This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2248**

Unique Paper Code : **32351201** **IC**

Name of the Paper : **Real Analysis**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

There are internal choices in Q. Nos. **2-5**.

1. Prove or disprove : 6×2½=15

(*a*) If $x \in R$, $x > 0$, then $\frac{1}{x} > 0$.

(*b*) If s is an upper bound of a non-empty set S such that $s \in S$, then $s = \sup S$.

(*c*) A sequence (x_n) satisfying $\lim (|x_{n+1} - x_n|) = 0$ is convergent.

P.T.O.

(*d*) $\lim ((a^n + b^n)^{1/n}) = b$, where $0 < a < b$.

(*e*) The series $\sum_{n=1}^{\infty} (\cos nx)$ converges for all $x \in \mathbf{R}$.

(*f*) $\sum_{n=1}^{\infty} \frac{n2^n}{(n^2+1)}$ is a convergent series.

2. Answer any *three* parts : 3×5=15

(*a*) State and prove the Density Theorem for real numbers.

(*b*) (*i*) Let $a, b \in \mathbf{R}$ and suppose that for every $\varepsilon > 0$, we have $a \le b + \varepsilon$. Show that $a \le b$.

(*ii*) Let S be a non-empty subset of **R**. Show that $u \in \mathbf{R}$ is an upper bound of S if and only if the conditions $t \in \mathbf{R}$, $t > u$ imply $t \notin S$.

(*c*) Let S be a non-empty bounded set in **R** and let $b < 0$. Prove that $\inf (bS) = b (\sup S)$ and $\sup (bS) = b (\inf S)$.

(*d*) If S is a non-empty subset of **R**, show that S is bounded if and only if there exists a closed and bounded interval I of **R** such that $S \subseteq I$.

3. Answer any *three* parts : 3×5=15

(*a*) Find the limit of the following sequences whose nth term is given by :

(*i*) $x_n = \frac{n}{b^n}$, where $b > 1$

(*ii*) $y_n = \frac{\sin n}{n} + \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)$.

(*b*) Prove that if a sequence (x_n) is increasing and bounded above, then it converges to u where u is the least upper bound of the set $\{x_n : n \in \mathbf{N}\}$.

(*c*) If a sequence (x_n) of real numbers converges to a real number x, prove that every subsequence (x_{n_k}) of (x_n) converges to x.

(*d*) State the Cauchy Convergence Criterion for sequences. Use it to show that the sequence (x_n) defined by

$$x_n = \frac{1}{1!} - \frac{1}{2!} + \ldots\ldots + \frac{(-1)^{n+1}}{n!}$$

is convergent.

4. Answer any *three* parts : $3 \times 5 = 15$

(*a*) Use the integral test to check the convergence of the series :

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}.$$

(*b*) When do we say that the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent ? Show that the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(\sqrt{n+1} - \sqrt{n}\right)$$

is absolutely convergent.

(*c*) Test the convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$

(*ii*) $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}.$

(*d*) (*i*) Find all $x \in R$ for which the series $\sum_{n=1}^{\infty} e^{nx}$ converges.

(*ii*) Show that the series $\sum_{n=1}^{\infty} \log\left(\frac{n}{n+1}\right)$ is divergent.

5. (*a*) (*i*) Let X and Y be non-empty sets and let $h : X \times Y \to \mathbf{R}$ have bounded range in **R**. Let $f : X \to \mathbf{R}$ and $g : Y \to \mathbf{R}$ be defined by

$$f(x) = \sup \{h(x, y) : y \in Y\},$$

$$g(y) = \inf \{h(x, y) : x \in X\}.$$

Prove that :

$$\sup \{g(y) : y \in Y\} \leq \inf \{f(x) : x \in X\}.$$

(*ii*) Give an example of a set which has exactly two limit points. 4,1

Or

(*i*) Show that for any real numbers p, q and rational number r such that $r < p + q$, there exist rational numbers $r_1 < p$ and $r_2 < q$ such that $r = r_1 + r_2$.

(*ii*) Provide a bijection between **N** and the set of all odd integers greater than 49. 3,2 (*b*) (*i*) If $\lim (x_n) = x (\neq 0)$, prove that there is a positive number A and a natural number N such that $|x_n| > A$ for all $n \geq N$.

(*ii*) Is the sequence (x_n) where

$$x_n = \frac{n^3 + 3n^2}{n+1} - n^2$$

bounded ? Justify. 3,2

Or

Is the sequence (x_n) where

$$x_n = \frac{n^2}{n^3 + n + 1} + \frac{n^2}{n^3 + n + 2} + \ldots\ldots + \frac{n^2}{n^3 + 2n}$$

convergent ? If yes, find its limit. 5

(*c*) State the Alternating Series Test. Show that the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$$

is conditionally convergent. 5

Or

If the series $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ are convergent, then prove that the series

$$\sum_{n=1}^{\infty} a_n b_n$$

is convergent where $a_n \geq 0$ and $b_n \geq 0$ for all $n \in \mathbf{N}$.

