (a) Find the general solution of the differential equation :

$$z(xp - yq) = y^2 - x^2.$$ 6.5

(b) Find the complete integral of the differential equation :

$$p^2 y(1 + x^2) = qx^2.$$ 6.5

(c) (i) Determine the region in which the given equation is elliptic : $xu_{xx} + u_{yy} = x^2$. 2.5

(ii) Find the partial differential equation arising from the following surface : 4

$$z = (x + a)(y + b).$$

6. (a) Find the complete integral of the differential equation :

$$\sqrt{p} + \sqrt{q} = 2x.$$ 6

(b) Eliminate the arbitrary function f from the equation $z = xy + f(x^2 + y^2)$ to find the corresponding partial differential equation. 6

(c) Find the general solution of the partial differential equation : 6

$$(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial z} = 0.$$

[This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1414**

Unique Paper Code : **62357502** I

Name of the Paper : **Differential Equations**

Name of the Course : **B.A. (Prog.) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* the questions by selecting any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$\left(3x^2y^4 + 2xy\right)dx + \left(2x^3y^3 - x^2\right)dy = 0;\ y(1) = 2.$$

(*b*) Solve : 6

$$x = y + a \ln p.$$

(*c*) Solve : 6

$$(y + x + 5)dy - (y - x + 1)dx = 0.$$

P.T.O.

2. (a) Solve : 6.5

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = x.$$

(b) Solve : 6.5

$$(x+3)^2\frac{d^2y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = x.$$

(c) Consider the following differential equation : 6.

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$$

(i) Show that x, x^2 and x^4 are solution of abov differential equation.

(ii) Show that the solutions x, x^2 and x^4 are linearl independent.

(iii) Write the general solution of the above differenti equation.

3. (a) Using the method of variation of parameters to find th general solution of : 6.

$$\frac{d^2y}{dx^2} + y = \tan x.$$

(b) Use the method of undetermined coefficients to find the general solution of the differential equation : 6.5

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=e^x+2\cos 2x.$$

(c) Given that $y = x$ is a solution of the differential equation : 6.5

$$(x^2+1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

Find a linearly independent solution by reducing the order and write the general solution.

4. (a) Solve : 6

$$\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}.$$

(b) Solve : 6

$$(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0.$$

(c) Solve : 6

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0,$$

$$\frac{dy}{dt}+5x+3y=0.$$

5. (*a*) Find the general solution of the differential equation : 6.5

$$x^2 p + y^2 q = (x + y)z.$$

(*b*) Find the complete integral of the differential equation : 6.5

$$(p^2 + q^2)x^2 - qz = 0.$$

(*c*) (*i*) Classify the partial differential equation as elliptic, parabolic or hyperbolic : 2.5

$$4u_{xx} - 4u_{xy} + 5u_{yy} = 0.$$

(*ii*) Eliminate the parameters *a* and *b* from the following equation to find the corresponding partial differential equation : 4

$$z = x + ax^2 y^2 + b.$$

6. (*a*) Find the complete integral of the equation : 6

$$(p + q)(z - xp - yq) = 1.$$

(*b*) Eliminate the arbitrary function *f* from the equation $z = x + y + f(xy)$ to find the corresponding partial differential equation. 6

(*c*) Find the general solution of the partial differential equation : 6

$$y^2 p - xyq = x(z - 2y).$$

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 897 I

Unique Paper Code : 32355101

Name of the Paper : Calculus

Name of the Course : Mathematics : G.E. for Honours

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION 1

1. Given $f(x)=2x-2,\ x_o=-2,\ \varepsilon=0.02$. Find $L=\lim_{x\to x_o} f(x)$.

   Then find a number $\delta>0$ such that for all $x,\ 0<|x-x_o|<\delta \Rightarrow |f(x)-L|<\varepsilon$.

2. Find a linearization of $f(x)=\sqrt{x^2+9}$ at $x=-4$.

3. The radius of a circle is increased from 2.00 to 2.02 m. Estimate the resulting change in area. Also express th estimate as a percentage of the circle's original area.

4. Evaluate the limit $\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$.

5. Use L'Hôpital's rule to find $\lim_{x \to 0^+} \frac{\ln(e^x - 1)}{\ln x}$.

6. Sketch the graph of a function $f(x) = x^3 - 3x + 1$.

7. Find the volume of the solid generated by revolving th region between the y-axis and curve $x = 2/y$, $1 \le y \le 4$, abo the y-axis.

SECTION 2

8. Use the shell method to find the volume of the solid generat when the region R in the first quadrant enclosed betwe $y = x$ and $y = x^2$ is revolved about the y-axis.

