[This question paper contains 5 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7462** **J**

Unique Paper Code : 32351102 - OC

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Algebra

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Attempt any **two** parts from each questions.

(iii) **All** questions are compulsory.

1. (a) Find the polar representation for the complex number 6

$$z = 1 - \text{Cos}a + i\ \text{Sin}a, \quad a \in [0, 2\pi)$$

(b) Solve the equation $(2 - 3i)z^6 + 1 + 5i = 0$. 6

(c) Compute $z^n + \frac{1}{z^n}$, if $z + \frac{1}{z} = \sqrt{3}$. 6

P.T.O.

2. (a) Define ~ on $\mathbb{Z}$ by a~b if and only if $2a + 3b = 5n$ for some integer n. Prove that ~ defines an equivalence relation on $\mathbb{Z}$. 6

(b) Define $f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 3x^3 - x$.

(i) Is f one-to-one ?

(ii) Is f onto ?

Justify each answer. 6

(c) Show that the open intervals (0, 1) and (1, 2) have the same cardinality. 6

3. (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that $17369x + 5472y = 1$. 6

(b) (i) Show that $3^6 \equiv 1 \pmod{7}$ and hence evaluate $3^{60} \pmod{7}$.

(ii) Find all integers $x \pmod{12}$ that satisfy $9x \equiv 3 \pmod{12}$. 6

(c) Use the Principle of Mathematical Induction to prove $2^{2n} - 1$ is divisible by 3, $\forall n \geq 1$. 6

4. (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_2 - 6x_3 = -3$$

(b) Let $A=\begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$. Show that the equation $Ax = b$ may not be consistent for every $b=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Also describe the set of all vectors b for which $Ax = b$ is consistent. 6.5

(c) Determine h and k such that the solution set of the given system 6.5

$x_1 + 3x_2 = k$

$4x_1 + h\,x_2 = 8$

(i) is empty.

(ii) contains a unique solution.

(iii) contains infinitely many solutions.

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is $B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$.

Balance the chemical equation using the vector equation approach. 6.5

P.T.O.

(b) Find the value of h for which the following vectors are linearly dependent. Also find a linear dependence relation among them. 6.5

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$$

(c) A linear transformation $T : R^2 \rightarrow R^2$ first performs a vertical shear that maps e_1 into $e_2 - 2e_1$, leaves the vector e_2 unchanged and then reflects point through the line $x_2 = x_1$

(i) Find Matrix A such that $T(x) = Ax$, $x \in R^2$.

(ii) Find x such that $T(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. 6.5

6. (a) Given :

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

(i) Show that the matrix A is row equivalent to I_3.

(ii) Find inverse of A and hence find inverse of A^T. 6.5

(b) Find a basis for column space for the matrix A 6.5

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$$

(c) Is $\lambda = 4$ an eigen value of the matrix A ?

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

If so, find eigen space of A corresponding to eigen value $\lambda = 4$. 6.5

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8597**

Unique Paper Code : **32351101** **J**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

Attempt any *four* questions from Section I.

1. State Leibnitz's theorem for finding *n*th derivative of product of two functions. If $y = a \cos(\ln x) + b \sin(\ln x)$, prove that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$.

P.T.O.

2. Evaluate the following limit :

$$\lim_{x \to 0^+} x^{\sin x}.$$

3. Find the intervals of increase and decrease of the following function, discuss its concavity and then sketch its graph $y = (x+1)^2 (x-5)$.

4. Sketch the graph of the polar curve $r = 3\cos 2\theta$.

5. A manufacturer estimates that when 'x' units of a particular commodity are produced each month, the total cost (in dollars) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to the maximum profit.

Section II

Attempt any *four* questions from Section II.

6. Find a reduction formula for $\int \text{cosec}^n x\, dx$, $n \geq 2$ is an integer. Evaluate $\int \text{cosec}^4 x\, dx$.

7. Find the volume of the solid generated when the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the x-axis.

8. The base of a certain solid is enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross-section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

9. Find the arc length of the parametric curve :

$$x = (1 + t)^2,\ y = (1 + t)^3,\ 0 \leq t \leq 1.$$

10. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the x-axis.

Section III

Attempt any *three* questions from Section III.

11. Find the equation of the parabola whose focus is $(-1, 4)$ and directrix is $x = 5$.

12. Find the equation of the hyperbola whose foci are $(1, 8)$ and $(1, -12)$ and vertices are 4 units apart.

13. Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

14. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

Section IV

Attempt any *four* questions from Section IV.

15. Evaluate :

$$\lim_{t \to 0^+} \left[\frac{\sin 3t}{\sin 2t}\hat{i} + \frac{\log(\sin t)}{\log(\tan t)}\hat{j} + (t \log t)\hat{k}\right].$$

16. The acceleration of a moving particle is $\vec{A}(t) = 24t^2\hat{i} + 4\hat{j}$. Find the particle's position as a function of t if $\vec{R}(0) = \hat{i} + 2\hat{j}$ and $\vec{v}(0) = 0$.

17. If a shot putter throws a shot from a height of 5 ft with an angle of $46°$ and initial speed of 25 ft/sec, what is the horizontal distance of the throw ?

18. Find $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$ at $t = \frac{\pi}{4}$.

19. Show that the curvature of the polar curve $r = e^{\alpha\theta}$ is inversely proportional to r.

This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : **8617**

Unique Paper Code : **32351102** **J**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. 5

   (*b*) Find all the rational roots of $96y^3 - 16y^2 - 6y + 1 = 0$. 5

   (*c*) (*i*) Find the geometric image of the complex numbers z, such that $|z + i| \geq 2$.

   (*ii*) Find the polar representation of the complex number $z = -4i$ and find $Arg\ z$. 2,3

2. (*a*) Find all complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$. 5

P.T.O.

(*b*) Solve the equation : 5

$$z^4 = 5(z-1)(z^2 - z + 1).$$

(*c*) Show that : 5

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

3. (*a*) For (x, y) and (u, v) in R^2, define $(x, y) \sim (u, v)$ if $x^2 + y^2 = u^2 + v^2$.

Prove that $\sim$ defines an equivalence relation on R^2.

Find equivalence classes of $(1, 0)$ and $(1, 1)$. 6

(*b*) Suppose $f : A \to B$ and $g : B \to C$ are functions :

(*i*) If gof is one-to-one and f is onto, prove that g is one-to-one.

(*ii*) If gof is onto and g is one-to-one, prove that f is onto. 3,3

(c) Prove that the intervals $(0, 1)$ and $(0, \infty)$ have the same cardinality. 6

4. (*a*) (*i*) Suppose a and b are integers and p is a prime such that $p|ab$. Then prove that $p|a$ or $p|b$.

(*ii*) Find the quotient q and the remainder r as defined in division algorithm. If $a = -517$ and $b = 35$. 3½,3

(b) Using Euclid's Algorithm, find integers x, y such that $150x + 284y = 4$. 6½

(c) Using Principle of Mathematical Induction prove that for any $x \in \mathbb{R}$, $x > -1$, $(1+x)^n \geq 1 + nx$, $\forall n \in \mathbb{N}$. 6½

5. (a) Consider the following system of linear equations :

$$x_1 + 3x_2 + x_3 = 1$$
$$-4x_1 - 9x_2 + 2x_3 = -1$$
$$-3x_2 - 6x_3 = -3$$

Write the matrix equation and the vector equation of the above system of equations. Find the general solution in parametric vector form by reducing the augmented matrix to echelon form. 7½

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that :

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$$

(i) Find standard matrix of T.

(ii) Is T one-to-one ? Is T onto ? Justify your answers.

(iii) Find X such that T(X) = (–1, 4, 9). 7½

(c) (i) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find an eigenvector corresponding to an eigenvalue $\lambda = 3$.

(*ii*) Show that if λ is an eigenvalue of A and $p(t) = c_0 + c_1 t + c_2 t^2 + \ldots.. + c_n t^n$, then one eigenvalue of $p(\mathrm{A})$ is $p(\lambda)$. 5,2½

6. (*a*) (*i*) Using homogeneous coordinates, find the 3×3 matrix that produce the following composite transformation : Reflect points through the x-axis, and then rotate 30° about the origin.

(*ii*) Show that $\mathrm{H} = \{(a, b, c) \in \mathrm{R}^3 \mid b = 2a + 3c\}$ is a subspace of R^3. 5,2½

(*b*) Let $\mathrm{S} = \{v_1, v_2, v_3, v_4\}$ where $v_1 = (1, 2, 2)$, $v_2 = (3, 2, 1)$, $v_3 = (11, 10, 7)$, $v_4 = (7, 6, 4)$. Find a basis for the subspace W = span S of R^3. What is dim W ? 7½

(*c*) Compute the rank and nullity of the matrix A. Show that rank A + nullity A = number of columns of A.

$$\mathrm{A} = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -1 & 3 \\ 7 & -8 & 3 \\ 5 & -7 & 0 \end{bmatrix}.$$ 7½

[This question paper has 7 printed pages]

Sr. No. of Question Paper : [redacted] 1298

Unique Paper Code : 203161

Name of the Paper : English Higher Qualifying

Name of the Course : B.A (hons)/B.Sc.(hons.) Mathematics

Semester : I/III

Duration: 3 hours Maximum Marks: 100

Instructions for candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.

SECTION I

1. Read the passage and answer the questions that follow:

Forests are among the top natural resources given to mankind. They provide us with both tangible and intangible resources without which the existence of many living things would be threatened.

To fulfil the rising demands for housing, cultivation among other human needs forests are being destroyed at an alarming rate. By cutting down trees, animals are forced to venture into regions of human habitat endangering not only their lives but the humans safety. A long lasting solution needs to be therefore formulated for a sustainable solution.

Forests have been important since ancient times. Forests covered 60 percent of the earth but with the rising population, extensive areas of forests have been cleared to allow farming, roads, mining, and other activities. In the present day only about 30 percent of forests, cover the earth.

Forests play an important role in improving the quality of the environment. They help to keep the environment healthy, clean, and beautiful. Trees release oxygen in the air and take in carbon dioxide, which if let to accumulate, could turn fatal to the human existence. Trees prevent soil erosion and reduce floods.

Strict measures need to be made to punish those who violate the set rules. Forests are vital for human existence and if nothing is done to conserve them all living things are in danger of extinction.

a. Answer whether the following are true or false: (5*1)

i. Forest is a man made resource.

ii. We cannot live without forests.

iii. Trees give us air to breathe.

iv. The forests make up 60% of the total world area.

v. Trees give us oxygen.

b. Match the words in Column 1 with their meanings in Column 2: (5*1).

	Column 1	Column 2
i.	Ancient	large
ii.	Extensive	risking
iii.	Alarming	old
iv.	Endangering	worrying
v.	Reduce	decrease

c. Answer the following questions in 2 or 3 sentences: (4*3)

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AI DUAL CAMERA

2020/10/8 23:31

i. What do forests provide us with?

ii. How do forests increase the quality of the environment?

iii. Why do wild animals move towards human habitat?

iv. Why are forests being cut?

2. Read the following passage and answer the questions that follow: (4*2)

Vijaya: Oh, Mr. Roy! I didn't hear you come in. That was Mrs Mukherjee.

Roy: So I heard. She's a good lady, but she always calls at the wrong time.

Vijaya, we have a problem. Damodar says my car won't start........the other cars are out, and we have to meet Dr. Dass at the airport.

Vijaya: Shall I call a taxi?

Roy: No, Vijaya. Call Mr. Patil. After all, he did offer to help.

i. Where is Mrs. Mukherjee calling from?

ii. Why is the car not working?

iii. Who is Roy meeting at the airport?

iv. Why is Patil interested in helping Mr. Roy?

3. Answer any four of the following questions in about 40-60 words each: (4*3)

i. Why was David sent to Trivandrum by Mr. Roy?

ii. How do Shiva and David Blake know each other?

iii. What role does Vayu play in the text *The Tiger's Eye*?

iv. Mrs Mukherjee is the recipient of a special award? What is it?

v. What is the role of Gurusamy in the play?

SECTION II

4. Change the following into reported speech: (4*2)

REDMI NOTE 7S
AI DUAL CAMERA
2020/10/8 23:31

i. He said, " I've lived here for a long time."

ii. Sheela said to her teacher, " May I go out now."

iii. David said to the guide, "Do you speak English?"

iv. He said to the Policeman, " I was playing football when the accident occurred."

5. Complete the dialogues: (5*2)

Rama : Where are you going this summer?

Shyam: ______________________ to Paris.

Rama: Oh that's wonderful. ______________________?

Shyam: I plan to be there for 10 days.

Rama: Who else is going with you.

Shyam: ______________________

Rama: Have you made a list of places you plan to visit?

Shyam: ______________________

Rama: That sounds fun. Hope you have a great time

Shyam: ______________________

SECTION III

Fill in the blanks with the [illegible] form of the verbs given.

i. Maya ________ for you for over an hour.(wait)

ii. It is not worth ____________ so much money for this play. (pay).

iii. When I reached the airport, the entry gates ________ (close).

iv. I ________ Jaipur last month. (visit).

v. The criminal ________ the victim with a blunt object. (attack).

EDMI NOTE 7S
DUAL CAMERA
2020/10/8

vi. His company is greatly ______________ after.(seek)

vii. His courage ______________ him (forsake).

viii. The terrified people ______________ to the mountains. (flee).

7. Fill in the blanks with the correct article (a. an or the): (9*1)

i. Gold is .... precious metal.

ii. Honest men speak .... truth.

iii. He looks as stupid as .... owl.

iv. I would like to meet Brad Pit; .... actor.

v. What did you do with .... camera I lent you?

vi. He is going out with .... German girl.

vii. He remained .... bachelor all his life.

viii. 'What is that noise?' 'I think it is .... helicopter.'

ix. The poor man fell asleep on ______________ a tree.

8. Fill in the blanks with the contracted form of the underlined words: (8*1)

i. You should not talk so much.

ii. They have written the text.

iii. Let us go home.

iv. He did not play cards.

v. I could not find my pen.

vi. Here is your book.

vii. I would ask him.

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2020/

viii. Who is this girl?

9. Fill in the blanks with the correct preposition given: (8*1)

1. I slept .... nine o'clock.

to

till

until

2. I commenced work .... 1st May.

since

from

for

3. We walked .... the end of the street.

till

to

for

4. The child has been missing .... yesterday.

since

from

for

5. He traveled seventy miles .... two hours.

for

in

by

6. I received this message......................... 7 o'clock .... the morning.

in, at

at, at

at, in

7. I saw him felling a big tree .... a hatchet.

by

6

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with

off

8. An old feud existed .... the two families.

between

among

within

10. Correct the following sentences(any five): (5*2)

i. It is raining when I got home last night.

ii. My sister is annoying today, but usually she is nice.

iii. I have not ate anything today.

iv. If I am a child, I would play outside.

v. Everyone have seen that movie.

vi. If we will be late, they will be angry.

vii. My father is thinking that I should stop smoking.

viii. Look! It is snow.

[This question paper contains 8 printed pages.]

(5)

Your Roll No..............

Sr. No. of Question Paper : 7279 **J**

Unique Paper Code : 32353301

Name of the Paper : Latex and HTML

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : III

Duration : 2 Hours Maximum Marks : 38

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

1. Fill in the blanks **(Any 4)** : $(4 \times \frac{1}{2} = 2)$

   (i) To create a hyperlink in HTML .... element is used.

   (ii) LaTeX is a .... language.

   (iii) The command .... draws a circle with center (2,2) and radius 1.

P.T.O.

(iv) Boldface text on a webpage is obtained with the .... element.

(v) The command to produce name of institute in a beamer presentation is ....

2. Answer any **eight** parts from the following : (8×2=16)

(i) Describe three different ways in LaTeX to write in math mode.

(ii) What is wrong with the following input:

$\$theta = pi\$$, then $\$ \sin theta = 0\$$.

(iii) What is the output of the following command :

\[\left(\frac{a+b}{x+y}\right)^{1/3}\]

(iv) Make the following equation in LaTex:

$$R_\theta = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

(v) Give any two attributes of the img tag in HTML.

(vi) Typeset a code in LaTeX for the following :

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(vii) Give the output of the command

```
\psarc(1,1){3}{0}{50}
```

(viii) Write a LaTeX code to produce $p^q + q^p + z^z$ as the output.

(ix) Write the output of the following HTML code :

```
<h3> Ordered list with Arabic numerals </h3>
<ol type = "1">
<li> Analysis </li>
<li> Algebra </li>
</ol>
```

(x) Write the postfix notation in standard form: x sin 1 add 2 exp 1 x sub div.