Hence or otherwise show that the series $\sum_{n=1}^{\infty} \frac{a_n}{n}$ is convergent whenever $\sum_{n=1}^{\infty} a_n^2$ is convergent. 5

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2249**

Unique Paper Code : **32351202** **IC**

Name of the Paper : **Differential Equations**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use of non-programmable scientific calculators is allowed.

Section–I

1. Attempt any *three* parts. Each part is of **5** marks.

(*a*) Solve the initial value problem :

$$(x^2+1)\frac{dy}{dx}+4xy=x,\ y(2)=1.$$

(*b*) Determine the constant A in the following equation such that the equation is exact, and solve the resulting exact equation :

$$(x^2+3xy)dx+(Ax^2+4y)dy=0.$$

P.T.O.

(c) Solve the differential equation :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=4x.$$

(d) Solve the differential equation :

$$2xy\frac{dy}{dx}=x^2+2y^2.$$

2. Attempt any *two* parts. Each part is of **5** marks.

(a) A cylindrical tank with length 5 ft and radius 3 ft is situated with its axis horizontal. If a circular bottom hole with a radius of 1 inch is opened and the tank is initially half full of xylene, how long will it take for the liquid to drain completely ?

(b) Suppose that sodium pentobarbital is used to anesthetize a dog. The dog is anesthetized when its bloodstream contains at least 45 milligrams (mg) of sodium pentobarbital per kilogram of the dog's body weight. Suppose also that sodium pentobarbital is eliminated exponentially from the dog's bloodstream, with a half life of 5 hours. What single dose should be administered in order to anesthetize a 50 kilogram dog for 1 hour ?

(*c*) Suppose that a motorboat is moving at 40 ft/sec when its motor suddenly quits, and that 10 seconds later the boat has slowed to 20 ft/sec. Assume that the resistance it encounters is proportional to its velocity. How far will the boat cast in all ?

Section–II

Attempt any *two* parts. Each part is of **7.5** marks.

(*a*) Consider the American system of two lakes : Lake Erie feeding into Lake Ontario. Assuming that volume in each lake to remain constant and that Lake Erie is the only source of pollution for Lake Ontario.

(*i*) Write down a differential equation describing the concentration of pollution in each of two lakes, using the variables V for volume, F for flow, $c(t)$ for concentration at time t and subscripts 1for Lake Erie and 2 for Lake Ontario.

(*ii*) Suppose that only unpolluted water flows into Lake Erie. How does this change the model proposed ?

(*iii*) Solve the system of equations to get expression for the pollution concentration $c_1(t)$ and $c_2(t)$.

(*b*) In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X.$$

(*i*) Find the non-zero equilibrium population.

(*ii*) At what critical harvesting rate can extinction occur ?

(*c*) Consider the population of the country. Assume constant per capita birth and death rates and that the population follows an exponential growth (or decay) process. Assume there to be significant immigration and emigration of people into and out of the country.

(*i*) Assuming the overall immigration and emigration rates are constant, formulate a single differential equation to describe the population size over time.

(*ii*) Suppose instead that all immigration and emigration occurs with a neighbouring country, such that the net movement from one country to the another is proportional to the population difference between the two countries and such that people move to the country with the larger population. Formulate a coupled system of equations as a model for this situation.

Section–III

4. Attempt any *four* parts. Each part is of **5** marks.

(*a*) Find general solutions (for $x > 0$) of the Euler's equation :

$$x^2y'' + 7xy' + 25y = 0.$$

(*b*) Solve the initial value problem by using the method of undetermined coefficients :

$$y'' + y = \sin x\,;\ \ y(0) = 0, y'(0) = -1.$$

(*c*) Use the method of variation of parameters to find the solution of the differential equation :

$$y'' + 3y' + 2y = 4e^x.$$

(*d*) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -10$ m/s. Find the amplitude, period, and frequency of the resulting motion.

(*e*) A body of mass $m = 2$ kg is attached to both a spring with a spring constant $k = 4$ and a dashpot with a damping constant $c = 3$. The mass is set in motion with initial position $x_0 = 2$ and initial velocity $v_0 = 0$. Find the position function $x(t)$ and determine whether the motion is overdamped, critically damped or underdamped. If it is underdamped, find its pseudofrequency, pseudoperiod of oscillation and its time varying amplitude.