9. Sketch the graph of $r = 1 - 2\cos\theta$ and identify its symmetri

10. Find the area of the surface generated by revolving t curve $y = \sqrt{x}$, $0 \le x \le 1$, about the x-axis.

. Suppose a person on a hang glider is spiraling upward due to rapidly rising air on a path having acceleration vector $a(t) = -3\cos t\, i - 3\sin t\, j + 2k$. It is also known that initially (at time $t = 0$), the glider departed from the point $(3,0,0)$ with velocity $v(0) = 3j$. Find the glider's position as a function of t.

. Find the unit tangent vector of the curve

$$r(t) = t^2 i + 2\cos t\, j + 2\sin t k.$$

. Determine whether $\int_{-\infty}^{-1} \frac{1}{x} dx$ converges?

. Find the arc length parameterization of the helix

$$r(t) = \cos 4t\; i + \sin 4t\; j + 3t\, k, \quad 0 \le t \le 2\pi.$$

SECTION 3

. Show that the ellipse $x = a\cos t$, $y = b\sin t$, $a > b > 0$, has its largest curvature on its major axis and its smallest curvature on its minor axis.

. Find the binormal vector $\vec{B}$ and the torsion function τ for the space curve

$$\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3\hat{k}$$

17. Show that the function

$$f(x, y) = \frac{xy}{|xy|}$$

has no limit as (x, y) approaches $(0, 0)$.

18. If $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v \sin u$, $y = ue^v \cos u$ &

$z = ue^v$, find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using chain rule at the point (u, v)

=

$(-2, 0)$.

19. Find the directions in which the funct $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increase and decrease m rapidly at the point $P_0(1, 1, 1)$. Then find the derivative the function in those directions.

20. Find parametric equations for the line tangent to the cu of intersection of the surfaces

$$x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0 \text{ and } x^3 + y^2 + z^2 = 11$$

at the point $(1, 1, 3)$.

21. Find the absolute maxima and minima of the function

$$T(x, y) = x^2 + xy + y^2 - 6x + 2$$

on the rectangular plate. $0 \le x \le 5$; $-3 \le y \le 0$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1026 **I**

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : **Generic Elective for Hons. : Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Using exactness, solve the following differential equation

$$\cos(x+y)dx+\left(3y^2+2y+\cos(x+y)\right)dy=0. \qquad (6)$$

(b) Solve the initial value problem

$$y'\tan x=2y-8, \quad y\left(\frac{\pi}{2}\right)=0. \qquad (6)$$

P.T.O.

(c) Find the orthogonal trajectories of the given family of curves

$y^2 = 2x^2 + c$. (6)

2. (a) Solve the differential equation :

$y' = Ay - By^2$. (6)

(b) Solve the initial value problem :

$y'' + 0.4y + 9.04y = 0, \quad y(0) = 0, \quad y'(0) = 3$. (6)

(c) Show that the functions e^{-2x}, e^{-x}, e^{x} and e^{2x} form a basis of a differential equation on any interval. (6)

3. (a) Find a homogeneous linear ordinary differential equation for which two functions x^{-3} and $x^{-3}\ln x$ are solutions. Show also their linear independence by considering their Wronskian. (6)

(b) Use the method of Variation of Parameters to find a general solution of the following non-homogeneous ordinary differential equation : (6)

$y'' - 2y' + y = e^{x}\sin x$.

(c) Solve the following differential equation :

$(xD^2 + 4D)y = 0$.

Also find the solution satisfying $y(1)=12,\ \ y'(1)=-6$.

(6)

(a) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$y''+4y'+5y=25x^2+13\sin 2x\ .$$

Also find its general solution. (6.5)

(b) Find the solution of the linear system

$$\frac{dx}{dt}=5x+3y,$$

$$\frac{dy}{dt}=4x+y,$$

that satisfies the initial conditions $x(0)=0, y(0)=8$.

(6.5)

(c) Reduce the equation to canonical form and obtain the general solution :

$$u_x-yu_y=u+1\ .$$ (6.5)

(a) Find a power series solution, in powers of x of the differential equation :

$$y''-y'=0\ .$$ (6.5)

(b) Find the solution of quasi-linear partial differential equation :

$$x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$$

with Cauchy data $u=1$ on $x+y=0$.

(6

(c) Find the general solution of the linear partial different equation :

$$x^2u_x + y^2u_y + z(x+y)u_z = 0.$$

(6

6. (a) Find the solution of the following partial differen equation by the method of separation of variables :

$$u_x - u_y = u, \quad u(x,0) = 4e^{-3x}.$$

(6

(b) Reduce the equation

$$yu_{xx} + 3yu_{xy} + 3u_x = 0, y \neq 0$$

to canonical form and hence find its general soluti

(

(c) Form partial differential equations by

(i) eliminating the arbitrary constants a and b f the relation

$$(x-a)^2 + (y-b)^2 + z^2 = r^2.$$

(ii) eliminating the arbitrary function f from relation

$$z = xy + f(x^2+y^2).$$

(

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1494**

Unique Paper Code : **62355503** **IC**

Name of the Paper : **General Mathematics-I**

Name of the Course : **B.A. (Prog.) Mathematics : G.E.**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions as per directed question wise.