3. Answer any **five** parts from the following : (5×4=20)

(i) Write a code in LaTeX for typesetting the following expression:

$$A_n = \begin{bmatrix} n & n^2 & n^3 \\ 3 & 9 & 27 \\ 4 & 16 & 64 \\ 11 & 121 & 1331 \end{bmatrix}$$

(ii) Find the errors in the following LaTeX source, write a corrected version and write its output :

```
\documentclass{article}

\usepackage{amsmath}

\title{My Document}

\author{ABC}

\date{today}

\maketitle

\begin{document}

\[ lim_{n \rightarrow infinity} \frac{sin 2 x}{x} \]

\end{document}
```

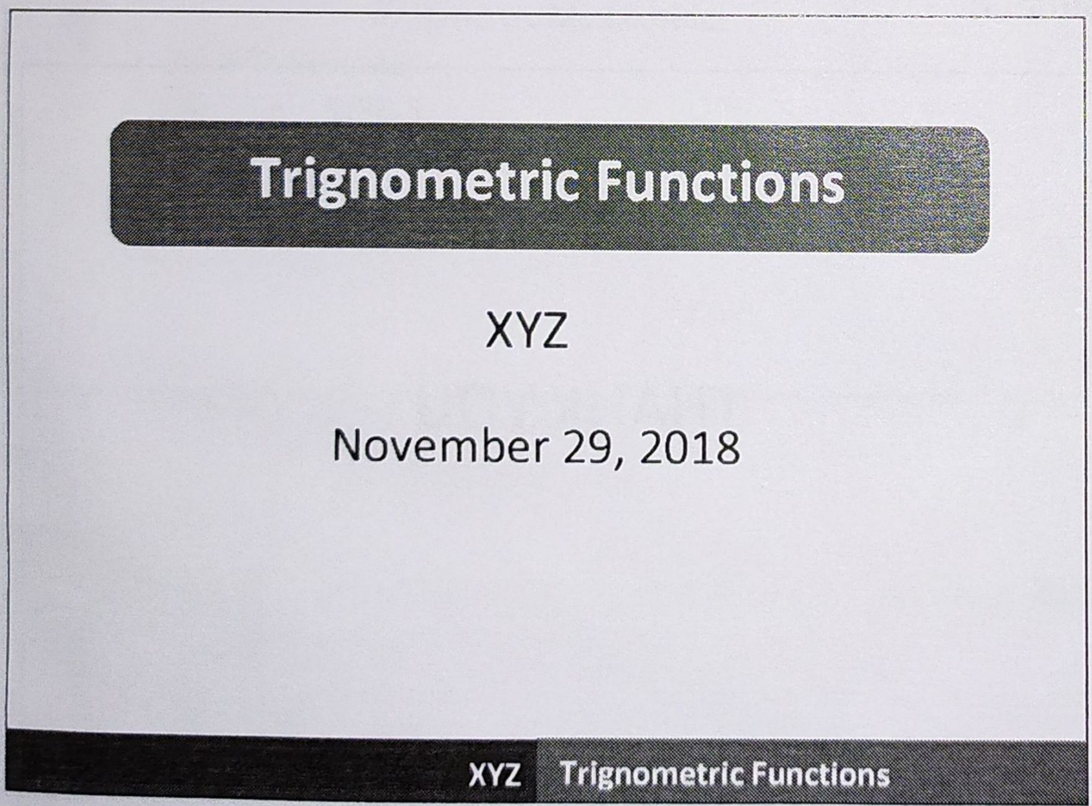
(iii) Write the code in LaTeX to plot the functions $y = \sqrt{x}$ and $y = x^2$ on the same coordinate system, for $0 \leq x \leq 1$. Show the sine function as a solid curve and the cosine function as a dotted curve.

(iv) Write a code in LaTeX for typesetting the following expression :

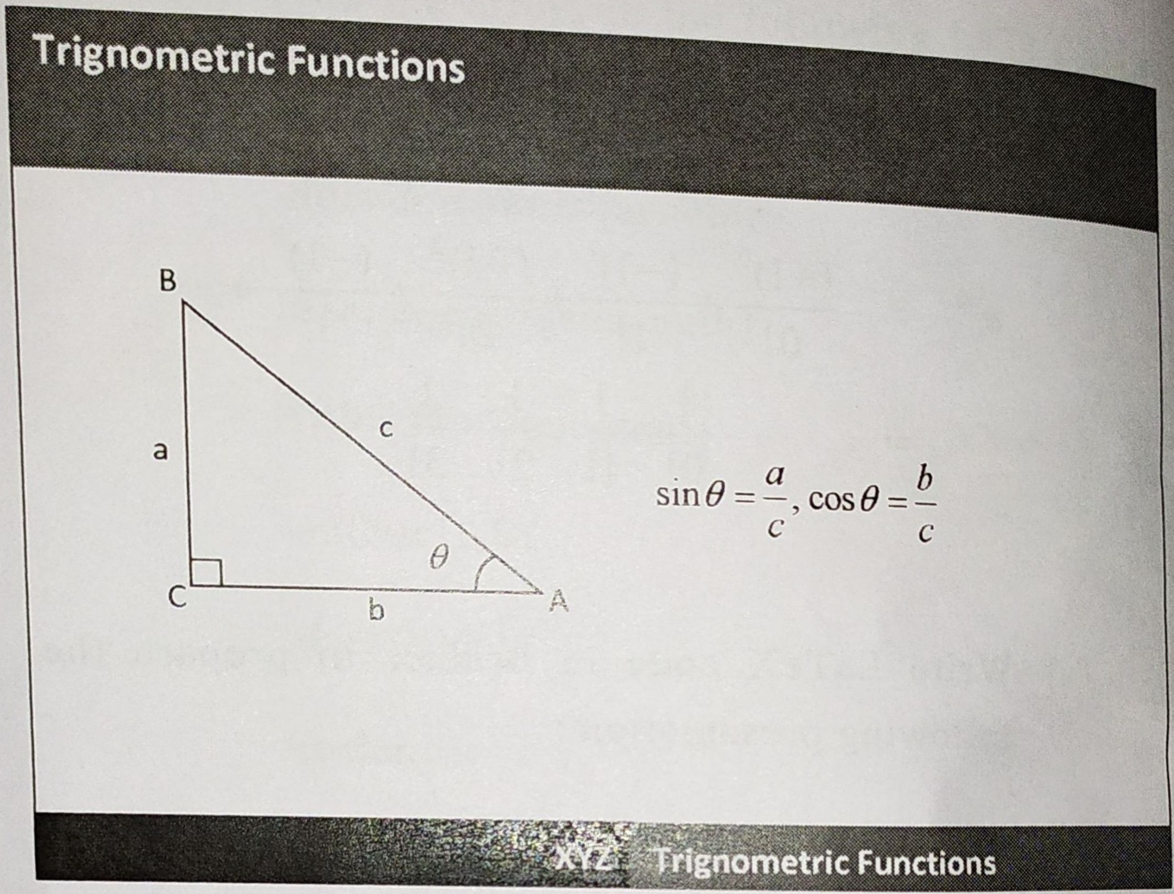
$$
\begin{aligned}
e^x &= \frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \\
e^{-1} &= \frac{(-1)^0}{0!}+\frac{(-1)^1}{1!}+\frac{(-1)^2}{2!}+\frac{(-1)^3}{3!}+\cdots \\
&= \frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots
\end{aligned}
$$

(v) Write LaTeX code in beamer to prepare the following presentation :

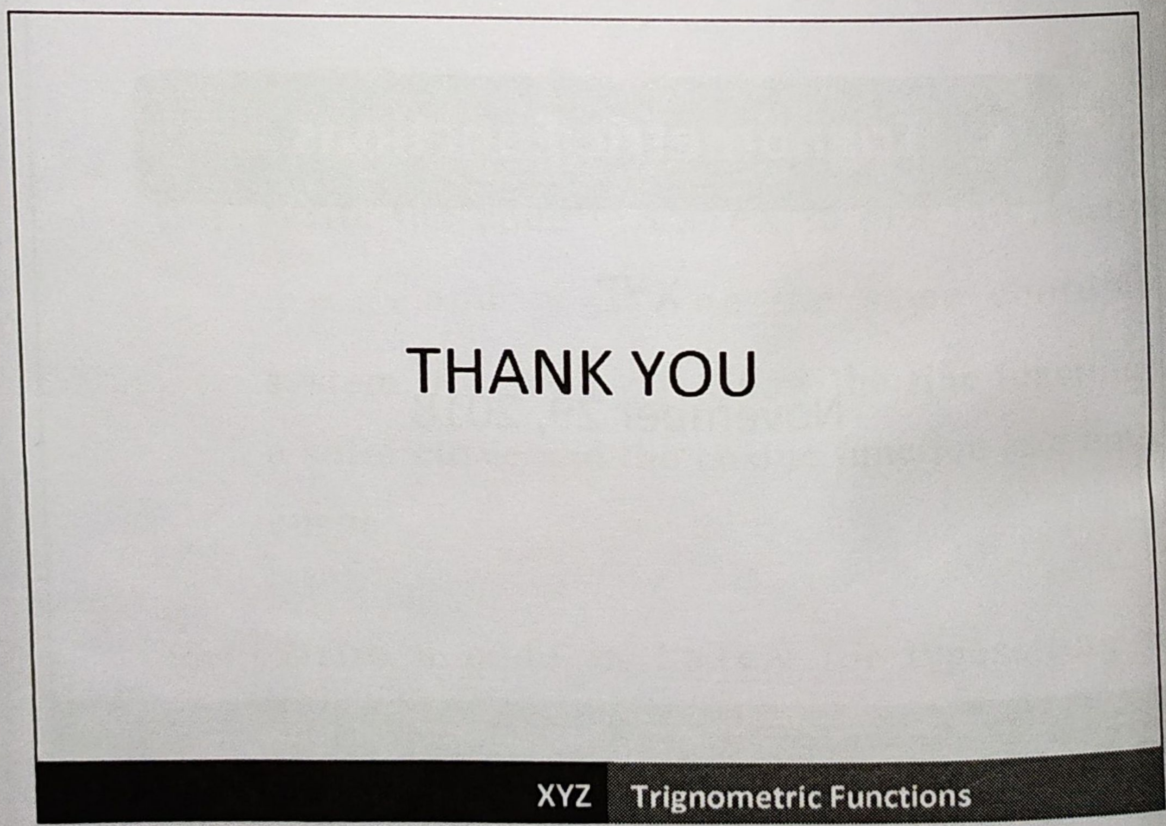
Slide 1:



Slide 2:



Slide 3:



(vi) Write an HTML code to generate the following web page:

University of Delhi

Department of Mathematics

The list of options for DSE papers offered in B.Sc.(H)-Mathematics:

1. Vth Semester
    a. DSE-1
        i. Numerical Methods
        ii. Mathematical Modelling and Graph Theory
    b. DSE-2
        i. Mathematical Finance
        ii. Discrete Mathematics
2. VIth Semester
    a. DSE-3
        i. Probability Theory & Statistics
        ii. Mechanics

Keep the following in mind while writing the code :

(i) Font face of the text should be Arial.

(ii) Text color of the main heading should be purple.

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7463** **J**

Unique Paper Code : 32351301

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Theory of Real Functions

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **three** parts from each question.

(c) **All** questions carry equal marks.

1. (a) Find the following limit and establish it by using $\in-\delta$ definition of limit :

$$\lim_{x \to -1} \frac{x+5}{2x+3}$$

(b) State and prove the sequential criterion for limits of a real valued function.

(c) Determine whether the following limit exists in R :

$$\lim_{x\to 0} \operatorname{sgn}\left(\sin 1/x^2\right)$$

(d) Show that :

$$\lim_{x\to 0^-} e^{\frac{1}{x}} = 0$$

and establish that

$$\lim_{x\to 0^+} e^{\frac{1}{x}}$$

does not exist in R .

2. (a) Let $c \in R$ and f be defined on (c, ∞) and $f(x) > 0$ for all $x \in (c, \infty)$. Show that

$$\lim_{x\to\infty} f(x) = \infty$$

if and only if

$$\lim_{x\to\infty} \frac{1}{f(x)} = 0$$

(b) Evaluate the following limit by using the appropriate definition :

$$\lim_{x\to 1^+} \frac{x}{x-1}$$

(c) Determine the points of continuity of the function f (x) = x [x] where [.] denotes the greatest integer function.

(d) A function $f : R \to R$ is such that f (x+y) = f(x) + f(y) for all x,y in R. Prove that if f is continuous at some point x_0, then it is continuous at every point of R.

3. (a) Let $A \subseteq R$ and $f : A \to R$ and let $f(x) \geq 0$, for all $x \in A$. Let $\sqrt{f}$ be defined as $\sqrt{f}(x) = \sqrt{f(x)}$ for $x \in A$. Show that if f is continuous at a point $c \in A$, then $\sqrt{f}$ is continuous at c.

(b) Suppose that $f : R \to R$ is continuous on R and that f(r) = 0 for every rational number r. Prove that f(x) = 0 for all $x \in R$.

(c) Let f be a continuous and real valued function defined on a closed and bounded interval [a, b]. Prove that f is bounded. Give an example to show that the condition of boundedness of the interval cannot be dropped.

(d) State the intermediate value theorem. Show that $x_2^k = 1$ for some $x \in]0, 1[$.

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4. (a) Show that the function $f(x) = x^2$ is uniformly continuous on $[-2, 2]$, but it is not uniformly continuous on R.

(b) Prove that if f and g are uniformly continuous on $A \subseteq R$ and if they both are bounded on A, then their product fg is uniformly continuous on A.

(c) Show that the function

$$f(x) = |x + 1| + |x - 1|$$

is not differentiable at -1 and 1.

(d) Prove that if $f : R \to R$ is an even function and has a derivative at every point, then the derivative f' is an odd function.

5. (a) State Darboux theorem. Let I be an interval and $f : I \to R$ be differentiable on I. Show that if the derivative f' is never zero on I, then either $f'(x) > 0$ for all $x \in I$ or $f'(x) < 0$ for all $x \in I$.

(b) Find the Taylor's series for cos x and indicate why it converges to cos x for all $x \in R$.

(c) Prove that $e^x \geq 1 + x$ for all $x \in R$, with equality occurring if and only if $x = 0.5$.

(d) Is $f(x) = |x|$, $x \in R$, a convex function ? Is every convex function differentiable ? Justify your answer.

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[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7464** **J**

Unique Paper Code : 32351302

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Group Theory - I

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) All questions carry equal marks.

1. (a) Let $G=\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in R, a \neq 0\right\}$. Show that G is a group under matrix multiplication.

(b) Let G be a group and H be a subset of G. Prove that H is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$. Hence prove that H = {$A \in G$: det A is a power of 3} is a subgroup of GL (2, R).

P.T.O.

(c) (i) Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have ?

(ii) If $|a| = n$ and k divides n, prove that $|a^{n/k}| = k$.

$6 \times 2 = 12$

2. (a) Let G = <a> be a cyclic group of order n. Prove that G = $\langle a^k \rangle$ if and only if gcd (k, n) =1. List all the generators of Z_{20}.

(b) (i) If a cyclic group has an element of infinite order, how many elements of finite order does it have.

(ii) List all the elements of order 6 and 8 in Z_{30}.

(c) Suppose that a and b are group elements that commute and have orders m and n. If $\langle a \rangle \cap \langle b \rangle = \{e\}$, Prove that the group contains an element whose order is the least common multiple of m and n. Show that this need not be true if a and b do not commute.

$6.5 \times 2 = 13$

3. (a) Let G be a group. Is $H = \{x^2 : x \in G\}$ a subgroup of G ? Justify.

(b) Prove that any two left cosets of a subgroup H in a group G are either equal or disjoint.

(c) Show that (Q ,+) has no proper subgroup of finite index.

$6 \times 2 = 12$

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4. (a) Prove that every subgroup of index 2 is normal. Show that A_5 is normal subgroup of S_5.

(b) Let G be a group and H be a normal subgroup of G. Prove that the set of all left cosets of H in G forms a group under the operation aH.bH = abH where $a, b \in G$.

(c) If H is a normal subgroup of G with $|H| = 2$, prove that $H \subseteq Z(G)$. Hence or otherwise show that A_5 cannot have a normal subgroup of order 2. $6.5 \times 2 = 13$

5. (a) Let C be the set of complex numbers and

$$M = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in R \right\}.$$

Prove that C and M are isomorphic under addition and $C^* = C \setminus \{0\}$ and $M^* = M \setminus \{0\}$ are isomorphic under multiplication.