Section–IV

5. Attempt any *two* parts. Each part is of **7.5** marks.

(*a*) Consider a simple model for a battle between two armies. Assumed that the probability of a single bullet hitting its target is constant. Suppose that the soldiers from the red army are visible to the blue army. But the soldiers from the blue army are hidden.

(*i*) Develop the model for describing the rate of change of number of soldiers in each of the armies.

(*ii*) By making appropriate assumptions, extend the model to include the reinforcements if both of the armies receive reinforcements at constant rates.

(*b*) Consider a disease where all those who are infected remain contagious for life. Assume that there are no births and deaths.

(*i*) Write down suitable word equations for the rate of change of number Susceptible and Infective and hence develop a pair of differential equations.

(*ii*) Use the chain rule to find a relationship between the number of susceptibles and the number of infectives.

(*iii*) Draw a sketch of typical phase-plane trajectories. Deduce the direction of travel along the trajectories providing reasons.

(c) A model of a three species interaction is :

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ,$$

$$\frac{dY}{dt} = a_2XY - b_2Y,$$

$$\frac{dZ}{dt} = a_3XZ - b_3Z.$$

Where a_i b_i c_i for $i = 1, 2, 3$ are all positive constants. Here $X(t)$ is the prey density and $Y(t)$ and $Z(t)$ are the two predator species densities.

(i) Find all possible equilibrium populations. Is it possible for all three populations to coexist in equilibrium ?

(ii) What does this suggest about introducing an additional predator into an ecosystem ?

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2293**

Unique Paper Code : **42351201** **IC**

Name of the Paper : **Calculus and Geometry**

Name of the Course : **B.Sc. Mathematical Science/ B.Sc. (Prog.)**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

Marks of each part are indicated.

1. (*a*) Use (ε, δ) definition to prove that $\lim_{x \to -1} (7x + 5) = -2$. Find a $\delta > 0$ if $\varepsilon = 0.01$. 6

(*b*) Let the function be defined as :

$$f(x) = \begin{cases} 2x + 3, & x \le 4 \\ 7 + \dfrac{16}{x}, & x > 4 \end{cases}$$

Find the value of x (if any) at which f is not continuous. 6

P.T.O.

(c) Define uniform continuity of a function on the interval I. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on]0, 1]. 6

2. (a) Discuss the derivability of the function :

$$f(x) = \begin{cases} 2x - 3, & 0 \le x < 2 \\ x^2 - 3, & 2 \le x \le 4 \end{cases}$$

at $x = 2$. 6

(b) (i) State Lagrange's Mean Value Theorem and give its geometrical interpretation.

(ii) If f is a function satisfying all the hypothesis of Lagrange's Mean Value Theorem in $[a, b]$ and $f'(x) > 0 \ \forall x \in (a, b)$, then prove that f is increasing on $[a, b]$. 4,2

(c) Find the asymptotes of the curve :

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0.$$ 6

3. (a) Determine the intervals of concavity and points of inflexion of the curve

$$y = 2x^4 - 3x^2 + 2x + 1.$$ 6

(b) Prove that the curve $y^2 = (x - a)^2 (x - b)$ has at $x = a$, a node if $a > b$, a cusp if $a = b$ and a conjugate point if $a < b$. 6

(c) Trace the curve $y^2(a^2 - x^2) = x^4$. 6

4. (*a*) Trace the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \le \theta \le 2\pi$. 6.5

(*b*) Show that :

$$\int \sin^m x \cos^n x \, dx = \frac{-\cos^{n+1} x \sin^{m-1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \, dx$$

m, n being positive integers. 6.5

(*c*) Find the length of the loop of the Cardioid :

$$r = a(1 + \cos\theta).$$

Or

Evaluate the definite integral :

$$\int_0^{2a} x^2 \sqrt{2ax - x^2} \, dx.$$ 6.5

5. (*a*) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x-axis. 6

(*b*) Describe the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$ 6

(c) (i) Find an equation for the ellipse with foci $(0, \pm 2)$ and major axis with end-points $(0, \pm 4)$.

(ii) State reflection property of a parabola. 4,2

6. (a) Rotate the coordinate axes to remove the xy-term from the equation $x^2 - xy + y^2 - 2 = 0$. Then identify the type of conic and sketch its graph. 7

(b) (i) Find the divergence and the curl of the vector field :

$$\vec{F}(x, y, z) = x^2 y\hat{i} + 2y^3 z\hat{j} + 3z\hat{k}.$$

(ii) Sketch the ellipsoid :

$$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} = 1.$$ 4,3

(c) (i) Calculate $\frac{d}{dt}\left(\vec{F} \times \vec{G}\right)$ for vector functions :

$$\vec{F}(t) = 2t\hat{i} + 3t^2\hat{j} + t^3\hat{k} \text{ and } \vec{G}(t) = t^4\hat{k}.$$

(ii) Verify $\nabla \| \vec{r} \| = \frac{\vec{r}}{\| \vec{r} \|}$ for the radius vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$ 4,3

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3199 **IC**

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : **B.A. (Program) Mathematics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Show that the set $W = \{(a_1, a_2, 0) : a_1, a_2 \in \mathbb{R}\}$ is a subspace of the vector space $\mathbb{R}^3(\mathbb{R})$.