Section I

1. Write short notes on the life and contributions of any *three* of the following mathematicians :

(*a*) Poisson

(*b*) Fourier

(*c*) Euler

(*d*) Lagrange

(*e*) Laplace. 15

P.T.O.

Section II

2. Attempt any *six* questions. Each question carries *five* marks :

(*a*) What do you understand by prime numbers ? Also give the definition of twin primes with examples.

(*b*) Define Goldbach conjectures and Pythagorean Triples with examples.

(*c*) Explain unit fraction and express 25/13 and 9/10 as unit fractions.

(*d*) Find the number of combinations in the word 'NUMBERS' selecting at a time :

(*i*) 2 letters

(*ii*) 6 letters.

(*e*) Explain the Fifteen Puzzle.

(*f*) Define Mersenne Numbers and Mersenne Primes. Give examples.

(*g*) State Prime Testing Method given by Fermat. Is the converse true. Justify your answer.

Section III

3. Do any *three* questions. Each question carries *six* marks :

(*a*) If :

$$A = \begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$$

show that :

$$(AB)^T \neq A^T B^T.$$

(*b*) Decompose the matrix :

$$\begin{pmatrix} 1 & 0 & -4 \\ 3 & 3 & -1 \\ 4 & -1 & 0 \end{pmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

(*c*) Determine whether :

$$A = \begin{pmatrix} 5 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 6 & 3 & 1 \\ 1 & -15 & -5 \\ -2 & -1 & 10 \end{pmatrix}.$$

Commute (i.e. AB = BA) or not.

(*d*) Find the inverse of the matrix :

$$\begin{pmatrix} 2 & -6 & 5 \\ -4 & 12 & -9 \\ 2 & -9 & 8 \end{pmatrix}.$$

4. Do any *two* questions. Each question carries *six* marks :

(*a*) If $A = \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & -1 & -2 \end{pmatrix}$, find determinant of A.

(*b*) If $A = \begin{pmatrix} -1 & 4 & 1 \\ 2 & 0 & 3 \\ -1 & -1 & 2 \end{pmatrix}$, verify that $|A| = |A^T|$.

(*c*) Use Cramer's Rule to solve the following system :

$$-5x + 6y + 2z = -16$$

$$3x - 5y - 3z = 13$$

$$-3x + 3y + z = -11.$$

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : 146 I

Unique Paper Code : 42343306

Name of Paper : Office Automation Tools

Name of Course : B.Sc (Prog.) / B.Sc. Math. Sc. : SEC

Semester : III

Duration : 2 hours

Maximum Marks : 25

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A consists of 10 questions of 1 mark each (MCQ). All questions are compulsory. In Section B answer any three questions.

SECTION A

1. (a) Which feature allows you to see how next slide appears after the previous one in Powerpoint presentation?

(i) Slide Show

(ii) Slide Transition

(iii) Slide Animation

(iv) Slide View

(b) Which representation is **not** available to represent negative numbers in Binary arithmetic?

(i) 1's complement

(ii) 2's complement

(iii) 3's complement

(iv) Signed Magnitude

(c) $(1011)_2$ is a number in base:

(i) 2

(ii) 8

(iii) 10

(iv) 16

(d) Which font is **not** a valid font in Word?

(i) Calibri

(ii) Arial

(iii) Comic Sans

(iv) Vfont

(e) Which type of charts are **not** available in MS Excel?

(i) Bar Chart

(ii) Histogram

(iii) Pie Chart

(iv) Line Chart.

(f) Ctrl+Home takes you to:

(i) Beginning of page

(ii) Cell A1

(iii) Cell 1A

(iv) Beginning of row.

(g) Short cut key for inserting a new slide is:

(i) Ctrl+M

(ii) Ctrl+N

(iii) Ctrl+S

(iv) Ctrl+P

(h) Which button allows you to add merge fields in Word documents?

(i) Merge to PDF

(ii) Insert Merge Field

(iii) Preview Results

(iv) Finish and Merge.

(i) Which function is **not** available in Spreadsheet?

(i) IF

(ii) SumIF

(iii) CountIF

(iv) ListIF

(j) Cell after Z1 in the same row is:

(i) Z2

(ii) AA1

(iii) ZA1

(iv) Z11

SECTION B

2\. (a) What are the various alignments available in Word? 2

(b) What feature would you use to write x^2 in Word? 1

(c) Write 2 ways to create a table in Word. 2

3. (a) Write statement in Excel using IF function to find larger of 3 numbers A, B and C. 2

(b) What is Pivot Table in Excel? 1

(c) How do you embed an Excel Worksheet in a Word file? 2

4. (a) What is use of Powerpoint presentation? 1

(b) Differentiate between animation and transition. 2

(c) What are various views available in Powerpoint? 2

5. (a) Write hexadecimal equivalent of 26. 1

(b) Add $(11101)_2$ and $(100111)_2$. 2

(c) Convert $(56{\cdot}2)_{10}$ to $(?)_2$ 2

6. (a) What are header and footer in Word?