(b) Prove that a finite cyclic group of order n is isomorphic to the group $Z_n = \{0, 1, 2, \dots, n-1\}$ under addition modulo n.

(c) (i) Suppose that φ is an isomorphism from a group G onto a group G^*. Prove that G is cyclic if and only if G^* is cyclic.

P.T.O.

(ii) Show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition. $6 \times 2 = 12$

6. (a) Let φ be a group homomorphism from a group G to a group G^* then prove that :

(i) $|\varphi(x)|$ divides $|x|$, for all x in G.

(ii) φ is one-one if and only if $|\varphi(x)| = |x|$, for all x in G.

(b) State and prove the Third Isomorphism Theorem.

(c) (i) Let G be a group. Prove that the mapping $\phi(g) = g^{-1}$, for all $g \in G$, is an isomorphism on G if and only if G is Abelian.

(ii) Determine all homomorphisms from Z_n to itself. $6.5 \times 2 = 13$

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[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7465** **J**

Unique Paper Code : 32351303

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Multivariate Calculus

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** Sections are compulsory.

(iii) Attempt any **five** questions from each **Section.**

(iv) All questions carry equal marks.

Section- I

1. Given that the function

$$f(x,y)=\begin{cases}\dfrac{3x^3-3y^3}{x^2-y^2} & \text{for } x^2\neq y^2\\ B & \text{otherwise}\end{cases}$$

is continuous at the origin, what is B ?

2. In physics, the *wave equation* is :

$$\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is :

$$\frac{\partial z}{\partial t}=c^2\frac{\partial^2 z}{\partial x^2}$$

Determine whether z = sin5ct cos5x satisfies the wave equation, the heat equation, or neither.

3. The radius and height of a right circular cone are measured with errors of at most 3% and 2%, respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.

4. If $f(x, y, z) = xy^2e^{xz}$ and $x = 2 + 3t$, $y = 6 - 4t$, $z = t^2$. Compute $\frac{df}{dt}(1)$.

5. Sketch the level curve corresponding to $C = 1$ for the function $f(x,y) = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ and find a unit normal vector at the point $P_0\left(2\sqrt{3}\right)$.

6. Find the point on the plane $2x + y - z = 5$ that is closest to the origin.

Section - II

7. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

8. Sketch the region of integration and then compute the integral $\int_0^1 \int_x^{2x} e^{y-x}\, dy\, dx$ in 2 ways:

(a) with the given order of integration

(b) with the order of integration reversed

9. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.

10. Find the volume of the tetrahedron bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.

11. Compute $\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

12. Use the change of variables to compute $\iint_D \frac{(x-y)^4}{(x+y)^4}\,dydx$, where D is the triangular region bounded by the line $x + y = 1$ and the coordinate axes.

Section - III

13. Find the work done by the force field $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\vec{i} - \frac{y}{\sqrt{x^2+y^2}}\vec{j}$ when an object moves from (a,0) to (0,a) on the path $x^2+y^2=a^2$.

14. Verify that the following line integral is independent of the path $\oint(3x^2+2x+y^2)$ $dx + (2xy + y^3)\,dy$ where C is any path from (0,0) to (0,1).

P.T.O.

15. Use Green's theorem to evaluate $\oint_C \left(x \sin x dx - \exp\left(y^2\right)dy\right)$ where C is the closed curve joining the points (1,-1) (2,5) and (-1,-1) in counterclockwise direction.

16. State Stoke's theorem and use it to evaluate $\iint_S \text{curl}\vec{F}.d\vec{S}$ where $\vec{F} = xz\vec{i} + yz\vec{j} + xy\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

17. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F}.\vec{N}dS$, where $\vec{F} = \left(x^2 + y^2 - z^2\right)\vec{i} + yx^2\vec{j} + 3z\vec{k}$; S is the surface comprised of the five faces of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, missing z = 0.

18. Evaluate $\iint_S 2x\,dS$ where S is the portion of the plane $x + y + z = 1$ with $x \geq 0$, $y \geq 0$, $z \geq 0$.

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Your Roll No. : ....

Sl. No. of Q. Paper : **7466** **J**

Unique Paper Code : 32351501

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Metric Spaces

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

1. (a) Define a metric space. Let $p \geq 1$. Define $d_p : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ as $d_p(x, y) = \left(\sum_{i=1}^{n} |x_i - y_i|^p\right)^{1/p}$, $x (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Show that $(\mathbb{R}^n, d_p)$ is a metric space. 6.5

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(b) When is a metric space said to be complete ? Is discrete metric space complete ? Justify. 6.5

(c) Let (X, d) be a metric space. Define $d_1 : X \times X \to \mathbb{R}$ by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$, for all $x, y \in X$.

Prove that d_1 is a metric on X and d_1 is equivalent to d. 6.5

2. (a) Prove that every open ball in a metric space (X, d) is an open set in (X, d). What about the converse ? Justify. 6

(b) Define a homeomorphism from a metric space (X, d_1) to a metric space (Y, d_2). Show that the function $f : \mathbb{R} \to]-1, 1[$ defined by $f(x) = \frac{x}{1+|x|}$ is a homeomorphism. 6

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(c) Let (X, d) be a metric space and let A, B be non-empty subsets of X. Prove that : 6

(i) $(A \cap B)^0 = A^0 \cap B^0$

(ii) $\overline{A \cup B} = \bar{A} \cup \bar{B}$

(a) Let (X, d) be a metric space and $F \subseteq X$. Prove that the following statements are equivalent :

6

(i) $x \in \bar{F}$

(ii) $S(x, \varepsilon) \cap F \neq \phi$, for every open ball $S(x, \varepsilon)$ centred at x

(iii) There exists an infinite sequence $\{x_n\}$ of point (not necessarily distinct) of F such that $x_n \to X$.

(b) Let (X,d) be a metric space and $F \subseteq X$. Prove that F is closed in X if and only if F^C is open in X, where F^C is complement of F in X.

6

(c) Let (X,d) be a metric space such that for every nested sequence $\{F_n\}_{n\geq 1}$ of non-empty closed subsets of X satisfying $d(F_n) \to 0$ as $n \to \infty$, the intersection $\bigcap_{n=1}^{\infty} F_n$ contains exactly one point. Prove that (X,d) is complete. 6

4. (a) Let f be a mapping from a metric space (X, d_1) to a metric space (Y, d_2). Prove that f is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y. 6.5

(b) Let (X, d_1) and (Y, d_2) be two metric spaces. Prove that the following statements are equivalent : 6.5

(i) f is continuous on X

(ii) $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, for all subsets B of Y

(iii) $f(\overline{A}) \subseteq \overline{f(A)}$, for all subsets A of X.

(c) Define uniform continuity of a function f from a metric space (X, d_1) to a metric space (Y, d_2). Let (X, d) be a metric space and A be a non-empty subset of X. Show that the function $f : (X, d) \to \mathbb{R}$ defined as $f(x) = d(x, A)$, for all $x \in X$, is uniformly continuous on X. 6.5

5. (a) State and prove contraction mapping theorem. 6

(b) (i) Let Y be a non-empty subset of a metric space (X, d) and (Y, d_y) be complete, where d_y is restriction of d to $Y \times Y$. Prove that Y is closed in X. 3

(ii) Let A be a non-empty bounded subset of a metric space (X, d). Prove that $d(A) = d(\overline{A})$. 3

 P.T.O.

(c) Let (X, d) be a metric space. Then prove that following statements are equivalent : 1.5×4=6

(i) (X, d) is disconnected.

(ii) There exist two non-empty disjoint subsets A and B, both open in X, such that $X = A \cup B$.

(iii) There exist two non-empty disjoint subsets A and B, both closed in X, such that $X = A \cup B$.

(iv) There exists a proper subset of X, which is both open and closed in X.

6. (a) Let ($\mathbb{R}$, d) be the space of real numbers with the usual metric. Show that a connected subset of $\mathbb{R}$ must be an interval. Give example of two connected subsets of $\mathbb{R}$ such that their union is disconnected.

(b) Let (X, d) be a metric space and Y be a subset of X. If Y is compact subset of (X, d), then prove that Y is closed. 6.5

(c) Let f be a continuous function from a compact metric space (X,d_1) to a metric space (Y, d_2). Prove that f is uniformly continuous on X. 6.5

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This question paper contains 4 printed pages]

'our Roll No.	: ....
l. No. of Q. Paper	: **7467** **J**
nique Paper Code	: 32351502
ame of the Course	: **B.Sc.(Hons.) Mathematics**
ame of the Paper	: Group Theory - II
emester	: V

ime : 3 Hours ***Maximum Marks : 75***

structions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) All questions carry equal marks.

(a) Let Inn (D_8) denotes the group of inner automorphisms on the dihedral group D_8 of order 8. Find Inn (D_8). 6

(b) Define inner automorphism of a group G induced by $g \in G$. Then prove that the set Inn(G) of all inner automorphism of a group G is a normal subgroup of the group Aut(G) of all automorphisms of G. 2+4

(c) Let G be a cyclic group of order n. The prove that Aut(G) is isomorphic to U(n). Her Aut(G) denotes the group of automorphism on G and U(n) = {m ∈ N : m < n and gc (m, n) = 1} is a group under multiplicatio modulo n.

2. (a) Prove that every characteristic subgroup (a group G is a normal subgroup of G. Is th converse true ? Justify. 4+

(b) Let G_1 and G_2 be finite groups. If $(g_1, g_2) \in$ ($\oplus G_2$, then prove that

$$|(g_1, g_2)| = \text{lcm}(|g_1|, |g_2|)$$

where $|g|$ denotes order of an element g i a group G.

(c) Prove that D_8 and S_3 cannot be expressed a an internal direct product of two of its prope subgroups. Here D_8 and S_3 denote th dihedral group of order 8 and the symmetri group on the set {1, 2, 3} respectively. 3+

3. (a) State Fundamental Theorem for Finit Abelian Groups. Find all Abelian groups (up isomorphism) of order 1176. 2+

(b) Let G be an Abelian group of order 120 an G has exactly three elements of order Determine the isomorphism class of (

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(c) For a group G, let the mapping from $G \times G \to G$ be defined by $(g, a) \to gag^{-1}$. Then prove that this mapping is a group action of G on itself. Also, find kernel of this action and the stabilizer G_x of an element $x \in G$. 2+2+2

4. (a) Let $G=\{1,a,b,c\}$ be the Klein 4-group. Label the group elements 1,a,b,c as integers 1,2,3,4 respectively. Compute the permutation σ_a, σ_b and σ_c induced by the group element a, b, c respectively under the group action of G on itself by left multiplication. 6.5

(b) Let G act on a set A. If $a, b \in A$ and $b = g.a$ for some $g \in G$, then prove that $G_b = gG_ag^{-1}$ where G_a is the stabilizer of a. Deduce that if G acts transitively on A then kernel of the action is $\bigcap_{g \in G} g\, G_a\, g^{-1}$. 3+3.5

(c) Let G be a group acting on a non empty set A and $a \in A$. Then prove that the number of elements in orbit containing a is equal to index of the stabilizer of a. 6.5

5. (a) State the class equation for finite groups. Find conjugacy classes of the quaternion group Q_8 and hence verify the class equation for Q_8. 2+3+1.5

(b) Let p be a prime and P be a group of prime power order p^α for some $\alpha \geq 1$. Then prove that P has a non trivial centre. Deduce that a group of order p^2 is an Abelian group. 4+2.5

(c) Let G be a non-Abelian group of order 231. Then prove that a Sylow 11-subgroup is normal and is contained in the centre of G. 2.5+4

6. (a) Let G be a group of order pq such that $p < q$ and p does not divide $(q - 1)$. Then prove that G is a cyclic group. Hence deduce that a group of order 33 is cyclic. 4.5+2

(b) Define a simple group. Prove that groups of order 72 and 56 are not simple. 1 + 2.5 + 3

(c) Let G be a group such that $|G|=2n$, where $n \geq 3$ is an odd integer. Then prove that G is not simple. 6.5

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This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7941**

Unique Paper Code : **32357501** **J**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics : DSE-1**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (*a*) Find a root of the equation $x\sin(x) + \cos(x) = 0$ using Newton-Raphson Method for starting approximation $x_0 = \pi$. 6.5

P.T.O.

(*b*) A real root of the equation $x^3 - 5x + 1 = 0$ lies]0, 1[. Perform three iterations of Bisection Method obtain the root. 6

(*c*) Prove that fixed point method converges at a linear ra using $g(x) \equiv x^2 - 2x - 3 = 0$ and starting approximatio $x_0 = 4$. 6

2. (*a*) Using Secant Method, find a real root of the equatio $xe^x - 1 = 0$ correct upto four decimal places. Given th root lies between 0 and 1, perform three iterations.

(*b*) Consider the function $g(x) = 1 + x - \frac{x^3}{8}$. Analyticall verify that this function has unique fixed point on the rea line. Perform three iterations starting at $x_0 = 0$ to locate the fixed point.

(c) Evaluate order of convergence of Newton Method numerically using $f(x) \equiv x^5 + 2x - 1 = 0$ and initial approximation $x_0 = \frac{1}{2}$. 6

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. 6.5

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$4x_1 + x_2 + 2x_3 = 4$$

$$3x_1 + 5x_2 + x_3 = 7$$

$$x_1 + x_2 + 3x_3 = 3$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6.5

(c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. 6.5

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data : 6

x	-1	0	1
$f(x) = e^x$	e^{-1}	e^0	e^1

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	-1	0	1	2
y	5	1	1	11

Hence, estimate the value of $f(0.5)$. 6

(c) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval [0, 1] so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}. 6

5. (a) Define the forward difference operator Δ, the central difference operator δ and the averaging operator μ.

Prove that :

(i) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$

(ii) $\mu = \left(1+\frac{1}{2}\Delta\right)(1+\Delta)^{-\frac{1}{2}}$. 6.5

(b) Use the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ taking $h = 1, 0.1, 0.01$. What is the order of approximation ? 6.5

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(c) Derive the following forward difference approximation for the second derivative : 6.5

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}.$$

6. (a) Define degree of precision of a quadrature rule. Show that the degree of precision of the Trapezoidal rule is 1. 6

(b) Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's 1/3rd rule. 6

(c) Use Euler's method to approximate the solution of the initial value problem.

$x' = tx^3 - x$, $x(0) = 1$, $0 \le t \le 1$ taking 4 steps. 6

This question paper contains 4+1 printed pages]

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S. No. of Question Paper : **7942**

Unique Paper Code : **32357502** **J**

Name of the Paper : **Mathematical Modelling & Graph Theory**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-1**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *ten* parts from Question No. **1**.

Attempt any *two* parts from Question Numbers **2** to **5**.

1. (*a*) Draw 2-dimensional cube Q_2. 2

(*b*) What is the number of edges and the number of vertices in $K_{p,q}$? 2

(*c*) What is the difference between a trail and a path ? 2

(*d*) What is the degree of each vertex in Q_{20} ? 2

P.T.O.

(e) Define Path graph P_n. Draw P_{11}.

(f) Determine whether $x = 0$ is an ordinary point, a regu singular point or an irregular singular point of the differen equation $3x^3y'' + 2x^2y' + (1 - x^2)y = 0$.