(b) Show that the vectors $\{(1,1,-1), (2,-3,5), (-2,1,4)\}$ in $\mathbb{R}^3(\mathbb{R})$ are linearly independent.

(c) Let $\{a,b,c\}$ be a basis for $\mathbb{R}^3(\mathbb{R})$. Show that the set $\{a+b, b+c, c+a\}$ is also a basis of $\mathbb{R}^3(\mathbb{R})$.

$(2\times6=12)$

P.T.O.

2. (a) Using elementary transformations find rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}.$$

(b) Solve the following system of equations :

$$x - y + 2z = 4$$
$$3x + y + 4z = 6$$
$$x + y + z = 1.$$

(c) Find the characteristic roots and the characteristic vectors for the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$ (2×6½=13)

3. (a) Prove that $128\sin^2\theta\cos^6\theta = -\cos 8\theta - 4\cos 6\theta - 4\cos 4\theta + 4\cos 2\theta + 5$.

(b) Solve the equation $x^4 - 5x^3 + 10x^2 - 10x + 4 = 0$, given that the product of two of its roots is equal to the product of other two roots.

(c) Let α, β, γ be the roots of the equation $x^3 + qx - r = 0$. Find the values of :

$$\sum \frac{\beta+\gamma}{\alpha^2} \text{ and } \sum \alpha^2 .$$ (2×6=12)

4. (a) Solve the equation $z^4 - z^3 + z^2 - z + 1 = 0$ using De Moivre's theorem.

(b) If α, β, γ are the roots of $x^3 + px^2 + qx + p = 0$, prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$ radians, except in one particular case.

(c) Find the sum of the series

$$\sin\alpha + c\sin(\alpha+\beta) + \frac{c^2}{2!}\sin(\alpha+2\beta) + \ldots + \infty$$

(2×6½=13)

5. (a) Show that the set of positive rational numbers Q^+ forms an abelian group under the operation '*' defined by: a*b = ab/2. Also, write the inverse of ½ with respect to this operation.

(b) Draw the diagram associated with the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 8 & 1 & 6 & 3 & 7 & 2 & 4 & 9 \end{pmatrix}.$$

(c) Show that inverse of each element in a group is unique. (2×6=12)

6. (a) Let G be a group and H be a non-empty subset of G. Then show that the set H is a subgroup of G if and only if $ab^{-1} \in H$ for all $a,b \in H$.

(b) Show that under the usual operations of matrix addition and multiplication, the set

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \mathbb{Z} \right\} \text{ is a ring.}$$

(c) With proper justification, give an example of a subring of $(\mathbb{Z},+,\cdot)$. (2×6½=13)

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2250**

Unique Paper Code : **32351401** **IC**

Name of the Paper : **Partial Differential Equations**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Section-I

1. Attempt any *three* parts out of the following :

(*a*) Determine the integral surfaces of the equation :

$$x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$$

with the data $x+y=0,\ \ u=1.$ 6

(*b*) Apply the method of separation of variables to solve the initial-value problem :

$$x^2u_{xy}+9y^2u=0,\ \ u(x,0)=\exp\left(\frac{1}{x}\right).$$ 6

(*c*) Reduce the following equation into canonical form and find the general solution :

$$u_x+u_y=u.$$ 6

P.T.O.

(*d*) Solve the initial-value problem :

$$u_t + uu_x = 0$$

with the initial curve $x = \frac{\tau^2}{2}, t = \tau, u = \tau.$ 6

Section-II

2. Attempt any *one* part out of the following :

(*a*) Show that the equation of motion of a vibrating string is :

$$u_{tt} = c^2 u_{xx}, \text{ where } c^2 = T/\rho.$$ 6

(*b*) Derive the wave equation of a string :

$$u_{tt} + au_t + bu = c^2 u_{xx},$$

where the damping force is proportional to the velocity, the restoring force is proportional to the displacement of a string, and a and b are constants. 6

3. Attempt any *two* parts out of the following :

(*a*) Determine the general solution of the equation :

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$

by reducing it into canonical form. 7

(*b*) Transform the equation to the form $v_{\xi\eta} = cv$, $c =$ constant,

$$u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$$

by introducing the new variable $v = ue^{-(a\xi + b\eta)}$, where a and b are undetermined coefficients. 7