(b) Give the use of Count function in Excel. 1

(c) What do you mean by slideshow? 1

(d) Subtract 5 from 10 using 1's complement in 8 bits. 2

This question paper contains 4 printed pages.

Your Roll No. ....

. of Paper	: 255	I
ue Paper Code	: 42353327	
e of the Paper	: **Mathematical Typesetting System: LaTeX**	
e of the Course	: **B.Sc. (Prog.) : SEC**	
ster	: **III**	
tion	: **2 hours**	
mum Marks	: **38**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

. Fill in the blanks in any *four* parts of the following:

4×0.5=2

(i) The preamble in a LaTeX document begins with a ------------------ command whose argument is predefined article class.

(ii) The title page of a LaTeX document is generated by a -------------- command which comes ---------- the \begin{document}.

(iii) ---------- command is used to produce $\mathbb{Z}$ in math mode of LaTeX document.

(iv) The ---------- command typesets its a
in LR mode and then produces its mirro

(v) In LaTeX, the command ----------
centered ellipsis.

2. Answer any *eight* parts from the following:

(i) Write the output of the command:

\(f(x)\stackrel{def}{=}x^2-1

(ii) Write the code in LaTeX to o

expression:

$$f(x)=\begin{cases}0 & \text{for } x=0;\\ x\sin\left(\frac{1}{x}\right) & \text{for } x\neq 0.\end{cases}$$

(iii) Write two differences between LaT

and PSTricks picture environment.

(iv) Write the command to draw an arro

of length 15 units in the direction of (

(v) Use \graphpaper command to get a

left corner at (-2, 3) width 50 and

with one line every 8 units.

(vi) Write the code in LaTeX to produce

If $|x-y|<\delta,\ \forall\ \delta>0$, then $x =$

(vii) Typeset the following in LaTeX:

$$\tan\left(\frac{\pi}{4}+\theta\right) = \frac{\tan(\pi/4)+\tan\theta}{\tan(\pi/4)-\tan\theta}$$

$$= \frac{1+\tan\theta}{1-\tan\theta}$$

(viii) Write the following postfix expression in standard form:

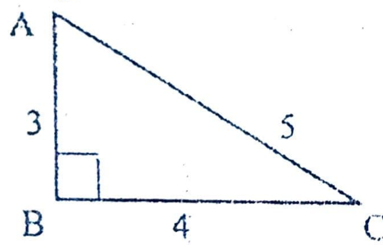
x x 2 exp 1 sub mul x 2 exp 1 add div

(ix) Give a command to draw an arc of a circle of radius 2 units centered at the point (2, 2), making an angle of 30 degree.

3. Answer any *three* parts from the following:

4+4+4=12

(a) In the picture environment make a 3 – 4 – 5 Pythagorean triangle:



(b) Write the code in LaTeX using delimiters or otherwise:

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

(c) Write the code for the following in LaTeX environment:

$$F_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n}\right)^{1/r}, \quad r \in \mathbb{R}\backslash\{0\},$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in (\mathbb{R})^n$.

P. T. O.

(d) Plot the oscillating function $y = \sin(1/x)$.

4. Write a presentation containing in beamer with the following content:

Slide-1: Title: There is No Largest Prime Number,
Author: ABC

Slide-2: Proof. We shall prove the result in four steps:

Step-1. Suppose the number of primes is finite.

Slide-3: **Step-2.** Let p be the number of all primes

Slide-4: **Step-3.** Then $p + 1$ is *not* divisible by any prime.

Slide-5: **Step-4.** Therefore, $p + 1$ is also a prime, a contradiction.

8

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1346**

Unique Paper Code : **62353325** **I**

Name of the Paper : **Latex and HTML**

Name of the Course : **B.A. (Prog.) : Mathematics—SEC**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks (any *four*) : 4×½=2

(*i*) .... tells LaTeX to start a new paragraph.

(*ii*) .... command adds name of the author to a LaTeX document.

(*iii*) Matrices can be created using .... environment in LaTeX.

(*iv*) Enumerated list are created using .... element in HTML.

P.T.O.

(v) The .... element is used to include images to a web page.

2. Answer any *eight* parts from the following : 8×2=16

(i) Describe *three* different ways in LaTeX to write in math mode.

(ii) Write the LaTeX command for the symbols :

α, π, Σ, $\geq$, ∞.