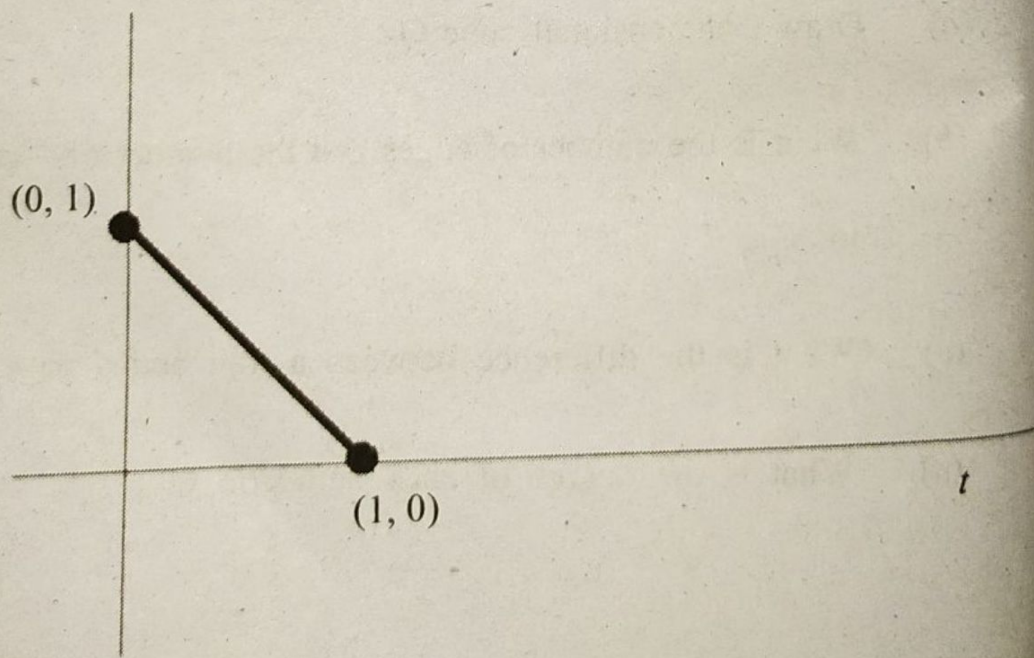
(g) Find the Laplace transform of the functi

$$f(t) = 3t^{\frac{5}{2}} - 4t^3.$$

(h) Find the inverse Laplace transform of the functi

$$F(s) = \frac{9 + s}{4 - s^2}.$$

(i) Apply the definition to find directly the Laplace transfo of the function described in the following figure :



(j) Use Middle-Square method to generate 7 random number using the seed $x_0 = 1009$. 2

(k) Suppose the feasible region of a linear program is a non-empty and bounded convex set. Then the objective function must attain both a maximum and a minimum value occurring at .... points of the region. 2

2. (a) Prove that in a bipartite graph every cycle has an even number of edges. Does the converse hold ? Justify. 6.5

(b) Prove that, if G is a simple graph with at least two vertices, then G has two or more vertices of the same degree. 6.5

(c) Show that there is no knight's tour on 5 × 5 chessboard. 6.5

3. (a) Use Laplace transforms to solve the initial value problem : 7

$$x'' + 4x' + 13x = te^{-t};\ \ x(0) = 0,\ x'(0) = 2.$$

(*b*) Find two linearly independent Frobenius series solutions of 7

$$2xy'' + (1 - 2x^2)y' - 4xy = 0.$$

(*c*) Solve the initial value problem : 7

$$(x^2 - 6x + 10)y'' - 4(x - 3)y' + 6y = 0; y(3) = 2, y'(3) = 0.$$

4. (*a*) Using Monte-Carlo simulation, write an algorithm to find the area under the curve $y = \cos x$ over the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. 7

(*b*) Explain Linear Congruence Method for generating random numbers. Does this method have any drawback ? Explain by taking a suitable example. 7

(*c*) Solve using Simplex Method : 7

Maximize $3x_1 + x_2$

subject to

$$2x_1 + x_2 \le 6$$

$$x_1 + 3x_2 \le 9$$

$$x_1, x_2 \ge 0.$$

5. (*a*) By finding an Eulerian trail in K_7, arrange a set of twenty eight dominoes (from 0-0 to 6-6) in a ring. 7

(*b*) Use the factorization : 7

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

to derive the inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{1}{s^4 + 4a^4}\right\} = \frac{1}{4a^3}(\cosh at \sin at - \sinh at \cos at).$$

(*c*) Solve the problem : 7

Maximize $25x_1 + 30x_2$

subject to

$$20x_1 + 30x_2 \leq 690$$

$$5x_1 + 4x_2 \leq 120$$

$$x_1, x_2 \geq 0.$$

Suppose now the objective function is changed to $C_1x_1 + 30x_2$. Show that at $C_1 = 20$ and $C_1 = 37.5$, alternative optimal solutions exist.

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8081**

Unique Paper Code : **32357501** **J**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (*a*) A real root of the equation $x^3 - 5x + 1 = 0$ lies in $]0, 1[$. Perform three iterations of Regula Falsi Method to obtain the root. 6

P.T.O.

808

(b) Perform three iteration of Bisection Method to obtain root of the equation $\cos(x) - xe^x$ in]0, 1[.

(c) Discuss the order of convergence of the Secant method and give the geometrical interpretation of the method. 6

2. (a) Verify $x = \sqrt{a}$ is a fixed point of the function $h(x) = \frac{1}{2}\left(x + \frac{a}{x^2}\right)$. Determine order of convergence of sequence $p_n = h(p_{n-1})$ towards $x = \sqrt{a}$. 6.5

(b) Use Secant method to find root of $3x + \sin(x) - e^x = 0$ in]0, 1[. Perform three iterations. 6.5

(c) Prove that Newton's Method is of order two using $x^3 + 2x^2 - 3x - 1 = 0$ and initial approximation $x_0 = 2$. 6.5

3. (a) Define a lower and an upper triangular matrix. Solve the system of equations :

$$-3x_1 + 2x_2 - x_3 = -12$$

$$6x_1 + 8x_2 + x_3 = 1$$

$$4x_1 + 2x_2 + 7x_3 = 1$$

by obtaining an LU decomposition of the coefficient matrix A of the above system. 6

(*b*) For Jacobi method, calculate T_{jac}, C_{jac} and spectral radius of the following matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$ 6

(*c*) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$4x_1 + 2x_2 - x_3 = 1$$

$$2x_1 + 4x_2 + x_3 = -1$$

$$-x_1 + x_2 + 4x_3 = 1$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6

4. (*a*) Construct the Lagrange form of the interpolating polynomial from the following data :

x	1	2	3
***f*(x) = ln x**	ln 1	ln 2	ln 3

6.5

(*b*) Prove that for $n + 1$ distinct nodal points $x_0, x_1, x_2, \ldots x_n$, there exists a unique interpolating polynomial of at most degree n. 6.5

(*c*) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval $[0, 1]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}. 6.5

5. (*a*) Define the backward difference operator ∇ and the Newton divided difference. Prove that :

$$f[x_0, x_1, -----, x_n] = \frac{\nabla^n f_n}{n! h^n} \text{ where } h = x_{i+1} - x_i .$$ 6

(*b*) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	–7	–5	–4	–1
y	10	5	2	10

Find the approximation of y for $x = -3$. 6

(*c*) Use the formula

$$f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}$$

to approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 1$ taking $h = 1, 0.1, 0.001$. What is the order of approximation ? 6

6. (*a*) Approximate the value of $\int_0^1 e^{-x}dx$ using the Trapezoidal rule and verify that the theoretical error bound holds for the same. 6.5

(*b*) State Simpson's 1/3rd rule for the evaluation of $\int_a^b f(x)dx$ and prove that it has degree of precision 3. 6.5

(*c*) Use Euler's method to approximate the solution of the initial value problem.

$x' = (1+x^2)/t$, $x(1) = 0$, $1 \le t \le 4$ taking 5 steps. 6.5

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8082**

Unique Paper Code : **32357502** **J**

Name of the Paper : **Mathematical Modelling and Graph Theory**

Name of the Course : **B.Sc. (H) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *ten* parts from Question No. **1**.

Attempt any *two* parts from Question Nos. **2** to **5**.

1. (*a*) Draw a simple connected graph with degree sequence (1, 1, 2, 3, 3, 4, 4, 6) 2

(*b*) What is the number of edges in Q_3 ? 2

(*c*) What is the sum of degrees of vertices of K_{30} ? 2

(*d*) State Ore's Theorem. 2

(*e*) How many 2017- regular graphs with 2019 vertices exist ? Justify. 2

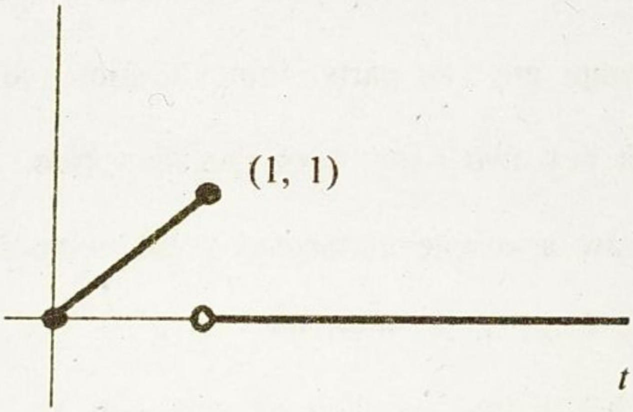
(*f*) Determine whether $x = 0$ is an ordinary point, a regular singular point or an irregular singular point of the differential equation $x^2y'' + (6\sin x)y' + 6y = 0$. 2

P.T.O.

(g) Find the Laplace transform of the function $f(t) = (1+t)^3$.

(h) Find the inverse Laplace transform of the function $F(s) = 2s^{-1}e^{-3s}$.

(i) Apply the definition to find directly the Laplace transform of the function described in the following figure : 2



(j) Many computers use the Linear Congruence Method for generating pseudorandom numbers. The method is given by the rule :

$$x_{n+1} = (ax_n + b) \bmod c$$

What value of c is usually taken by computers to avoid cycling ? 2

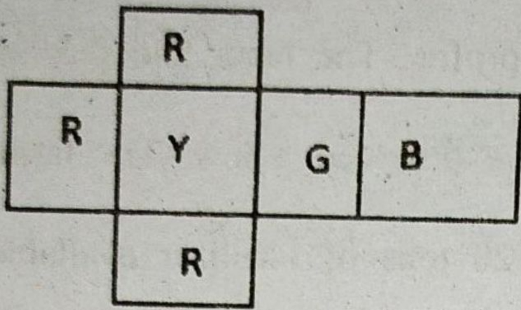
(*k*) A Montana farmer owns 45 acres of land. She is planning to plant each acre with wheat or corn. Each acre of wheat yields \$200 in profits, whereas each acre of corn yields \$300 in profits. The labor and fertilizer requirements for each are provided below. The farmer has 100 workers and 120 tons of fertilizer available.

	Wheat	**Corn**
Labor (workers)	3	2
Fertilizer (tons)	2	4

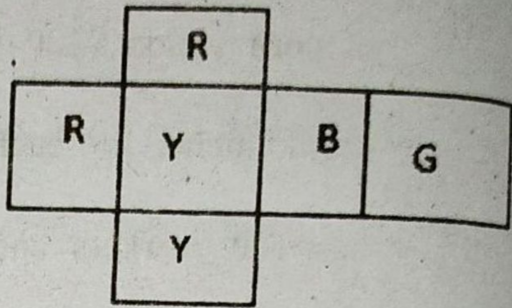
The farmer wants to plant wheat and corn such that her profit maximizes. Formulate the linear programming problem, clearly specifying the objective function and the constraints. 2

2\. (*a*) Let G be a simple connected graph with n vertices, where $n \geq 3$ and deg $v \geq n/2$ for each vertex v. Use Ore's Theorem to show that G is Hamiltonian. Give an example of a Hamiltonian graph that does not satisfy the conditions of Ore's Theorem. 6.5

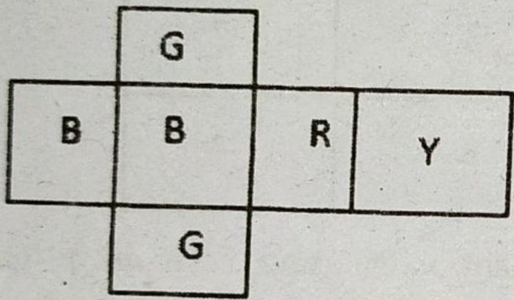
(*b*) Find solution to the four-cubes problem for the following set of cubes :



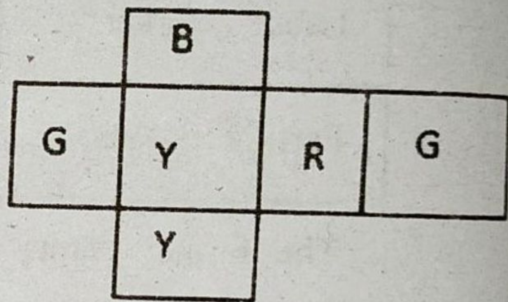
cube 1



cube 2



cube 3



cube 4

(*c*) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes (from 0–0 to 4–4) in a ring. 6.5

3. (*a*) Use Laplace transforms to solve the initial value problem : 7

$$x'' + 6x' + 18x = \cos 2t; \quad x(0) = 1, \; x'(0) = -1.$$

(*b*) Find two linearly independent Frobenius series solutions of : 7

$$3x^2y'' + 2xy' + x^2y = 0.$$

(c) Solve the initial value problem : 7

$$(x^2+6x)y'' + (3x+9)y' - 3y = 0;\ y(-3) = 0,\ y'(-3) = 2.$$

4. (*a*) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any one time. Ships arrive for unloading of cargo at the harbor, and the time between the arrival of successive ships varies from 15 to 145 minutes. The unloading time for ships varies from 45 to 90 minutes. For 5 ships, the data is as given :

	Ship 1	**Ship 2**	**Ship 3**	**Ship 4**	**Ship 5**
Time between successive ships	20	30	15	120	25
Unloading Time	55	45	60	75	80

Draw time-line for each ship and hence answer the following questions :

(*i*) What is the average waiting time for the ships ?

(*ii*) For how much time the harbor remains idle ? 7

(*b*) Using Monte-Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle :

$$Q : x^2 + y^2 = 1,\ x \geq 0,\ y > 0$$

where the quarter circle is taken to be inside the square

$$S : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1.$$ 7

(*c*) Solve using Simplex Method :

Maximize $10x + 35y$

Subject to

$4x + 3y \leq 24$

$4x + y \leq 20$

$x, y \geq 0.$ 7

5. (*a*) Show that there is no knight's tour on 3×6 chessboard. 7

(*b*) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

to derive the inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^2}{s^4 + 4a^4}\right\} = \frac{1}{2a}(\cosh at \sin at + \sinh at \cos at).$$ 7

(c) Solve the problem :

Maximize $25\ x_1 + 30\ x_2$

subject to

$$20x_1 + 30x_2 \le 690$$

$$5x_1 + 4x_2 \le 120$$

$$x_1,\ x_2 \ge 0$$

Suppose the second constraint is changed to :

$$5x_1 + 4x_2 \le b_2$$

What is the change in the value of the objective function as b_2 increases by one unit in the range $92 \le b_2 \le 172.5$? 7

(12)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8525**

Unique Paper Code : **32355101** **J**

Name of the Paper : **Calculus**

Name of the Course : **Mathematics : G.E. for Honours**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) Find the open interval on which $f(x) = x^3 - 3x + 3$ is concave up and concave down. Also determine points of inflection, if any.

(*b*) Find the interval in which the function $f(x)$ is (*i*) increasing (*ii*) decreasing $f(x) = 2x^3 - 9x^2 + 12x$.

(*c*) Evaluate :

$$\lim_{x \to -\infty} \frac{3x^2 - 5x + 7}{7x^2 + 2x - 3}.$$ 6+6

P.T.O.

2\. (*a*) Find the volume of the solid that results when the re[illegible] enclosed by $y = x^2, x = 0, x = 2, y = 0,$ is revolved ab[illegible] x-axis.

(*b*) Use cylindrical shells to find the volume of the so[illegible] generated when the region enclosed between $y =$ [illegible] $x = 1, x = 4, y = 0$ is revolved about y-axis.

(*c*) Find the area of the surface that is generated by revolv[illegible] the portion of the curve $y = x^2$ between $x = 1$ [illegible] $x = 2$ about y-axis. 6½

3\. (*a*) Evaluate :

$$\lim_{x \to 0} \frac{10(\sin x - x)}{x^3}.$$

(*b*) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

(*c*) Find the asymptotes of the graph of the function :

$$f(x) = -\frac{8}{x^2 - 4}.$$

6

4. (*a*) Identify the symmetries of the curve $r^2 = \cos\theta$ and then sketch the curve.

(*b*) Solve the initial value problem and find $\vec{r}$ as a vector valued function of t.

$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{\frac{1}{2}}\,\hat{i} + e^{-1}\hat{j} + \frac{1}{t+1}\hat{k},\;\; \hat{r}(0) = \hat{k}.$$

(*c*) Find a unit tangent and unit normal vector for space curve :

$$\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3\hat{k}.$$ 6+6

5. (*a*) Write acceleration $\vec{a}$ in the form $a_T\,\mathrm{T} + a_N\,\mathrm{N}$ without finding T and N for :

$$\vec{r}(t) = (t+1)\hat{i} + 2t\,\hat{j} + t^2\,\hat{k} \text{ at } t = 1.$$

(*b*) Show that :

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & \end{cases}$$

is continuous at every point except at origin.

(c) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$, where $w = x + 2y + z^2$, x

$y = r^2 + \ln s$, $z = 2r$.

6. (*a*) Find the direction in which $f(x, y) = xe^y + z^2$

(*i*) increases most rapidly at $P\left(1, \ln 2, \frac{1}{2}\right)$

(*ii*) decreases most rapidly $P\left(1, \ln 2, \frac{1}{2}\right)$.