(c) Classify the equation and reduce it to canonical form : 7

$$y^2 u_{xx} + 2xyu_{xy} + 2x^2 u_{yy} + xu_x = 0.$$

Section-III

4. Attempt any *three* parts out of the following :

(a) Determine the solution of the initial boundary-value problem : 7

$$\begin{aligned} u_{tt} &= 4u_{xx}, && 0 < x < \infty, t > 0, \\ u(x,0) &= x^4, && 0 \le x < \infty, \\ u_t(x,0) &= 0, && 0 \le x < \infty, \\ u(0,t) &= 0, && t \ge 0. \end{aligned}$$

(b) Find the solution of the initial boundary-value problem : 7

$$\begin{aligned} u_{tt} &= u_{xx} && 0 < x < 2, \quad t > 0, \\ u(x,0) &= \sin(\pi x / 2), && 0 \le x \le 2, \\ u_t(x,0) &= 0, && 0 \le x \le 2, \\ u(0, t) &= 0, \quad u(2, t) = 0, && t \ge 0. \end{aligned}$$

(c) Determine the solution of the Goursat problem : 7

$$u_{tt} = c^2 u_{xx},$$
$$u(x,t) = f(x) \text{ on } x - ct = 0$$
$$u(x,t) = g(x) \text{ on } t = t(x),$$

where $f(0) = g(0)$.

(*d*) Determine the solution of the initial boundary-value problem : 7

$$u_{tt} = c^2 u_{xx}, \quad x > 0, \; t > 0,$$
$$u(x,0) = f(x), \quad x \ge 0,$$
$$u_t(x,0) = g(x), \quad x \ge 0,$$
$$u_x(0,t) = q(t), \quad t \ge 0.$$

Section-IV

5. Attempt any *two* parts out of the following :

(*a*) Determine the solution of the initial boundary-value problem by the method of separation of variables : 8

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < 1, \; t > 0,$$
$$u(x, 0) = x(1-x), \quad 0 \le x \le 1,$$
$$u_t(x,0) = 0, \quad 0 \le x \le 1,$$
$$u(0, t) = u(1, t) = 0, \quad t \ge 0.$$

(*b*) Prove the uniqueness of the solution of the problem : 8

$$u_t = k u_{xx}, \quad 0 < x < l, \; t > 0,$$
$$u(x,0) = f(x), \quad 0 \le x \le l,$$
$$u_x(0,t) = u_x(l,t) = 0, \quad t \ge 0.$$

(c) Determine the solution of the initial-boundary value problem : 8

$$u_t = k u_{xx}, \quad 0 < x < \pi, \; t > 0,$$
$$u(x,0) = \sin^2 x, \quad 0 \le x \le \pi,$$
$$u(0,t) = u(\pi,t) = 0, \quad t \ge 0.$$

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2251**

Unique Paper Code : **32351402** **IC**

Name of the Paper : **Riemann Integration and Series of Functions**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Find the upper and the lower Darboux integrals for $f(x) = 2x^3$ on [0, 1]. Is f integrable on [0,1] ? Justify. 6

(*b*) Prove that a bounded function f on $[a, b]$ is Riemann integrable if and only if it is Darboux Integrable and the value of integrals agree. 6

(*c*) Prove that if f is integrable on $[a, b]$ then $|f|$ is integrable on $[a, b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$. What about the converse ? Justify your answer. 6

2. (*a*) Prove that every bounded piecewise monotonic function f on $[a, b]$ is integrable. 6.5

P.T.O.

(*b*) State Fundamental Theorem of Calculus II. Let f be defined as follows :

$$f(t)=\begin{cases}0 & \text{for } t<0\\ t & \text{for } 0\le t\le 1\\ 4 & \text{for } t>1\end{cases}$$

Determine the function $F(x)=\int_0^x f(t)\,dt$. Discuss the continuity and differentiability of F and also calculate F′. 6.5

(*c*) Let f be a bounded function on $[a, b]$. If P and Q are partitions of $[a, b]$ such that $P \subseteq Q$, show that :

$$L(f, P) \le L(f, Q) \le U(f, Q) \le U(f, P).$$ 6.5

3. (*a*) Examine the convergence of the following improper integrals :

(*i*) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

(*ii*) $\int_{-1}^1 \frac{dx}{x^3}$. 6

(*b*) Prove that :

$$\int_0^\infty \frac{\sin x}{x}\,dx$$

is convergent. 6

(*c*) Examine the convergence of the improper integral :

$$\int_0^1 x^{a-1}(1-x)^{b-1}\,dx.$$ 6

4. (a) Let (f_n) be a sequence of functions on $[a, b]$ converging uniformly to f on $[a, b]$. Prove that f is integrable on $[a, b]$ and

$$\int_a^b f = \lim \int_a^b f_n.$$ 6.5

(b) Let $f_n(x) = \dfrac{nx}{1+n^2x^2}$ $\forall\, x \in \mathbf{R}$.