(iii) What is the output of the command :

`$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$`

(iv) Write the LaTeX command to typeset :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

(v) Correct the following input :

⟨*img* "smiley.gif" *alt* = *smileyface height* = 42 *width* = 42⟩

(vi) What are delimiters ? Explain with an example.

(vii) Write the output of the command :

\pswedge(2,2){1.5}{0}{60}.

(*viii*) Correct the LaTeX code :

(\Frac{a+b}c+d)^1/3.

(*ix*) Name the basic elements needed to create a simple web page.

(*x*) Write the postfix notation in standard form :

x sin 1 add 2 exp 1 x sub div.

3. Answer any *five* questions from the following : 5×4=20

(*i*) Draw an ellipse with a shaded sector.

(*ii*) Write LaTeX code to typeset the following :

Let $x = (x_1, x_2, \ldots\ldots, x_n)$ where x_i are non-negative real numbers. Set :

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \ldots\ldots + x_n^r}{n}\right)^{1/r}, \quad r \in R\backslash\{0\}$$

and

$$M_0(x) = (x_1 x_2 \ldots\ldots x_n)^{1/n}.$$

(*iii*) Find errors in the following code and write the corrected version and its output :

```
\Documentclass{article}

\begin{document}

\begin{enumerate}

\item Suppose that x = 137.

\item If theta = pi, then sin theta = 0.

\item The curve y = sqrt{x}, where x >= 0, is concave
downward.

\end{document}
```

(*iv*) Write a code in LaTeX to typeset the following :

Define,

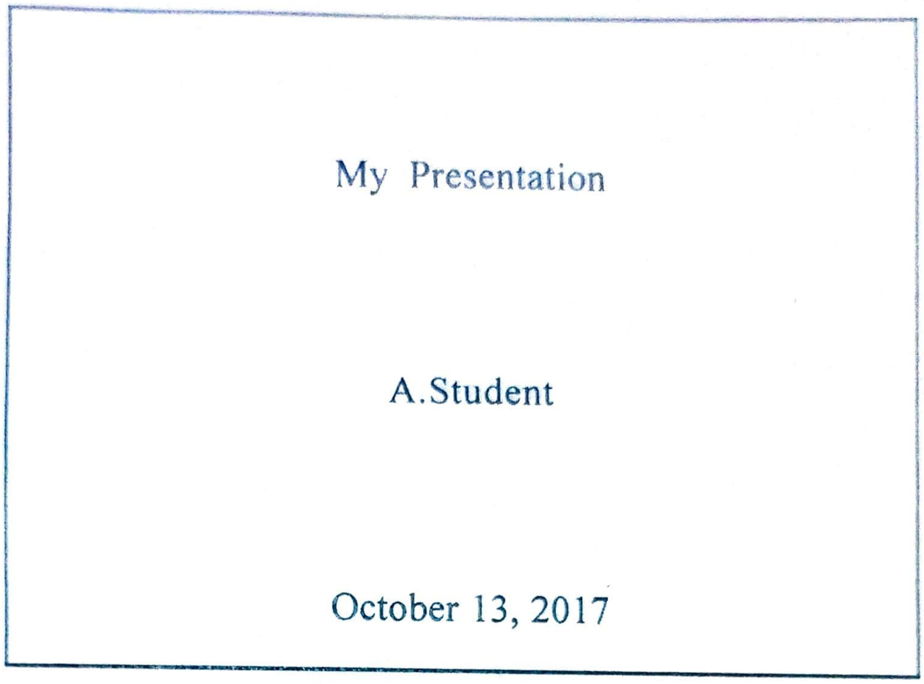
$$F_j(u) = \lim_{t \to \infty} F_j(t, u), \qquad j = 1, 2, \ldots\ldots t, x \geq 0$$