(*b*) Find equations of tangent plane and normal lines f

curve $z^2 - 2x^2 - 2y^2 - 12 = 0$ at $P(1, -1, 4)$.

(*c*) Find all the local maxima, local minima and saddle p

of the curve :

$$f(x, y) = 4xy - x^4 - y^4.$$

6½

This question paper contains 5 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7273 **J**

Unique Paper Code : 42351101 – OC

Name of the Paper : Calculus and Matrices

Name of the Course : **B.Sc. (Mathematical Sciences) / B.Sc. (Prog.)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** questions from each section.

SECTION – I

1. (a) Verify that the set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis of R^4. (6)

P.T.O.

(b) Is $W = \left\{ \begin{bmatrix} x \\ y \\ 2x \end{bmatrix} : xy > 0 \right\}$ a subspace of R^3? Justi

your answer.

2. (a) Let $T: R^3 \rightarrow R^3$ be defined by

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y-z \\ x-z \end{bmatrix}$$

Show that T is a linear transformation. Also fi a matrix representation for T. (

(b) Find the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}.$$

(

3. (a) Determine the unique solution of the followi system of equations

$$x + y + z = 6$$
$$2x + 3y + 4z = 20$$
$$x + y = z.$$

(6

(b) Find the rank of the matrix

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}.$$ (6)

SECTION – II

4. (a) (i) Determine whether the sequence $\left\{1+\frac{(-1)^n}{n}\right\}$ is bounded and monotonic.

(ii) Compute $\lim_{n\to\infty}\left\{\frac{\cos n}{n}\right\}$. (6)

(b) Find the n^{th} derivative of $y = e^{3x}\sin(4x+1)$. (6)

(c) If $y = (\sin^{-1}x)^2$, prove that

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$$ (6)

5. (a) Sketch the graph of $y = e^{-x} + 1$. (6)

(b) According to Newton's Law of Cooling, the rate at which a substance cools in air is proportional to the difference between the temperature of the substance and that of the surrounding air. If the temperature of the air is 30°C and the substance cools from 100°C to 80°C in 20 minutes, find when the temperature will be 40°C. (6)

(c) Find the Taylor series generated by $f(x)=\frac{1}{x}$ at $x = 2$. (6)

6. (a) Draw the level curves for the surface $z=9x^2+25y^2$ at heights k = 1, 2, 3. (6)

(b) Find all the second order partial derivatives of $f(x,y)=e^{x-3y}$. (6)

(c) Verify that $z=e^x\sin(y)+e^y\cos(x)$ is a solution of the Laplace equation. (6)

SECTION – III

7. (a) Prove that the product of all the n^{th} roots of unity is $(-1)^{n-1}$. (4)

(b) Represent graphically the set $\{z : |z| \leq |z-1|\}$. (3½)

(a) Evaluate $\dfrac{(\text{Cos}\,\alpha + i\,\text{Sin}\,\alpha)^4}{(\text{Sin}\,\beta + i\,\text{Cos}\,\beta)^5}$. (4)

(b) State fundamental theorem of algebra. Form an equation in lowest degree with rational coefficients having $\sqrt{3}+2$ and $\sqrt{5}-2$ as two of its roots. (3½)

9. (a) Find the equation of the circle described on the join of the points given by $-1-3i$ and $5+7i$ as extremities of one of its diameters. (4)

(b) Find the equation of the straight line joining the points whose affixes are $2-5i$ and $1-i$. (3½)

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8562**

Unique Paper Code : **42351101** **J**

Name of the Paper : **Calculus and Matrices**

Name of the Course : **B.Sc. Mathematical Sciences/B.Sc. (Prog.)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* questions from each section.

Section I

1. (*a*) Examine the existence of the limit of the function :

$$g(t) = \begin{cases} t-2 & : \quad t<0 \\ t^2 & : \quad 0 \le t \le 2 \\ 2t & : \quad t>2 \end{cases}$$

at $t = 0, 2$.

(*b*) Find a value of the constant k, if possible, that will make the function continuous everywhere.

$$f(x) = \begin{cases} 7x-2 & x \le 1 \\ kx^2 & x > 1 \end{cases}.$$

P.T.O.

(c) If $y = x^2 \sin x$, then prove that :

$$\frac{d^n y}{dx^n} = (x^2 - n^2 + n)\sin\left(x + \frac{n\pi}{2}\right) - 2nx\cos\left(x + \frac{n\pi}{2}\right).$$ 4+4+4

2. (a) Show that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and $(0, \infty)$ but has no derivative at $x = 0$.

(b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that :

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

(c) Find the Maclaurin series for the function $f(x) = \frac{1}{x+1}$ assuming the validity of expansion. 4+4+4

3. (a) State and prove Lagrange's mean value theorem. Also discuss its geometrical significance.

(b) Find the value of c for the following function that satisfies the hypotheses of the Lagrange's mean value theorem :

$$f(x) = x^2 + 2x - 1,\ a = 0,\ \ b = 1.$$

(c) Prove that :

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0.$$ 5+4+3

(*a*) Sketch the graphs of the following functions (any *two*) :

(*i*) $y = 1 + \sqrt{x-1}$

(*ii*) $y = \sin 2x$ in $[0, 2\pi]$

(*iii*) $y = e^{-|x|} - 1$.

(*b*) Given the function $f(x) = |x|$. The graph of the function $f(x)$ is shifted vertically down 3 units and horizontally right 2 units followed by a reflection across x-axis. Sketch the original function $f(x)$ along with the new graph. Also write the equation for the new graph.

6+6

Section II

5. (*a*) Sketch the contour plot of $f(x) = x^2 + y^2$ using the level curves at heights k = 0, 3, 5.

(*b*) Let $f(x, y) = x^2 + y^2 - 2$. Find an equation of the level curve that passes through the point $(1, -2, 0)$.

(*c*) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenvectors of the matrix A.

4+4+5

6. (*a*) Verify that $u(x, t) = \sin(x - 4t)$ is a solution of the wave equation.

(*b*) Row reduce the matrix A to reduced row echelon form. Circle the pivot positions in the final matrix and hence determine its rank :

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}.$$

(*c*) For what value of λ and μ do the following system of linear equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have :

(*i*) a unique solution

(*ii*) no solution

(*iii*) an infinite number of solutions. 4+4+5

REDMI NOTE 7S
AI DUAL CAMERA

7. (*a*) Let

$$f(x, y) = x^2y + 5y^3.$$

Find the slope of the surface $z = f(x, y)$ in x-direction at the point $(1, -2)$.

(*b*) Check whether the set $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ is linear independent or not.

(*c*) Check whether the transformation $T : R^2 \to R^2$ defined as $T(x, y) = (x + 4y, y)$ is linear Sketch the image of the unit square with vertices (0, 0), (0, 1), (1, 1), (1, 0) under the given transformation. 4+4+5

8. (*a*) Find the standard matrix of the reflection about xz plane.

(*b*) Find the polar representation of the following numbers :

(*i*) $z_1 = -1 - i$.

(*ii*) $z_2 = 1 - i\sqrt{3}$.

(*c*) If $z_1 = 1 - i$ and $z_2 = \sqrt{3} + i$, then find $Arg(z_1 z_2)$ and $|z_1 z_2|$.

5+4+4

9. (*a*) Find the equation of the circle whose radius is 3 and whose center has affix $1-i$.

(*b*) Find the equation of the straight line joining the points whose affixes are $z_1 = 1-i$ and $z_2 = 2-5i$.

(*c*) Compute $(1+i)^{1000}$.

(*d*) Solve the equation using De Moivre's theorem $z^7 + z = 0$. 3+3+3+4

REDMI NOTE 7S
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[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8564** **J**

Unique Paper Code : 42341102

Name of the Course : **B.Sc. (Prog.)/B.Sc. Mathematical Sciences**

Name of the Paper : Problem Solving Using Computers

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **Section-A** is compulsory.

(c) Answer any **five** questions from **Section-B**.

(d) Answer **all** parts of a question together.

Section - A

1. (a) What do you understand by Byte. Write number of bytes in each of the following : 3

(i) Megabyte

(ii) Gigabyte

(b) Given a=5, b=6 and c=4, find the value of following expressions 3

(i) b//c *a

(ii) b%c and a>c

(iii) len("hello")>a

(c) Identify the syntax errors in the following code : 3

```
def f1:?
    a=b+5
    return a
f1(5)
```

(d) Rewrite the following for loop using while loop : 3

```
sum=0
for i in range(1,7,2):
    sum+=i
```

(e) Write try and except block to handle exception related to text file "file1.txt" that is to be opened in read mode. Display message "file not found" on the occurrence of the appropriate exception related to files. 3

(f) Evaluate the following functions : 3

(i) math.ceil(8.6)

(ii) min("xx","xz","aaa")

(iii) abs(-5)

(g) Find the output of the following code : 2+2

```
(i) def fname(n):
        for i in range(n):
          if i==3:
          return 0
          else:
            print("i= ",i)
      print("output= ",fname(5))
```

```
(ii)   names={'DELHI':5,'BOMBAY':6,'GOA':3}
         print(list(names.keys())
      print(names.get('RANCHI','NONE'))
```

(h) Find the content of S after the execution of the following statements ? 3

```
S1="EXECUTION"
S=S1[0::3] *3
```

Section - B

2. (a) Given two lists L1=["red","green"] and L2=[3,5]. Write Python statement(s) to produce a list named **outlist** having following contents using L1 and L2: outlist=[('red', 3), ('green', 5)]. Also, display length of the outlist and its contents. 2+1

P.T.O.

(b) Given two lists NAMES1 and NAMES2 having names of students where the same name may appear in both lists. Write statements to do the following : 4

(i) Generate a list UnqNAMES to store all names without repetition from both NAMES1 and NAMES2.

(ii) Generate a list DupNAMES to have common names from both NAMES1 and NAMES2.

(c) Write PYTHON statements to accept positive integer only from the user. Use appropriate exception/assert statements for the same. 3

3. (a) Find the data type and content of the variable S2 after the execution of each of the following statements where S="Semester Examination" and S1="Finally" 5

(i) S2=S[:-12]+S1[:5]

(ii) S2=S1.find('l')+S1.rfind('l')

(iii) S2=S.split()

(b) Write a function that takes **str1** as parameter and replaces alternate character in str1 (starting from 0) with ' * ' and stores resultant string in **str2**. The function shall display the number of character replaced and shall return str2. For example if str1='Examination' then str2='*x*m*n*t*o*' and the number of character replaced is 6. 5

4. (a) Write a function named **fncompute** which takes **number** as parameter where number is a positive integer. The function returns the difference in maximum and minimum digit in the number i.e. if number is 7892, then difference is 9-2=7 5

(b) Write a function named **fnsearch** to search for an element X in a list L using linear search, where X and L are passed as parameters. The function should return all positions at which X is found else -1 if not found. Note that X may appear multiple times in the list. 5

P.T.O.

5. (a) Consider the following list **X** of numbers : 5

100, 34, 45, 56, 19

Show step by step iterations for arranging the given list in increasing order using insertion sort.

(b) Write a recursive function to compute nth term of the fibonacci series. Fibonacci series has 0 and 1 as first term and second term respectively. The third and subsequent terms are computed as sum of previous two terms. 5

6. (a) Write appropriate file handling statements to do the following : 2+3

(i) Open a text file '**exam.txt**' and append message **'Good Morning'** in it

(ii) Display contents of a CSV file students.csv where fields are seperated by delimiter *

(b) Write a function fnfile() which reads a text file '**sentences.txt**' having sentences of different length and write only those sentences whose length is less than 10 in a new file '**output.txt**.' 5

7\. (a) Write a function which accepts a list of names and returns a dictionary where key-value pairs are names and length of name respectively. 5

(b) Identify the local and global variables in the following code and find the output : 5

```
j=5
def fn(a,b=5):
    if a%b==0:
        print("Divisble")
    else:
        print("Non-Divisble")
    j=a+b
    print("in function j= ",j)
fn(15)
fn(16,3)
print("outside function j= ",j)
```

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P.T.O.

8. (a) Write Python statement(s) to store all common factors of given two numbers no1 and no2 in a variable of type set. 3

(b) Define a class **CIRCLE** having a single data member **radius**. Include following methods in the class definition : 7

(i) Constructor to initialize value to the data member.

(ii) Method **getperimeter** to return the perimiter of the circle which is defined as 2*pi*radius.

(iii) Method **getarea** to return the area of the circle which is defined as pi*radius^2.

After defining the class, create an object of CIRCLE with radius 5.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7277 **J**

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Prog.) / Mathematical Sciences**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

UNIT – I

1. (a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R}; a \neq 0 \right\}$

Show that G is an abelian group under matrix multiplication. (6)

(b) Describe the symmetries of a non-square rectangle. Construct the corresponding Cayley table. (6)

(c) Let H = $\{x \in U(20) : x \equiv 1 \bmod 3\}$. List all elements of H. Prove or disprove that H is a subgroup of U(20). (6)

2\. (a) Prove that an abelian group with two elements of order 2 must have a subgroup of order 4. (6)

(b) Define Cyclic Group. Is U(8) with the operation of multiplication modulo 8 a cyclic group? Justify. (6)

(c) Let $a, b \in S_n$. Prove that $a^{-1}b^{-a}ab$ is an even permutation. (6)

3\. (a) State Lagrange's theorem for finite groups. Prove that in a finite group, the order of each element of the group divides the order of the group. (6)

(b) Let H be a subgroup of G and $a, b \in G$. Prove that either $aH = bH$ or $aH \cap bH = \phi$. (6)

(c) Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} ; a, b, d \in \mathbb{R};\ ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2, \mathbb{R})$? Justify. (6)

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UNIT - II

(a) Define center of a ring R. Prove that center of a ring is a subring of R. (6.5)

(b) Define field and an integral domain. Prove that every field is an integral domain. Is the converse true? Justify. (6.5)

(c) Find all zero divisors in $\mathbb{Z}_{20}$. What is the relationship between the zero divisors and the units of $\mathbb{Z}_{20}$? (6.5)

UNIT - III

5. (a) Determine whether or not the set

$$\left\{\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\}$$

is linearly independent over $\mathbb{Z}_5$. (6.5)

(b) Let $V = \left\{\begin{pmatrix} a & b \\ b & c \end{pmatrix};\ a, b, c \in \mathbb{Q}\right\}$ be a vector space over $\mathbb{Q}$. Find a basis of V over $\mathbb{Q}$. (6.5)

(c) Which of the following is a subspace of $\mathbb{R}^3$? Justify.

P.T.O.

(i) $S=\{(a,b,c)\in\mathbb{R}^3 : 2a+3b=4c\}$

(ii) $T=\{(a,b,c)\in\mathbb{R}^3 : a^2+b^2=c^2\}$ (6.5)

6. (a) Which of the following function T from $\mathbb{R}^2$ into $\mathbb{R}^2$ is a linear transformation? Justify

(i) $T(a, b) = (a - b, 0)$

(ii) $T(a, b) = (a^2, b)$ (6.5)

(b) Let $T: \mathbb{C}^3 \to \mathbb{C}^3$ be a linear transformation defined by

$$T(x, y, z) = (x - y + 2z,\ 2x + y,\ -x - 2y + 2z).$$

If $(a, b, c) \in \mathbb{C}^3$, what are the conditions on a, b and c so that the vector be in range of T? What is the rank of T? (6.5)

(c) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1,2) = (2,3)$ and $T(0,1) = (1,1)$. Find $T(a,b)$ for any $(a,b) \in \mathbb{R}^2$. (6.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 7293 **J**

Unique Paper Code : 42357501

Name of the Paper : Differential Equations

Name of the Course : **B.Sc. (Math Sci) / B.Sc. (Prog.)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve $\left[y^2(x+1)+y\right]dx+(2xy+1)dy=0.$ (6.5)

   (b) Solve $x\frac{dy}{dx}+y=(xy)^{3/2},\quad y(1)=4.$ (6.5)

   (c) Solve $(px-y)(py+x)=h^2 p.$ (6.5)

P.T.O.

2. (a) Solve the initial value problem

$$\frac{d^2y}{dx^2} + y = 3x^2 - 4\sin x, \quad y(0) = 0, \quad y'(0) = 1. \qquad (6.5)$$

(b) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln x. \qquad (6.5)$$

(c) For the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0,$$

show that x^2 and $\frac{1}{x^2}$ are solutions on the interval $0 < x < \infty$. Are these linearly independent? Justify.