(i) Show that (f_n) converges pointwise to $f(x) = 0$ on $\mathbf{R}$.

(ii) Does (f_n) converge uniformly to f on $[0, 1]$? Justify.

(iii) Prove that (f_n) converges uniformly to f on $[1, \infty[$. 6.5

(c) Let $f_n : [0, 1] \to \mathbf{R}$ be defined for $n \geq 2$ by

$$f_n(x) = \begin{cases} n^2 x & \text{for } 0 \leq x \leq 1/n \\ -n^2(x - 2/n) & \text{for } 1/n \leq x \leq 2/n \\ 0 & \text{for } 2/n \leq x \leq 1. \end{cases}$$

Show that the sequence (f_n) is not uniformly convergent. 6.5

5. (a) Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbf{R}$. If (f_n) converges uniformly to f on A, prove that f is continuous on A. 6

P.T.O.

(*b*) Prove that the series :

$$\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$$

is uniformly convergent on $[-a, a]$, $\forall\ a > 0$, but is not uniformly convergent on **R**. 6

(*c*) Prove that the series :

$$\sum \frac{\cos\left(x^2+1\right)}{n^3}$$

represents a continuous function on **R**. 6

6\. (*a*) Find the radius of convergence of the power series :

(*i*) $\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n$

(*ii*) $\sum_{n=1}^{\infty} x^{n!}$. 6.5

(*b*) Let $\sum a_n x^n$ has radius of convergence $R > 0$ and let :

$$f(x) = \sum a_n x^n \text{ for } |x| < R.$$

Then prove that f is differentiable on $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \text{ for } |x| < R.$$ 6.5

(*c*) Prove that :

$$\sum_{n=1}^{\infty} n^2 x^n = \frac{x(x+1)}{(1-x)^3}$$

for $|x| < 1$. Hence evaluate :

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}.$$ 6.5

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2252

Unique Paper Code : 32351403 IC

Name of the Paper : **Ring Theory and Linear Algebra-I**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Describe all the subrings of the ring of integers. 6½

(*b*) Let m and n be positive integers and let k be least common multiple of m and n. Show that $m\mathbf{Z} \cap n\mathbf{Z} = k\mathbf{Z}$. 6½

(*c*) Show that : 6½

$$\mathbf{Z}_3[x]/<x^2 + x + 1> \text{ is not a field.}$$

P.T.O.

2\. (*a*) Find all idempotent and nilpotent elements in $\mathbf{Z}_3 \oplus \mathbf{Z}_6$. 6

(*b*) Show that any finite field has order p^n, where p is a prime. 6

(*c*) Let R be commutative ring with unity, and let A be an ideal of R. Show that $\frac{R}{A}$ is an integral domain if and only if A is prime. 6

3\. (*a*) Let ϕ be an isomorphism from a ring R onto a ring S. Show that ϕ^{-1} is an isomorphism from S onto R. 6½

(*b*) Let R be a ring with unity e and let characteristic of R be n. If $n > 0$, show that R contains a subring isomorphic to $\mathbf{Z}_n$ and if $n = 0$, show that R contains a subring isomorphic to $\mathbf{Z}$. 6½

(*c*) Determine all ring homomorphisms from $\mathbf{Z}$ to $\mathbf{Z}$. 6½

4\. (*a*) Prove that a subset W of a vector space V is a subspace of V if and only if $0 \in W$ and $ax + y \in W$ whenever $a \in F$ and $x, y \in W$. 6

(*b*) Whether $-x^3 + 2x^2 + 3x + 3 \in \text{span}(\{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\})$. Justify. 6

(*c*) Let V be a vector space over a field of characteristic not equal to two and u, v and w be distinct vectors in V. Prove that $\{u, v, w\}$ is linearly independent if and only if $\{u + v, u + w, v + w\}$ is linearly independent. 6

5. (*a*) Let V be a vector space having a finite basis. Then show that every basis for V contains the same number of elements. 6½

(*b*) Check whether $\{x^2 + 3x - 2, 2x^2 + 5x - 3, -x^2 - 4x + 4\}$ is a basis for $P_2(\mathbf{R})$? 6½

(*c*) Let V and W be vector spaces and let $T : V \to W$ be linear. If V is finite-dimensional, then prove that *nullity* (T) + *rank* (T) = dim V. 6½

6. (*a*) Let β and γ be standard ordered bases for $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively and let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear transformation given by 6

$T(a_1, a_2, a_3) = (2a_1 + 3a_2 - a_3, a_1 + a_3)$. Compute $[T]_\beta^\gamma$.