Then the *Kolmogorov forward equation* is given by,

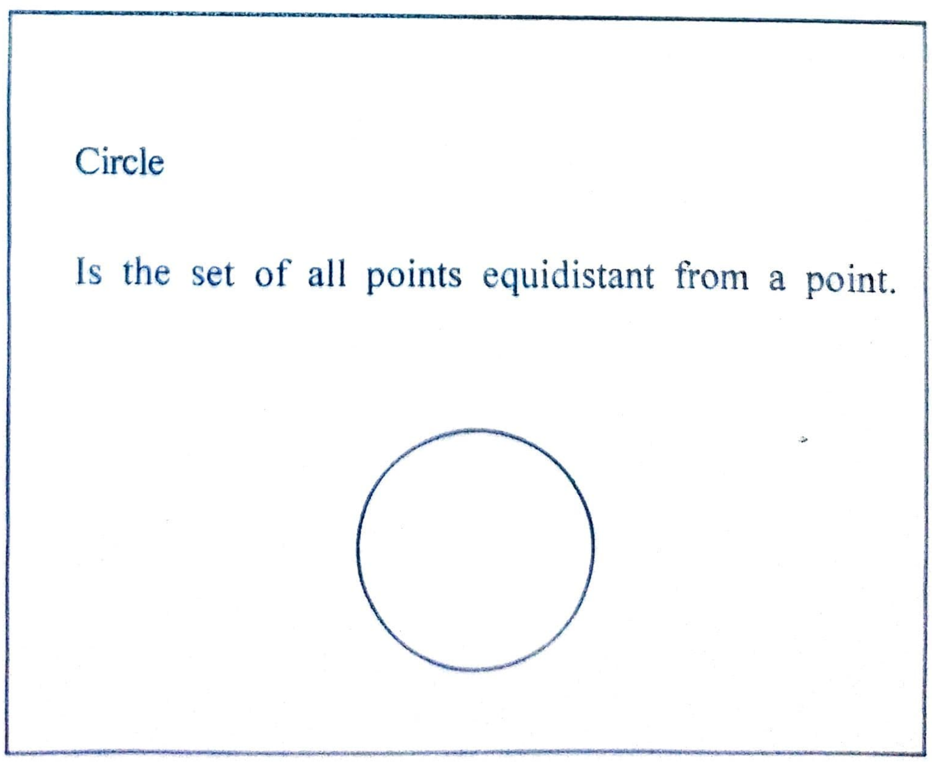
$$r_j \frac{dF_j(u)}{du} = \lambda_{j-1} F_{j-1}(u) - (\lambda_j + \mu_j) F_j(u) + \mu_{j+1} F_{j+1}(u). \quad (1)$$

(v) Create the following presentation in LaTeX :

Slide 1



Slide 2



(*vi*) Write an HTML code to generate the following web page :

UNIVERSITY OF DELHI • Skill Enhancement Courses (SEC) : 1. Sec-1 : LaTeX and HTML 2. Sec-2 Computer Algebra Systems • Discipline Specific Elective (DES) : 1. DSE-1 : Numerical Methods 2. DSE-2 : Discrete Mathematics

Note : Hyperlink one of the papers to a pdf document.

This question paper contains 2 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : **148**

I

Unique Paper Code : **42343501**

Name of Paper : **SEC-3 : System Administration and Maintenance**

Name of Course : **B.Sc. (Program) Mathematical Science : SEC**

Semester : **V**

Duration : **2 hours**

Maximum Marks : **25**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory. Attempt any three questions from Q. Nos. 2 to Q. 6.

1. (a) List any *two* services provided by an Operating System. Explain how each provides convenience to the user. 2

(b) Explain *three* different uses of cat command. 3

(c) What is the difference between the commands cd. and cd.. ? Explain with suitable examples. 2

(d) List any *two* control panel tools in Windows OS. 1

(e) What is the function of Synaptic Package Manager? 1

(f) What is the function of ipconfig command? 1

P. T. O.

2. (a) Compare the features of Windows and Linux OS. 3

   (b) Explain all components of the output given by 1S–1 command. 2

3. (a) Compare features of Windows7 and Windows XP Operating System. 3

   (b) Draw and explain the architecture of Linux Operating System. 2

4. List the function of each of the following commands:

   (a) kill

   (b) echo

   (d) ping

   (d) traceroute

   (e) netstat 5

5. (a) What is the difference between Kernel space and User space? Explain the dual mode operation of an Operating System. 3

   (b) Differentiate between Homegroup network type and Domain network type. 2

6. (a) Explain any *two* file systems supported in Windows 7. 3

   (b) What is an Active Directory? Explain the purpose it serves. 2

This question paper contains 5 printed pages.

Your Roll No. ....

S. No. of Paper	: **150**
Unique Paper Code	: **42353503**
Name of the Paper	: **Statistical Software R**
Name of the Course	: **B.Sc. (Math. Sc.) / B.Sc. (Prog.) : SEC**
Semester	: **V**
Duration	: **2 hours**
Maximum Marks	: **38**

I

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

All commands should be written using language R.

1. Do any *four* of the following: 1×4

State whether the following statements are true or false:

(i) R follows the BODMAS rule for the calculation of mathematical expressions.

(ii) c( ) command is easier than scan( ) command.

(iii) rm( ) is used to find the variables defined.

(vi) getwd( ) and setwd( ) are same commands.

(v) sort( ) command can perform on an entire data frame.

2. Do any *six* of the following: 1×6

Fill in the blanks:

(i) table( ) command shows the .... of the data. (frequency/density)

(ii) How many columns are present in a basis stem and leaf plot? (two/three)

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(iii) .... command is used to make bar charts. (boxplot( ) / barplot( ))

(iv) .... command is used to generate a sequence of 10 random numbers. (seq(10) / rseq(10))

(v) names( ) command is used for viewing .... (rows/columns)

(vi) To generate ten Poisson distributions with mean lemda=1, we use command:

(rpois(10,lemda=1), qpois(1,lemda=10)).