Also find the solution that satisfies the conditions $y(2) = 3$, $y'(2) = -1$. (6.5)

3. (a) Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + y = \sec^3 x$. (6)

(b) Given that $y = x$ is a solution of

$$(x^2 - x + 1)\frac{d^2y}{dx^2} - (x^2 + x)\frac{dy}{dx} + (x + 1)y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution. (6)

(c) Find the general solution of

$$(x+1)^2 \frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + 2y = 1,$$

given that $y = x + 1$ and $y = (x + 1)^2$ are linearly independent solutions of the corresponding homogeneous equation. (6)

4. (a) Solve

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}.$$ (6)

(b) Solve

$$2\frac{dx}{dt} + \frac{dy}{dt} + x + y = t^2 + 4t,$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = 2t^2 - 2t\ .$$ (6)

(c) Check the condition of integrability and solve

$$(y + z)dx + dy + dz = 0.$$ (6)

5. (a) Eliminate the arbitrary function f from the equation

$$z = f(x^2 - y^2)$$

to form the corresponding partial differential equation. (6)

(b) Find the general integral of the partial differential equation

$$(y + zx)p - (x + yz)q = x^2 - y^2.$$ (6)

(c) Show that the equations

$$xp - yq = x, \; x^2p + q = xz$$

are compatible and find their solution. (6)

6. (a) Find the complete integral of the equation

$$z^2 = pqxy$$ (6.5)

(b) Find the complete integral of the differential equation

$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2).$$ (6.5)

(c) Reduce the following differential equation to canonical form $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0.$ (6.5)

This question paper contains 6 printed pages.

Your Roll No. ....

No. of Ques. Paper	: 7300 J
Unique Paper Code	: 42347902
Name of Paper	: Analysis of Algorithm and Data Structure
Name of Course	: B.Sc. (Prog.) / B.Sc. Math. Sciences : DSE–2
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt any four of Question Nos. 2 to 8.

Parts of a question should be answered together.

SECTION A

Answer the following questions :

(a) An array A [1 ... 15, 1...10] is stored in the memory with each element requiring 4 bytes of storage. If the base address of the array A is 1500, determine the location of A [12] [9] when the array is stored columnwise. 5

(b) What is meant by the 'stack overflow' condition? Is it applicable to the linked list method of implementation of the stack? Give reasons. 4

(d) Arrange the following functions in increasing order growth :

$$n^2, n!, n \log n, n^n, 2^n.$$

(e) Why is linear implementation of queue using arr inefficient?

(f) A newspaper distribution route has recently be computerized. Information about each of the 100 custome is stored in individual records containing first name (of n more than 5 letters), last name (of no more than 5 letter and payment due. In order to ease out the process o accessing the customer records, following tasks need to be done :

(i) Sort the records based on customer's first name. Which algorithm is best suited amongst the following : radix sort, bubble sort, quick sort? Justify your choice. 2

(ii) After (i) above, what will be the maximum number of comparisons needed to find a particular customer's record? 2

(g) What is the time complexity of following algorithm : 2

```
a = 0
for i = 0 to N–1
{
    for j = N to i+ 1
      {
          a = a + i + j;
```

```
        }
}
```

(h) What is the maximum number of nodes in a binary tree :

(i) at height h.

(ii) at level i. 2+2=4

(i) Convert the infix expression a*b^c/d to postfix. Also, for the values a = 2, b = 4, c = 2, d = 6, evaluate the expression using a stack. 1+3=4

SECTION B

2. (a) Given the following sequence of letters and asterisks :

ALG*O*R*THM*S

(i) Consider the stack data structure, supporting two operations *push* and *pop*. Suppose that for the above sequence, each letter (such as E) corresponds to a *push* of that letter onto the stack and each asterisk (*) corresponds to a *pop* operation on the stack. Show the sequence of values in the stack. 3

(ii) Consider the queue data structure, supporting two operations *enqueue* and *dequeue*. Suppose that for the above sequence, each letter (such as E) corresponds to an *enqueue* of that letter into the queue and each asterisk (*) corresponds to a *dequeue* operation on the queue. Show the sequence of values in the queue. 3

(b) Write an algorithm to sort a list of elements using count sort. 4

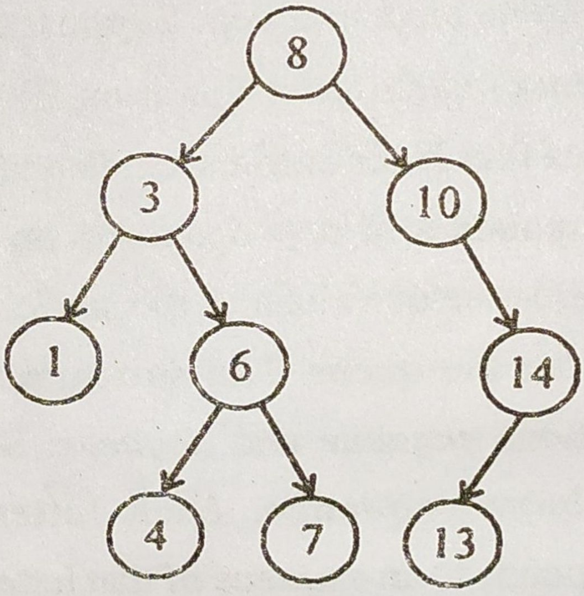
3. (a) Implement queue using linked list. Declare and define enqueue( ), dequeue( ), clear( ) and display( ) algorithms for inserting a value, removing a value, clearing queue and displaying contents of the queue. 8

(b) Arrange the following sorting algorithms according to their best case running time complexity : Quick sort, Selection sort, Count sort, Insertion sort. 2

4. (a) Create any possible binary search tree using the following values :

50, 75, 25, 12, 30, 20, 52, 11 4

(b) List the order in which the nodes in the below constructed tree will be traversed when each of the following algorithms is used : Inorder, Preorder and Postorder. 6



5. (a) A thief enters a house for robbing it. He can carry a maximal weight of 60 kg in his bag. There are 5 items in the house with the following weights and values. What items should thief take in order to maximize total value if he can even take the fraction of any item with him? 5

2	10	40
3	15	45
4	22	77
5	25	90

(b) Write an algorithm to search for an element and delete it if found, in a circular singly linked list. 5

6. (a) Given a singly linked list, write an algorithm to find middle of the singly linked list. For example, if given linked list is 1→2→3→4→5 then output should be 3. If there are even nodes, then there would be two middle nodes, we need to print second middle element. For example, if given linked list is 1→2→3→4→5→6 then output should be 4. 5

(b) Sort the following array using merge sort. Show each step.

3, 8, 4, 10, 1, 5, 6, 9 5

7. (a) Why do you need to represent the below mentioned matrix as a sparse matrix?

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 5 & 0 & 2 \\ 8 & 0 & 0 & 0 & 5 & 6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 6 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Show how the elements will be stored in one-dimensional array in row-major order and column-major order. 6

(b) What is the output of the following algorithm :

fun (integer n)

```
{
    if(n>0)
    {
        fun(n-1)
        print(n)
        fun(n-1)
    }
}
main()
{
    fun(4)
    return 0
}
```

Draw the tree showing all the calls generated by fun(4). 4

8. Write an algorithm to perform following operations :
   (a) Insert a value in front of a doubly linked list.
   (b) Delete a value from end of a circular singly linked list.
   (c) Print elements of a singly linked list in reverse order.

3+4+3

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 7168

Unique Paper Code : 62357502 J

Name of the Paper : Differential Equations

Name of the Course : B.A. (Prog.) Mathematics : DSE-2

Semester : V

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$(y+\sqrt{x^2+y^2})\,dx - x\,dy = 0;\;\; y(1) = 0.$$

(*b*) Solve : 6

$$(xy^2 - x^2)\,dx + (3x^2y^2 + x^2y - 2x^3 + y^2)\,dy = 0$$

(*c*) Solve : 6

$$y + px = x^4p^2$$

2. (*a*) Solve : 6.5

$$\frac{d^2y}{dx^2} + 4 = \cos 2x + \sin 2x$$

P.T.O.

(b) Solve :

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$$

(c) Consider the differential equation :

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$$

(i) Verify that $y_1 = e^x$ and $y_2 = e^{2x}$ are the sol of the above differential equation.

(ii) Find a particular solution of the form

$$y = c_1y_1 + c_2y_2$$

that satisfies the initial condition $y(0) =$ $y'(0) = 0$.

3. (a) Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\,e^x\log x,\quad (x>0)$$

(b) Given that $y = x + 1$ is a solution of different equation :

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+3y=0.$$

Find a linearly independent solution by reducing the order and write the general solution.

(c) Solve : 6

$$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^2 \log x + 3x$$

4. (a) Solve the following system of equations : 6.5

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,$$

$$\frac{dy}{dt} + 3y + 5x = 0.$$

(b) Solve : 6.5

$$\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{5z + \tan(y - 2x)}$$

(c) Solve : 6.5

$$(ydx + xdy)(a - z) + xydz = 0.$$

5. (a) Eliminate the arbitrary function f from the equation : 6

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

(b) Find the general solution of the differential equation : 6

$$x(y^2 - z^2)q - y(x^2 + z^2)p = (x^2 + y^2)z.$$

(c) Find the complete integral of the partial differential equation : 6

$$16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0.$$

6\. (*a*) (*i*) **Classify the following partial differential equa[tion]** into elliptic, parabolic or hyperbolic :

$$x(xy-1)r-(x^2y^2-1)s+y(xy-1)t+(x-\ldots$$
$$+(y-1)q=\ldots$$

where $r=\dfrac{\partial^2 z}{\partial x^2}$, $s=\dfrac{\partial^2 z}{\partial x\partial y}$, $t=\dfrac{\partial^2 z}{\partial y^2}$, $p=\dfrac{\partial z}{\partial x}$, $q=\dfrac{\partial z}{\partial y}$

(*ii*) Form a partial differential equation by eliminati[ng] constants a, b from the relation :

$$z=ax+by+cxy.$$

(*b*) Find the general solution of the differential equation : 6

$$x^2(y-x)q+y^2(x-y)p=z(x^2+y^2).$$

(*c*) Find the complete integral of 6

$$px+qy=pq.$$

[This] question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7282**

Unique Paper Code : **42343306** **J**

Name of the Paper : **Office Automation Tools**

Name of the Course : **B.Sc. (Prog.)/B.Sc. Math. Sciences : SEC**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **25**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A : *All* questions are compulsory.

Section B : Attempt any *three* questions.

Parts of question must be answered together.

Section A

10×1=10

(*a*) In a spreadsheet, which of the following functions is used to find the highest number out of a matrix of numbers :

(*i*) HIGH(B1:B10)

(*ii*) MAX(B1:B10)

(*iii*) BIG(B1:B10)

(*iv*) None of the above

P.T.O.

(*b*) Which of these toolbars allows us to change the Fonts and their sizes :

(*i*) Standard

(*ii*) Formatting

(*iii*) Options

(*iv*) None of the above

(*c*) Which of the following pair of short cuts is used for Cut and Paste :

(*i*) Ctrl +C, Ctrl +V

(*ii*) Ctrl +X, Ctrl +V

(*iii*) Ctrl +V, Ctrl +C

(*iv*) Ctrl +G, Ctrl +X

(*d*) Which of the following is not a font style ?

(*i*) Bold

(*ii*) Superscript

(*iii*) Regular

(*iv*) Italic.

(f) runif(10) creates ten random numbers. 1×5

2. Do any *five* of the following:

Fill in the blanks:

(a) For rotating data tables, we can use the command .... (trans(), t()).

(b) .... command produces the sum values for rows. (rowsums() / rowSums())

(c) hist() command is used for .... (history, histogram).

(d) .... command can sort the data. (order() / rank())

(e) .... command can be used to save one or more objects to a file. (load() / save())

(g) Tables can be summarized using the ...... command. (apply() / attach()) 1×5

3. Write the commands for the following:

(a) (i) Using scan command, enter the following data:

Week: sun, mon, tue, wed

(ii) Insert the items thu, fri, sat at the end of the above vector.

(b) (i) Write the command to print the object starting with 'ti'.

(*e*) Fill in the blank :

Landscape is ....

(*i*) A font style

(*ii*) Page Orientation

(*iii*) Paper Size

(*iv*) Page Layout

(*f*) What is the largest digit in the *octal* number system ?

(*g*) Give Hexadecimal digit for the decimal number 12.

(*h*) What is the value of sign bit for a negative number ?

(*i*) Give four digit ten's complement of the decimal number 0074.

(*j*) How many characters can be represented using ASCII-8 coding ?

Section B

2.

S.no	Employee Name	Basic Salary	HRA	DA	Allowance	Gross Salary
1.						
2.						
3.						

Write a formula for each of the following tasks :

(*a*) Calculate Gross Salary = Basic + HRA + DA +Allowance. 1

(*b*) Count the number of employees whose Basic Salary = 10,000. 1

(*c*) Calculate HRA = 30% of the Basic Salary. 1

(*d*) Calculate Allowance as : 2

₹ 5,000/-, if Basic is less than or equal to ₹ 10,000/-

₹ 7,500/-, if Basic is greater than ₹ 10,000/- and less than or equal to ₹ 20,000/-

₹ 10,000/-, if Basic is greater than ₹ 20,000/-

3. (*a*) Perform the following binary addition : 3

$$(10111)_2 + (11010)_2 + (111)_2.$$

(*b*) Give Truth Table for NAND gate. 2

4. (*a*) Write the steps to insert *fade* effect during slide transition in a presentation software. 3

(*b*) Write the steps to create a hyperlink on slide number 1 to open slide number 8 in a presentation software. 2

(a) Using 2's complement, perform the following operation using eight bits : 3

$$(+7) + (-19)$$

(b) Convert the hexadecimal number D3F4 to binary and octal. 2

6. (a) What is a *Filter* in a spreadsheet software ? Give an example of a filter. 3

(b) Give steps to insert a header in a document. 2

7. Differentiate between :

(a) Save and Save as 2

(b) New and Open 2

(c) Print and Print Preview. 1

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7280**

Unique Paper Code : **42353327** **J**

Name of the Paper : **Mathematical Typesetting System Latex**

Name of the Course : **B.Sc. (Prog.)/B.Sc. Math. SC : SEC**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(*i*) Boldfaced mathematical text is produced with a .... command.

(*ii*) The symbols .... may be used instead of pair of $ sign.

(*iii*) Emphasized text is produced with .... command.

(*iv*) For plotting a function with PSTricks, we need the .... package.

(v) Output of the command $\frac{d} {dx}\left(int_{0}^{x} f(t)\,dt\right)=f(x)$ is ....

2. Answer any *six* parts from the following : 2·5×6=15

(i) Write the command in LaTeX to obtain the expression :

$$\left(\frac{a+b}{x+y}\right)^{\frac{2}{3}}$$

(ii) Write the command in LaTeX to obtain the expression :

$$(a + b + a^2b + ab^2)^2.$$

(iii) Explain the command \pscircle(3,2.5){2.5}.

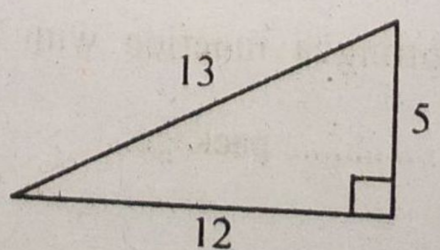
(iv) Explain the command \psarc(1,1){2}{0}{65}.

(v) What is the difference between the commands **\eqnarray** and **\eqnarray***.

(vi) Write a code in LaTeX to get the following :

$$\begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

(vii) Write the command in PSTricks to draw the following picture :



(*viii*) Write the command in PSTricks to plot the function $y = \sin x$.

(*ix*) What is wrong with the following input :

If \$theta = pi\$, then \$sin theta = 0\$

3. Answer any *four* parts from the following : 4·5×4=18

(*i*) Using beamer produce a presentation with the following content :

Slide 1 : Title of the presentation with authors name and date

Slide 2 : Some trigonometry identities :

$$\sin^2 \theta + \cos^2 \theta = 1$$

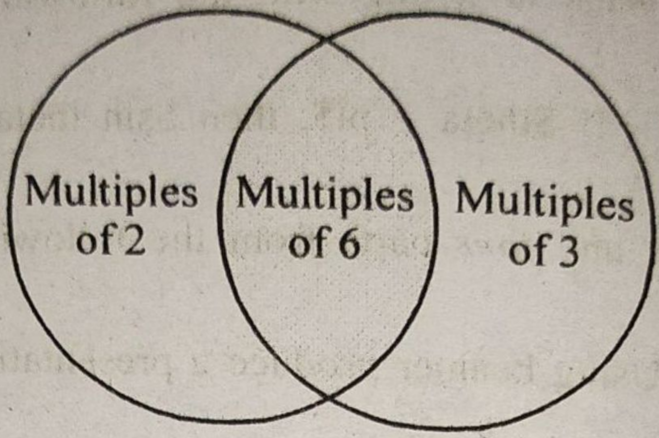
$$2 \sin \theta \cos \theta = \sin 2\theta$$

Slide 3 : Thank you

(*ii*) Write the code in LaTeX to typeset the following matrix :

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

(*iii*) Write the code in PSTricks to draw the following picture :



(*iv*) Write a code in LaTeX to get the following :

The general solution to the differential equation $y'' - 3y' + 2y = 0$ is

$$y = c_1 e^x + c_2 e^{2x}$$

(*v*) Write a code in LaTeX to get the following :

$$|x-2| = \begin{cases} x-2 & x \geq 2 \\ -x+2 & x < 2 \end{cases}$$

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This question paper contains 5 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : **7289**

J

Unique Paper Code : **42353503**

Name of Paper : **Statistical Software R**

Name of Course : **B.Sc. (Math. Sc.) / B.Sc. (Prog.) : SEC**

Semester : **V**

Duration : **2 hours**

Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

All commands should be written in software R.