(*b*) Let V and W be finite-dimensional vector spaces with ordered bases β and γ, respectively. Let $T : V \to W$ be linear. Then T is invertible if and only if $[T]_\beta^\gamma$ is invertible. Furthermore, $[T^{-1}]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$. 6

(*c*) Let $A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}$ and $\beta = \left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$ be an ordered basis of $\mathbf{R}^2$. Find $[L_A]_\beta$ and also find an invertible matrix Q such that $[L_A]_\beta = Q^{-1}AQ$, where L_A is a left-multiplication transformation. 6

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2297**

Unique Paper Code : **42354401** **IC**

Name of the Paper : **Real Analysis**

Name of the Course : **B.Sc. Mathematical Science/ B.Sc. (Prog.)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are *six* questions in this question paper.

Attempt any *two* parts from each question.

1\. (*a*) Suppose that S and T are sets such that $T \subseteq S$. Show that if S is a finite set, then T is also finite set.

(*b*) If A_m is a countable set for each $m \in N$, then show that the union $\bigcup_{m=1}^{\infty} A_m$ is countable.

(*c*) Define supremum and infimum of a set. Prove that every non-empty set of real numbers which is bounded below has an infimum. 6,6

P.T.O.

2. (*a*) Show that the set of rational numbers is not order complete.

(*b*) Define the limit of a sequence. Show that sequence $<r^n>$ converges to 0 if $|r| < 1$.

(*c*) Let $Y = <y_n>$ be defined inductively by $y_1 = 1$, $y_{n+1} = (2y_n + 3)/4$ for $n \geq 1$. Show that $\lim Y = 3/2$. 6,6

3. (*a*) State and prove Bolzano-Weierstrass theorem.

(*b*) Let $X = <x_n>$ be defined by $x_1 = 1$, $x_2 = 2$, $x_n = \frac{x_{n-1} + x_{n-2}}{2}$, for $n > 2$. Show that the sequence $<x_n>$ is convergent.

(*c*) Investigate the convergence or divergence of the following sequences :

(*i*) $\frac{\sqrt{n}}{n^2 + 1}$,

(*ii*) $\frac{\sqrt{n^2 + 1}}{n}$. 6,6

4. (*a*) Define the convergence of a series. Show that the series $\sum_{n=1}^{\infty} n^{p}$ converges when $p > 1$.

(*b*) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.

(*c*) State and prove D'Alembert's ratio test. 6,6

5. (*a*) Test for convergence and absolute convergence the series :

$$1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \text{.........}.$$

(*b*) State and prove M_n test for uniform convergence of sequence $<f_n>$ of real valued functions defined on $[a, b]$.

(*c*) Find the radius of convergence of the following power series :

(*i*) $\sum_{n=0}^{\infty} 2^{-n} x^{3n}$,

(*ii*) $\sum_{n=0}^{\infty} \frac{3^n}{n4^n} x^n$. 6,6

P.T.O.

6. (*a*) Define Riemann integral of a function. If $f \in R[a, b]$, then show that the value of integral of f is uniquely determined.

(*b*) Let $f(x) = 2$ if $0 \leq x < 1$ and $f(x) = 1$ if $1 \leq x \leq 2$. Show that $f \in R[0, 2]$ and evaluate its integral using the definition of Reimann integral.

(*c*) If $f : [a, b] \to R$ is monotone on $[a, b]$, then show that $f \in R[a, b]$. 7.5,7.5

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **3234**	**IC**
Unique Paper Code	: **62354443**	
Name of the Paper	: **Analysis**	
Name of the Course	: **B.A. (Prog.) Mathematics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.
Attempt any two parts from each question.

1. (a) Let A and B be two non empty bounded sets of positive real numbers and let

$$C = \{xy : x \in A \text{ and } y \in B\}.$$

Show that C is bounded and:

(i) $\text{Sup } C = \text{Sup } A \ \text{Sup } B$

(ii) $\text{Inf } C = \text{Inf } A \ \text{Inf } B$ 6

(b) If $y > 0$ is a real number, show that there exists a natural number n such that:

$$\frac{1}{2^n} < y.$$ 6

(c) Define limit point of a set $S \subseteq \mathbb{R}$. Find the limit points of the following sets:

P. T. O.

(i) $\mathbb{N}$

(ii) $\mathbb{R}$ 6

2. (a) If A and B are two open sets then prove that A ∩ B is also open. Is intersection of infinite sets of open sets again open? Justify. 6

(b) Test the continuity of function

$$f(x) = \begin{cases} \frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$. 6

(c) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2, 2]$. 6

3. (a) State Cauchy convergence criterion for sequences and hence show that the sequence $< x_n >$ defined by :