(vii) $ command is used for .... (copy a data, extract from a data).

3. Do the following questions: 2x8

(a) Write commands for the following:

(i) To remove all the variables beginning with 'e' defined.

(ii) To save the variables a=3, b=10 and c=5 in a different file.

(b) Write command to compute:

(i) $\frac{2+100}{5+e}$

(ii) $\tan^{-1}(1)$ in degree.

(c) Write the difference between lapply and sapply.

(d) Create scatter plot for two dimensional data with *one* example.

(e) Consider a matr_x X:

	Q1	Q2	Q3	Q4
R1	Jan	Apr	Jul	Oct
R2	Feb	May	Aug	Nov
R3	Mar	Jun	Sep	Dec

(i) Write command to change the names of rows with a, b, c and names of columns with A, B, C, D respectively.

(ii) Print all items of 2nd column.

(f) Rearrange the data in increasing order and draw a stem and leaf plot where data are:

X=3, 5, 7, 5, 3, 2, 6, 8, 5, 6, 9

(g) Make a score data file:

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Find the range, mean, median, standard deviations.

(h) By using data1 = 3, 5, 7, 6, 9, 2, 7, 1, write a sequence of items of data1 with:

(i) only even positioned items.

(ii) only odd positioned items.

4. Do any *four* of the following: 3×4

(a) Write the commands for the following:

(i) How to make a comment in R?

(ii) Create a vector

y: 12, 7. 5, 3, 4.2, 18, −21, NA, 6, NA.

(iii) Find the length of vector y.

(iv) Find mean of vector y by dropping NA values.

(v) Find the quartile of vector y.

(b) Consider the matrix:

>Marks

	Physics	Chemistry	Maths
Jim	73	84	82
Sui	75	68	58
Andy	90	85	73
Jojo	69	63	71
Pi	81	84	73

(i) Find the mean of the third column of Marks.
(ii) Find the median of all columns of Marks.
(iii) Find the column means of Marks.
(iv) Create a table of matrix Marks.
(v) How can you make a scatter plot of Physics *versus* Maths and display a line of best-fit?

(c) Make a dataframe file:

81	81	96
95	98	73
92	79	82
80	86	89
79	62	NA

Then convert this data into a matrix.

(d) If a data2 file is given as:

data2=3, 5, 8, 7, 9, 6, 8, 6, 3, 5, 4, 7, 3, 6, 2,

Which test would you apply to compare this sample to normal distribution? Also write command.

(e) Write a program in R for the following:

(i) Consider the given data:

x	5	6	13	4	12	10	16	5
y	4	4	16	18	19	12	16	20

(ii) Draw a scatter plot of data points $(x,\ y)$.

(iii) Find correlation between and x and y.

(iv) Compute a line of best fit for the data.

(v) Add the line of best fit to the scatter plot.

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : **271** **I**

Unique Paper Code : **42163512**

Name of Paper : **Ethnobotany**

Name of Course : **B.Sc. (Prog.) Botany : SEC**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions and all of their parts together.

1. (a) Define the following terms (any *five*): 1×5=5

(i) TKDL

(ii) Herbarium

(iii) Endangered taxa

(iv) Archaeoethnobotany

(v) IPR

(vi) Psychotropic drugs.

(b) Match the terms given in Column A with those in Column B: 1×5=5

Column A	Column B
(i) Indian ginseng	*Gloriosa superba*
(ii) Bio-pesticide	*Indigofera tinctoria*
(iii) Suicidal agent	*Vitex negundo*
(iv) Dye	*Withania somnifera*
(v) Chinese chastetree	*Azadirachta indica*

2. Write botanical name, family, part used and ethnobotanical use of any *four*: 2×4=8

(a) Sarpgandha

(b) Madagascar periwinkle

(c) Sweet wormwood

(d) Indian Beech

(e) Holy basil.

3. (a) Write short notes on any *two*: 2.5×2=5

(i) Forest management

(ii) Minor ethnic groups in India

(iii) *Tribulus Terrestris.*

(b) How can endangered taxa be conserved through forest management practices? 2

4. (a) Discuss how ethnobotanical knowledge can help in conservation of genetic resources. 4

(b) How are traditional medicines superior over modern medicines? 3

5. (a) Temples and sacred places serve as the source of ethnobotanical knowledge. Justify with the help of suitable examples. 4

(b) Mention different approaches used in ethnobotanical studies. 2

This question paper contains 6 printed pages.

Your Roll No. ....

I

S. No. of Paper	: 752
Unique Paper Code	: 32357501
Name of the Paper	: **Numerical Methods**
Name of the Course	: **B.Sc. (H) Mathematics : DSE-2**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions, selecting **two** *parts from each question. Use of non-programmable scientific calculator is allowed.*

1. (a) Given the following scheme for integration:

$$\int_a^b f(x)dx \approx \frac{h}{2} + [f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)],$$

write an algorithm to obtain the approximate value of the definite integral.