1. Do any *five* of the following:

State whether the following statements are true or false:

(a) The commands mean() and rowMeans() for a data frame give the same output.

(b) read.csv(choose.file()) is used to read a file.

(c) If we don't have any named object at all, then ls() command gives NA.

(d) getwd() and setwd() are same commands.

(e) $ syntax is used to copy a data.

P. T. O.

(f) runif(10) creates ten random numbers.

1x5

2. Do any *five* of the following:

Fill in the blanks:

(a) For rotating data tables, we can use the command .... (trans(), t()).

(b) .... command produces the sum values for rows. (rowsums() / rowSums())

(c) hist() command is used for .... (history, histogram).

(d) .... command can sort the data. (order() / rank())

(e) .... command can be used to save one or more objects to a file. (load() / save())

(g) Tables can be summarized using the ...... command. (apply() / attach()) 1x5

3. Write the commands for the following:

(a) (i) Using scan command, enter the following data:

Week: sun, mon, tue, wed

(ii) Insert the items thu, fri, sat at the end of the above vector.

(b) (i) Write the command to print the object starting with 'ti'.

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(ii) Find the length of the vector data_new.

(c) For the vector

production: 9, 10, 15, 10, 6, 8, 11, NA

(i) Find the largest of the given data, after removing the effect of NA.

(ii) Show the last three items of the vector.

(d) (i) Differentiate between data structure and a matrix.

(ii) Convert the following data into integer:

Data 7: 23.0 17.5 14.5 12.3 12.9

Consider the following data, for the questions (e) – (h):

Len : 12, 23, 45, 23, 16, 31

Speed : 12, 34, 16, 21

Noo : 2, 6, 5, 8

(e) Create a matrix mew.

(f) (i) Display the data for the columns Len, Speed.

(ii) Display the data of first and second row for the above sample.

(g) (i) Convert the above matrix into a data frame.

(ii) Convert the matrix into a list.

(h) Add the appropriate row heads. $2\times8=16$

4. Do any *four* of the following:

(a) (i) Create the following matrix:

>bird

	Garden	Hedgerow	Parkland
Blackbird	47	10	40
Chaffinch	19	3	5
Great Tit	50	0	10
Robin	9	3	8

(ii) Display the mean of second row and second column.

(iii) Display the sum of rows and columns of the matrix.

(b) (i) Make the data objects:

data1 : 3 3 8 4 2 7 1

data2 : a b c d e f g h

(ii) Create a Cleveland dot plot using above data. Set the background colour for the plotting symbols.

(iii) Set the character expansion factor for points and label the axes.

(c) Create mathematics1data file where,

Mathematics1 = 2, 9, 8, 4, 6, 2, 7, 5, 2, 7.

Also make a quantile-quantile plot.

(d) (i) Display the data frame:

> orchid

	closed	open
a	3	5
b	5	3
c	7	8
d	9	4
e	3	9

(ii) Scatter plot the columns of above data. Also, write down the command to use different plotting characters.

(iii) How do you get the best size and scale of each axis to fit the plotting area?

(e) (i) Make a vector:

Data2 : 3 3 8 4 2 7 1 5 7 2 8 7

(ii) Create a histogram for above data.

(iii) Specify the breaks of bars at nos. 2, 5, 6, 9.

(iv) Color the bars and suppress the main title for the histogram.

(28)

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7058** **J**

Unique Paper Code : 62351101-- OC

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : Calculus

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

1. (a) Discuss the existence of the limit of the function

$$f(x)=\frac{e^{1/x}-1}{e^{1/x}+1}$$

as $x \to 0$. 6

(b) Examine the continuity of the function

$f(x) = |x-2| + |x-3|$

at $x = 2$ and $x = 3$. 6

P.T.O.

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(c) Discuss the derivability of the function.

$$f(x)=\begin{cases}2x-3 & \text{if } 0\le x\le 2\\ x^2-3 & \text{if } 2<x\le 4\end{cases}$$

at $x = 2$. 6

2. (a) If $y = \sin(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$$ 6.5

(b) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{y}{x}$

where $x \neq 0$ and $y \neq 0$, prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}.$$ 6.5

(c) State Euler's theorem and using it prove that

if $z = \log\left(\frac{x^4+y^4}{x+y}\right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3$. 6.5

3. (a) Show that the length of the portion of the tangent to the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ intercepted between coordinate axes is constant. 6

(b) Find the equation of normal to the curve $y^2(x+a) = x^2(3a-x)$ at the point where $x = a$. 6

c) Find the radius of curvature at any point P(x, y) on the curve $x = e^t \cos t,\ y = e^t \sin t$. 6

(a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$ 6.5

(b) Find the position and nature of the double points on the curve

$x^3 - 2x^2 - y^2 + x + 4y - 4 = 0$ 6.5

(c) Trace the curve

$x(x^2 + y^2) = a(x^2 - y^2)$ 6.5

(a) Verify Lagrange's Mean Value Theorem for the function

$f(x) = (x-1)(x-2)(x-3)$ in $[1,4]$. 6

(b) Separate the intervals in which the following function is increasing or decreasing : 6

$f(x) = 2x^3 - 15x^2 + 36x + 1$

(c) Show that $\frac{x}{1+x} < \log(1+x) < x \ \forall x > 0.$ 6

 P.T.O.

6. (a) Find the maximum and minimum value of the function $f(x) = (x-1)(x-2)(x-3)$ 6.5

(b) Obtain Maclaurin's series expansion for the function $f(x) = e^x$ for all $x \in \mathbb{R}$. 6.5

(c) Evaluate $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$. 6.5

(29)

[This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : 8649

J

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : B.A. (Programme) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

Marks are indicated against each question.

1. (*a*) Evaluate : $\lim_{x\to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$. 6

(*b*) Examine for points of discontinuity of the function f defined on [0, 1] as follows :

$$f(x) = \begin{cases} 0, \text{ if } x = 0 \\ \frac{1}{2} - x, \text{ if } 0 < x < \frac{1}{2} \\ \frac{1}{2}, \text{ if } x = \frac{1}{2} \\ \frac{3}{2} - x, \text{ if } \frac{1}{2} < x < 1 \\ 1, \text{ if } x = 1 \end{cases}$$

State the types of discontinuity also. 6

P.T.O.

(c) Discuss the derivability of the function :

$$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 5x-4, & \text{if } 0 < x \le 1 \\ 4x^2-3x, & \text{if } 1<x<2 \\ 3x+4, & \text{if } x \ge 2 \end{cases}$$

at $x = 0, 1, 2.$ 6

2. (a) Show that $y = x + \tan x$ satisfies the differential equation

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0.$$ 6.5

(b) If $y = \cos(m \sin^{-1} x)$, show that : 6,5

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

(c) If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, use Euler's theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2} \tan u.$$ 6,5

3. (a) Find the point on the curve :

$$x = a(\theta + \sin\theta), y = a(1-\cos\theta),$$

where the tangent is perpendicular to x-axis. 6

(b) Find the equation of the normal at (a, a) to the curve : 6

$$x^2y^3 = a^5.$$

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(c) Find the radius of curvature at any point of the curve : 6

$$x = a(\cot t + t \sin t), y = a(\sin t - t \cos t)$$

(a) Find the asymptotes of the following curve : 6,5

$$xy^3 - x^3 = a(x^2 + y^2).$$

(b) Find the position and nature of the double points on the curve : 6,5

$$y^2 = 2x^2 y + x^4 y - 2x^4.$$

(c) Trace the curve : 6,5

$$9ay^2 = x(x - 3a)^2.$$

(a) Show that there is no real number 't' for which the equation $x^2 - 3x + t = 0$ has two distinct roots in [0, 1]. 6

(b) Show that : 6

$$\frac{x}{1+x} < \log(1 + x) < x \text{ for all } x > 0.$$

(c) State Lagrange's mean value theorem. Explain why Lagrange's mean value theorem is not applicable to the following function :

$$f(x) = \begin{cases} x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

for $x \in [-1, 1]$. 6

6. (*a*) Assuming the validity of expansion, find the Maclaurin's series expansion of $e^x \cos x$. 6.5

(*b*) Derermine the values of p and q for which

$$\lim_{x \to 0} \frac{x(1 + p\cos x) - q\sin x}{x^3}$$ exists and equals 1. 6.5

(*c*) Show that $x^4 - 4x^3 + 6x^2 - 4x + 1$ has a maximum at $x = 1$. 6.5

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[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7099** **J**

Unique Paper Code : 62354343

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : Analytical Geometry and Applied Algebra

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** questions are compulsory.

(iii) Attempt any **two** parts from each question.

1. (a) Identify and sketch the graph

$y = 4x^2 + 8x + 5$

Also label the focus, vertex and directrix. 6.5

(b) Describe the graph of the curve

$x^2 + 9y^2 + 2x - 18y + 1 = 0$

Find its foci, vertices and the ends of the minor axis. 6.5

P.T.O.

(c) Sketch the hyberbola

$16x^2 - y^2 - 32x - 6y = 57$

Also find its vertices, foci and asymptotes. 6.5

4.

2. (a) Find an equation for the parabola that has vertex (5, –3), axis parallel to the y-axis and passes through (9,5). 6

(b) Find an equation for the ellipse that has ends of major axis (±6,0) and passes through(2,3). 6

(c) Find an equation for a hyperbola that satisfies the given conditions :

Asymptotes $y = 2x + 1$ and $y = -2x + 3$ passes through the origin. 6

3. (a) Consider the equation $xy = -9$. Rotate the coordinate axes to remove xy-term. Then identify and sketch the curve. 6.5

(b) Let an x'y' coordinate system be obtained by rotating an xy - coordinate system through an angle of $\theta = 60^0$.

(i) Find the x'y'- coordinates of the point whose xy – coordinates are (–2,6)

(ii) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in x'y' - coordinates. 6.5

(c) (i) Describe the surface whose equation is given as

$x^2 + y^2 + z^2 + 2y - 6z + 25 = 0.$

(ii) Determine and sketch the surface represented by the equation $x^2 + y^2 = 25$ in 3-space. 6.5

4. (a) Find u and v if $5u + 2v = 6i - 5j + 4k$ and $3u - 4v = i + 2j + 9k$. Also find a vector of length 3 and oppositely directed to v. 6

(b) (i) Show that direction cosines of a vector satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(ii) Determine if $u = < 6, 1, 3 >$ and $v = < 4, -6, -7 >$ make an acute angle, an obtuse angle or are orthogonal ? Justify your answer. 6

(c) Find the volume of the parallelepiped with adjacent edges $u = 3i + 2j + k$, $v = i + j + 2k$ and $w = i + 3j + 3k$. Also find the area of the face determined by u and v. 6

5. (a) Find the distance from the point P(1,4,-3) to the line 6.5

$L : x = 2 + t,\ y = -1 - t,\ z = 3t.$

(b) Find the equation of the plane through the points $P_1\ (-2,1,4)$, $P_2\ (1,0,3)$ that is perpendicular to the plane $4x - y + 3z = 2$. 6.5

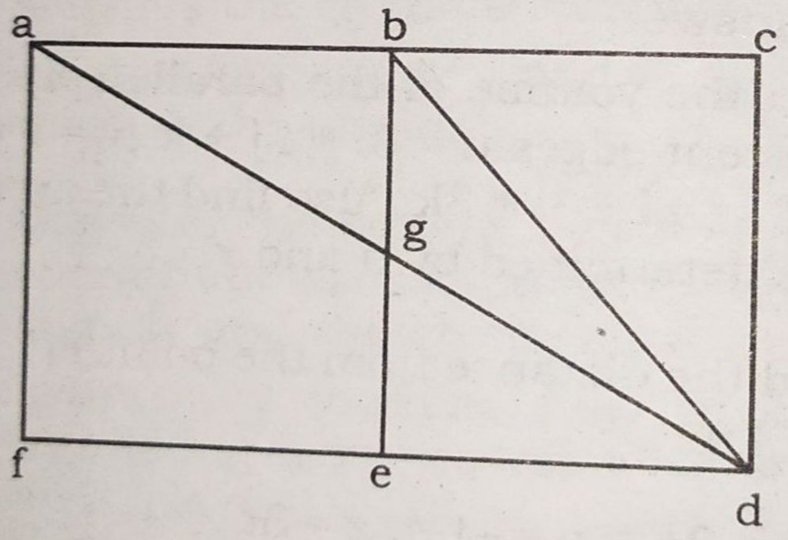
(c) Find the distance between the skew lines

$L_1 : x = 1 + 4t,\ y = 5 - 4t,\ z = -1 + 5t;$

$L_2 : x = 2 + 8t,\ y = 4 - 3t,\ z = 5 + t.$ 6.5

6. (a) Suppose there are three pitchers of water with capacity 8L,5L and 3L. Initially, the 8 L pitcher is full and the other two are empty. Is there a way to pour among pitchers to obtain exactly 4 litres in 5L pitcher or 3L pitcher ? If so, find the minimal sequence of pourings to get 4 litres in either 5L pitcher or 3L pitcher. 6

(b) For the following graph find a minimal edge cover and a maximal independent set of vertices 6

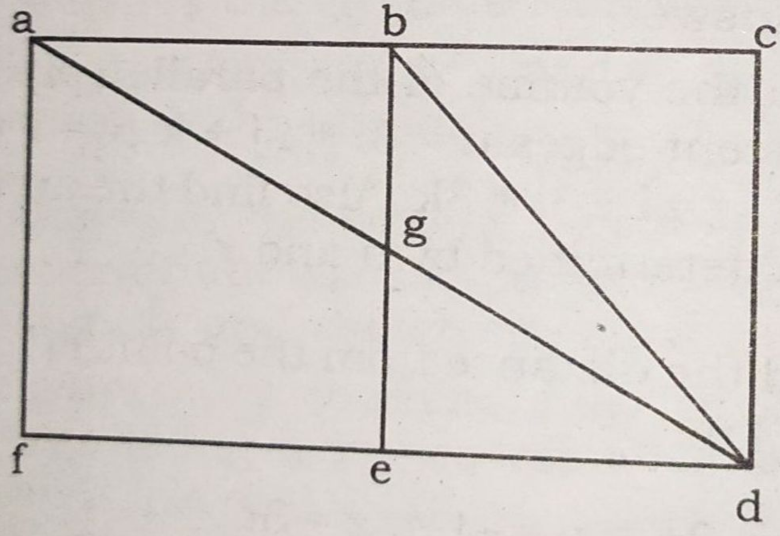


(c) A supermarket wishes to test the effect of putting cereals on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of the cereal. 6

7099

6. (a) Suppose there are three pitchers of water with capacity 8L,5L and 3L. Initially, the 8 L pitcher is full and the other two are empty. Is there a way to pour among pitchers to obtain exactly 4 litres in 5L pitcher or 3L pitcher ? If so, find the minimal sequence of pourings to get 4 litres in either 5L pitcher or 3L pitcher. 6

(b) For the following graph find a minimal edge cover and a maximal independent set of vertices 6



(c) A supermarket wishes to test the effect of putting cereals on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of the cereal. 6

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This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 7229

Unique Paper Code : 62353505 J

Name of the Paper : Statistical Software R

Name of the Course : B.A. (Prog.) Mathematics : S.E.C

Semester : V

Duration : 2 Hours Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

All commands should be written in software R.

1. Do any *five* of the following : 5×1=5

State whether the following statements are true or false :

(*i*) rm( ) command finds the defined variables.

(*ii*) colors( ) and colours( ) commands give the same output.

(*iii*) Quantile-Quantile plots are used for visualizing data in a straight line.

(*iv*) c (3 5 7 9) gives a vector.

P.T.O.

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(v) sample( ) command selects random elements from

(vi) ls.str( ) command finds the structure of all the d

objects.

2. Do any *five* of the following :

Fill in the blanks :

(i) ....command is used to make scatte

(splot( ) / plot( ))

(ii) ....command can be used to view the c

type of an object. (summary( ) / class( ))

(iii) names( ) command is used for viewing......................

(rows/columns)

(iv) ....command is used to generate a seque

10 random numbers. (seq(10)/ rseq(10))

(v) Command for $\cot^{-1}(x)$ is...................... (aco

arccot(x))

(vi) ....command rearranges the items in a vec

be in order (sort( ), order( )).

3. Write the commands in R for the following : 2×8=16

(*a*) (*i*) Read data from the file "hybrid .txt" .

(*ii*) Using scan function, enter the following data :

Subject : Eng Sociology Science History

(*b*) (*i*) List the object starting with b or ending with t.

(*ii*) Save the commands in a file with name "commands."

Use data : 2 3 7 2 4 3 2 5 6 3 1 3·7 8; for question(c) and (d)

(*c*) (*i*) Display the values less than 4 and greater than 6.

(*ii*) Count the items in the above sample.

(*d*) (*i*) Create a contingency table.

(*ii*) Create a stem and leaf plot.

Consider the following dataframe 'data', for questions (e)-(g) :

data :

data 1	data 2	data 3
23	25	34
43	32	56
23	65	21
34	76	78
32	67	32

(e) (i) Print the first, third and fifth rows.

(ii) Sort the above sample.

(f) Convert the above data frame into a matrix with 'consumers'. Also, determine the structure of it.

(g) Add the row names : FY2012 FY2013 FY201[illegible] FY2016 to the above dataframe.

(h) Explain the command :

data1[seq (1, length (data 1), 2)]

4. Do any *four* of the following :

(a) (i) Create the following data frame :

>bird

	Garden	Hedgero
Blackbird	47	10
Chaffinch	19	3
Great Tit	50	0
Robin	9	3

(ii) Plot a bar chart of above data.

(iii) Alter the scale of the y-axis and add axis lab[illegible]

(b) (i) Display the data frame :

> rainfall

	rain	Day
1	3	Mon
2	5	Tue
3	7	Wed
4	9	Thu
5	3	Fri

(ii) Plot above data, label the axes.

(iii) Enclose the whole plot in a bounding box.

(c) (i) Make a vector

data 1 : 3 5 7 6 5 7 3 3 8 4 2 7 1.

(ii) Display three quantiles 20%, 50% and 80%.

(iii) Display three quantiles by suppressing the headings.

(iv) What is the use of fivenum() command ?

(d) (i) Display the data frame :

> fw

	count	Speed
Taw	3	5
Torridge	5	3
Ouse	7	8
Exe	9	4
Pit	3	9

(*ii*) Display the mean values of each row and column in above sample data.

(*iii*) Explain the following :

> apply (fw, 1, mean, na.rm = TRUE).

(*e*) (*i*) Make a vector

Data 2 : 3 3 8 4 2 7 1 5 7 2 8 7.

(*ii*) Create a histogram for above data.

(*iii*) Specify the breaks of bars at nos. 2, 5, 6, 9.

(*iv*) Color the bars and suppress the main title for the histogram.

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Your Roll No. : ....

Sl. No. of Q. Paper : **7200 J**

Unique Paper Code : 62353326

Name of the Course : **B.A. (Prog.) Math : SEC**

Name of the Paper : Mathematical Typesetting System

Semester : III

Time : 2 Hours ***Maximum Marks : 38***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

1. Fill in the blanks : 1×5=5

(i) The symbols .... may be used instead of a pair of $ signs.

(ii) The output of $\sqrt[3]{x+y}$ is ......

(iii) The command \psset{unit=1.5} changes units from ....

(iv) The ...... commands is used to create a sector of a circle.

P.T.O.

(v) In beamer, the command ...... is used to sho the elements of a list one point at a time.

2. Answer any **six** parts from the following :

2.5×6=1

(i) Write the command in LaTeX to obtain th expression $\lim\limits_{x\to\infty}\frac{\pi(x)}{x/\log x}=1$.

(ii) Explain the command \psellipse(2,2)(1.5,1

(iii) Write the output of the command :

$$ \det V_n=\prod_{1 \leq i < j \leq n}(x_
x_i) $$

(iv) Typeset the following in LaTeX :

$$R_\theta=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

(v) Write the output for the following command

(a) \ast, (b) \notin, (c) \Leftarrow,

(d) \pm, (e) \cap .

(vi) Explain the command \pscircle(-2,1){2.5}.

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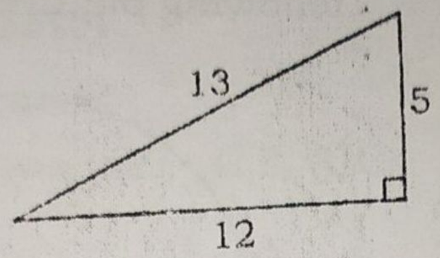
(vii) Write the command in PSTricks to draw the following picture



(viii) Write the command in PSTricks to plot the function y = cos (x).

3. Answer any **four** parts from the following :

4.5×4=18

(i) Write the code to make the following multi-line equations

$$\begin{aligned}(a+b)^2 &= (a+b)\,(a+b)\\ &= (a+b)a + (a+b)b\\ &= a\,(a+b) + b\,(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

(ii) Write the code to typeset the following :

$$f(x)\begin{cases} -x^2, & x < 0\\ x^2, & 0 \le x \le 2\\ 4, & x > 2\end{cases}$$

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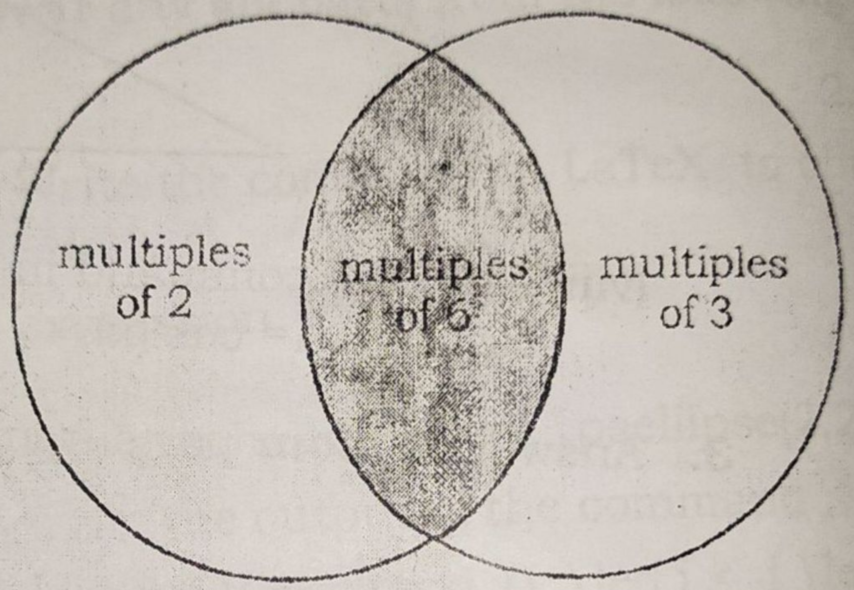
(iii) Write the code in PSTricks to draw the following picture



(iv) Write the code in PSTricks to plot the cardioid given by the parametric equations :

$x = \cos t\,(1-\cos t)$

$y = \sin t\,(1-\cos t),\ 0 \le t \le 2\pi$

(v) Write a code to make a beamer presentation of 5 pages (including title and thank you page) on any topic with diagram/picture.

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(73)

This question paper contains 4 printed pages.

Your Roll No. ....

. No. of Ques. Paper : 8227 J

nique Paper Code : 32355101

ame of Paper : Calculus

ame of Course : Mathematics : G.E. (OC)

emester : I

uration : 3 hours

laximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks, 5 each.

Attempt any five questions from each Section.

SECTION I

. Use $\in$–δ definition to show that $\lim_{x\to 0} x\sin\frac{1}{x} = 0$.

. Find the horizontal and vertical asymptotes of the curve $f(x) = \frac{x-1}{x^2+2}$.

. Find the linearization of $f(x) = \cos x$ at $x = -\frac{\pi}{2}$.

. For $f(x) = 4x^3 - x^4$:

(i) Find the intervals on which f is increasing and the intervals on which f is decreasing.

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(ii) Find where the graph of f is concave up and where it is concave down.

5. Use L' Hôpital's rule to find $\lim_{t \to 0} \frac{10(\sin t - t)}{t^3}$.

6. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

7. The radius r of a circle increases from $a = 10$ m to 10.1 m. Use dA to estimate the increase in the circle's area A. Estimate the area of the enlarged circle and compare your estimate to the true area.

SECTION II

8. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Use washer method to find the volume of the solid.

9. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

10. Is the area under the curve $y = \frac{(\ell n\, x)}{x^2}$ from $x = 1$ to $x = \infty$ finite? If so, what is it?

11. Use direct comparison test to determine whether $\int_1^\infty \frac{dx}{\sqrt{x} + e^{3x}}$ converges.

12. Sketch the graph of the curve $r = \cos 2\theta$ in polar coordinates.

13. If $r(t)$ is a differentiable vector-valued function of t of constant length, then show that $r(t)$ is orthogonal to $\frac{dr(t)}{dt}$ for all t. Verify this result for the function $r(t) = 3 \sin 5t\mathbf{i} + 9\mathbf{j} - 3 \cos 5t\mathbf{k}$.

14. Find the arc length parametrization for the helix $r(t) = 4 \cos t\mathbf{i} + 4 \sin t\mathbf{j} + 3t\mathbf{k}$, $0 \le t \le \frac{\pi}{2}$.

SECTION III

15. If $r(t) = \left(\frac{t^3}{3}\right)\mathbf{i} + \left(\frac{t^2}{2}\right)\mathbf{j}$, $t > 0$, find binomial vector and torsion.

16. Find the limit of f as $(x, y) \to (0, 0)$ and show that limit does not exist for the function $f(x, y) = \frac{x - y}{x + y}$.

17. If $f(x, y) = \cos^2 (3x - y^2)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

18. Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v if $z = 4e^x \ln y$, $x = \ln (u \cos v)$, $y = u \sin v$.

19. Find the directional derivative of the function f at P_0 in the direction of v where $f(x, y, z) = \cos xy + e^{yz} + \ln zx$, $P_0\left(1, 0, \frac{1}{2}\right)$ and $v = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

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20. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point :

Surfaces : $x^2 + y^2 = 4$, $x^2 + y^2 - z = 0$, Point : $\left(\sqrt{2}, \sqrt{2}, 4\right)$.

21. A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of cross-section) does not exceed 108 inches. Find the dimensions of the acceptable box of largest volume.

[This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8352**

Unique Paper Code : **32355301** **J**

Name of the Paper : **Differential Equations**

Name of the Course : **Generic Elective : Mathematics**

Semester : **III**

Duration : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions by selecting any *two* parts from each question.

1. (*a*) Show that the following first order ordinary differential equation :

$$(2x\cos y + 3x^2 y)dx + (x^3 - y - x^2 \sin y)dy = 0,$$

is exact and hence solve the equation with initial condition : $dy = 0$. 6.5

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(*b*) By finding an integrating factor, solve the initial value problem : 6.5

$$(2x^2 + y)dx + (x^2y - x)dy = 0, \ y(1) = 2.$$

(*c*) Solve the following Bernoulli equation : 6.5

$$\frac{dy}{dx} + (x+1)y = e^{x^2}y^3, \ y(0) = 0 \cdot 5.$$

2. (*a*) Find the orthogonal trajectories of the family of parabolas $y^2 = 2cx + c^2$. Is the orthogonal trajectories also a family of parabolas ? 6

(*b*) Solve the initial value problem :

$$y'' + 2y' + 2y = 0, \ y(0) = 1, \ y'(0) = -1.$$ 6

(*c*) Find a basis of the following differential equation $(xD^2 + 4D)y = 0$, where $D \equiv d/dx$. Also find the solution satisfying :

$$y(1) = 12, \ y'(1) = -6.$$ 6

3. (*a*) Solve by the method of variation of parameters :

$$y'' + 6y' + 9y = x^{-3}e^{-3x}, \ x > 0.$$ 6.5

(*b*) Solve the initial-value problem by the method of undetermined coefficients : 6.5

$$y'' + 3y' + 2 \cdot 25y = -10e^{-1.5x},\ y(0) = 1,\ y'(0) = -1.$$

(*c*) Find a homogeneous linear ordinary differential equation for which two functions $e^{-x}\cos x$ and $e^{-x}\sin x$ are solutions. Show also linear independence by considering their Wronskian. 6.5

4. (*a*) Solve the linear system that satisfies the stated initial conditions : 6

$$\frac{dy_1}{dt} = -3y_1 + 2y_2,\ y_1(0) = 1$$

$$\frac{dy_2}{dt} = y_1 - 3y_2,\ y_2(0) = -2.$$

(*b*) (*i*) Find the partial differential equation arising from the surface : 3

$$z = xy + f(x^2 + y^2).$$

(*ii*) Find the characteristics of the equation : 3

$$u_x - u_y = 1.$$

(c) Obtain the solution of the quasi-linear partial differential equation : 6

$$(y-u)u_x + (u-x)u_y = x-y,$$

with the condition $u = 0$ on $xy = 1$.

5. (a) Find a power series solution of the following differential equation : 6.5

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

(b) Find the general solution of the linear partial differential equation : 6.5

$$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0.$$

(c) Reduce the linear partial differential equation $u_x - yu_y - u = 1$ to canonical form, and obtain the general solution. 6.5

6. (a) Apply the method of separation of variables by taking $\log u(x,y) = f(x) + g(y)$, to solve the initial-value problem : 6

$$y^2u_x^2 + x^2u_y^2 = (xyu)^2, \quad u(x,0) = 3\exp(x^2/4).$$

(*b*) Determine the region in which the partial differential equation :

$$u_{xx} + xyu_{yy} + u_x + u_y + u = 1,$$

is hyperbolic, parabolic or elliptic, and transform the equation into canonical form for the parabolic region. 6

(*c*) Reduce the following partial differential equation with constant coefficients,

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$

into canonical form and hence find the general solution. 6

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 7264

Unique Paper Code : 62355503 J

Name of the Paper : General Mathematics—I

Name of the Course : Mathematics : Generic Elective

Semester : V

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions as per directed question wise.

Section I

1. Write a short note on the life and contributions of any *three* of the following mathematicians : 15

(*a*) Galois

(*b*) Reimann

(*c*) Poisson

(*d*) Cauchy

(*e*) Weierstrass.

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Section - II

2. Attempt any *six* questions. Each question carries *five* m

(*a*) Define Magic Square and state the propertie
Benjamin Franklin's Magic Square.

(*b*) What is the total number of matches in a Round-R
tennis tournament with 13 contestants ?

(*c*) What is a Perfect number ? Give Euclid's Formu
Perfect number. Is 28 a Perfect number ? Show

(*d*) Define Unit Fraction and express 2/5 and 98/100 a
fractions.

(*e*) State the Euclid's Algorithm. Using the above algo
find the greatest common divisor of 6237 and

(*f*) Show that the permutation (c e d b a) is an
permutation with the help of Inversions.

(*g*) Define Algebraic and Transcendental numbers.
transcendental numbers rational, irrational or both ?
examples.

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Section-III

3. Do any *three* questions. Each question carries *six* marks :

(*a*) If :

$$A = \begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix},$$

show that $(AB)^2 \neq A^2 B^2$.

(*b*) Decompose the matrix :

$$\begin{pmatrix} 1 & 0 & -4 \\ 3 & 3 & -1 \\ 4 & -1 & 0 \end{pmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

(*c*) If $A = \begin{pmatrix} 3 & -1 \\ 4 & 7 \end{pmatrix}$, find A^4.

(*d*) Find the adjoint and hence the inverse of the matrix :

$$\begin{pmatrix} -4 & 0 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

4. Do any *two* questions. Each question carries *six* m

(*a*) If :

$$A = \begin{pmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{pmatrix},$$

find determinant of A.

(*b*) If $A = \begin{pmatrix} 5 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 3 \\ 1 & -15 \\ -2 & -1 \end{pmatrix}$

then $|AB| = |A|\ |B|$?

(*c*) Use Cramer's Rule to solve the following syste

$$3x - y - z = -8$$

$$2x - y - 2z = 3$$

$$-9x + y = 39.$$