(b) Verify that the equation $x^5 - 2x - 1 = 0$ has a root in the interval (0, 1). Perform three iterations to approximate the zero of the equation by the Secant method using $p_0 = 0$ and $p_1 = 1$.

(c) Let f be a continuous function on the interval $[a, b]$ and suppose that $f(a)f($

Prove that the bisection method gene sequence of approximations $\{p_n\}$ converges to a root $p \in (a, b)$ with the pr

$$|p_n - p| \leq \frac{b-a}{2^n}.$$

Hence, find the rate of the convergence in method.

2. (a) Give the geometrical construction of the n of False Position to approximate the zer function. Further, write the algorithm f computation of the root approximated b method.

(b) Perform three iterations for finding the root (

$$f(x) = \frac{1}{x} - 37$$

by Newton's method starting with $p_0 =$ Further, compute the ratio

$$|p_3 - p|/|p_2 - p|^2$$

and show that this value approaches $|f''$ $2f'(p)|$, with $p = 1/37$.

(c) Let g be a continuous function on the closed interval $[a, b]$ with $g:[a,b]\to[a,b]$. And suppose that g' is continuous on the open interval (a,b) with $|g'(x)|\le k<1$ for all x belongs to (a, b). If $g'(p)\neq 0$, then prove that for any $p_0\in[a,b]$, the sequence $p_n=g(p_{n-1})$ converges only linearly to the fixed point p. 13

3.(a) Using LU decomposition, solve the system of equations $Ax=b$, where:

$$A=\begin{bmatrix}2 & 7 & 5\\ 6 & 20 & 10\\ 4 & 3 & 0\end{bmatrix} \text{ and } b=\begin{bmatrix}-3\\ -12\\ 6\end{bmatrix}.$$

(b) Use the SOR method with $\omega=0.9$ to solve the following system of equations:

$$2x_1-x_2=-1$$

$$-x_1+4x_2+2x_3=3$$

$$2x_2+6x_3=5$$

Use $x^{(0)}=0$ and perform three iterations

(c) (i) Compute the iteration matrix T_{gs} of the Gauss-Seidel method for obtaining the approximate solution of the system of equations $Ax=b$ where A is given as:

P. T. O.

$$\begin{bmatrix} 3 & 2 & - \\ -2 & -2 & 1 \\ 5 & -5 & 4 \end{bmatrix}$$

(ii) Determine the spectral radi

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix}.$$

4. (a) Let $x_0, x_1, x_2, \ldots, x_n$ be $n+1$ [a, b]. If f is continuous on continuous derivatives on $(a, b$ there exists $\xi \in (a, b)$ such that:

$$f[x_0, x_1, x_2, \ldots, x_n] = \frac{f^{n}}{n}$$

(b) Experimentally determined valu pressure of water vapor, p_A, a distance y, from the surface of are given below. Estimate the pa distance 2.1 mm from the surface

y (mm)	0	1	2	3
p_A (atm)	0.10	0.065	0.042	0.029

(c) (i) Define an interpolating polynomi set of data $(x_i, f(x_i))$, i Construct the Lagrange polynom through the points $(1, e)$, $(2, e^2)$ a

(ii) Define the backward difference operator and the central operator. Prove that:

$$\delta = \nabla(1-\nabla)^{-1/2}.$$ 12

5. (a) Derive the formula:

$$f''(x_0) \approx \frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2},$$

the second-order central difference approximation to the second order derivative of a function.

(b) Verify that:

$$f'(x) \approx \frac{f(x_0+h)-f(x_0-h)}{2h},$$

the difference approximation for the first order derivative provides the exact value of the derivative regardless of h, for the functions $f(x)=1, f(x)=x$ and $f(x)=x^2$, but not for the function $f(x)=x^3$.

(c) Use the formula:

$$f'(x) \approx \frac{f(x_0)-f(x_0-h)}{h}$$

to approximate the derivative of the function $f(x)=e^x$ at $x_0=0$, taking $h=1, 0.1, 0.01$ and 0.001. What is the order of approximation? 12

P. T. O.

6\. (a) Using Trapezoidal rule approximate the the integral:

$$\int_0^2 \tan^{-1} x \, dx .$$

Further verify the theoretical error boun

(b) Derive the closed Newton-Cotes rule (n the computation of the definite integral:

$$\int_a^b f(x) dx.$$

(c) Apply Euler's method to approximate the of the given initial value problem:

$$x' = \frac{1 + x^2}{t}, (1 \leq t \leq 4), x(1) = 0,$$

Further it is given that the exact solution is:

$$x(t) = \tan(\ln(t)).$$

Compute the absolute error at each step.