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots\dots + \frac{1}{2n-1}$$

does not converge. 6.5

(b) Show that $\lim_{n\to\infty}(r)^n = 0$ if $|r| < 1$. 6.5

(c) If $< x_n >$ and $< y_n >$ be two sequences such that $\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y$ then show that:

$$\lim_{n\to\infty}(x_n y_n) = xy.$$ 6.5

4. (a) State and prove limit comparison test for positive term series. 6.5

(b) Check the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\{(n^3+1)^{\frac{1}{3}} - n\}$

(ii) $\sum_{n=1}^{\infty} \frac{1.2.3 \ldots\ldots n}{7.10.13 \ldots\ldots (3n+4)}$ 6.5

(c) State Leibnitz test for convergence of an alternating series:

$$\sum_{1}^{\infty}(-1)^{n-1} u_n,\ u_n > 0\ \ \forall n$$

and check the convergence and absolute convergence of the series:

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \ldots\ldots\ldots$$ 6.5

5. (a) Prove that a sequence cannot converge to more than one limit. 6

(b) Show that the sequence $< x_n >$ defined by:

$$x_1 = 1, x_{n+1} = \frac{3+2x_n}{2+x_n},\ n \geq 2$$

is convergent. Also find $\lim_{n\to\infty} x_n$. 6

(c) Show that:

$$\lim_{n\to\infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + \ldots\ldots + n^{1/n}\right] = 1$$ 6

P. T. O.

6. (a) Apply Integral Test to examine the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ 6.5

(b) Prove that a continuous function is Riemann integrable. 6.5

(c) Discuss the integrability of the function f defined on $[0, 1]$ as follows:

$$f(x) = \begin{cases} \frac{1}{2^n}, & \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n} \quad (n = 0,1,2,3, \ldots) \\ 0, & x = 0 \end{cases}$$

6.5

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2253**

Unique Paper Code : **32351601** **IC**

Name of the Paper : **Complex Analysis**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all parts from Question No. **1**.

Each part carries **1½** marks.

Attempt any *two* parts from question Nos. **2** to **6**

Each part carries *six* marks.

1. State True *or* False. Justify your answer in brief :

(*a*) A point z_0 of a domain need not be an accumulation point of that domain.

(*b*) $\lim_{z \to 0} \frac{\bar{z}^2}{z} = 0$

P.T.O.

(c) The function $f(z) = e^z$ is periodic with period 2π.

(d) $\log(-ei) = 1 - \frac{\pi}{2}i$.

(e) The function $f(z) = |z|^2$ is analytic at $z = 0$.

(f) Let C denote the boundary of the triangle with vertices at the point 0, $3i$, and -4, oriented in the counterclockwise direction. Then $\left|\int_C (e^z - \overline{z})\,dz\right| \le 60$.

(g) If C is any simple closed contour, in either direction, then

$$\int_C \exp(z^3)\,dz = 0.$$

(h) If C is the positively oriented unit circle $|z| = 1$, then

$$\int_C \frac{\exp(2z)}{z^4}\,dz = \frac{8\pi i}{3}.$$

(i) $\text{Res}_{z=0} f(z) = -\frac{1}{3!}$, where $f(z) = z^2 \sin\left(\frac{1}{z}\right)$, $0 < |z| < \infty$.

(j) The function $f(z) = \dfrac{1}{\sin\left(\frac{\pi}{z}\right)}$ has no isolated singular point.

2\. (a) Prove that a finite set of points cannot have any accumulation point.

(b) Suppose that $f(z) = u(x, y) + iv(x, y)$ $(z = x + iy)$ and $z_0 = x_0 + iy_0$, $w_0 = u_0 + iv_0$. Then $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \to (x_0,y_0)} u(x, y) = u_0$ and $\lim_{(x,y) \to (x_0,y_0)} v(x, y) = v_0$.

(c) Define neighbourhood of the point at infinity. Show that a set S is unbounded if and only if every neighbourhood of the point at infinity contains at least one point in S.

3\. (a) Use Cauchy-Riemann equations to show that $f'(z)$ does not exist at any point if $f(z) = \exp(\overline{z})$. State sufficient conditions for differentiability of a function $f(z)$ at any point $z_0 = x_0 + iy_0 \in \mathbb{C}$.

(b) Suppose that a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate $\overline{f(z)} = u(x, y) - iv(x, y)$ are both analytic in a given domain D. Show that $f(z)$ must be constant throughout D.

(*c*) If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then show that $\int_C f(z)\,dz = 2\pi i\ \text{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right]$. Use it to show that $\int_C \frac{5z-2}{z(z-1)}\,dz = 10\pi i,$ where C is the circle $|z| = 2$, described counterclockwise.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2254

Unique Paper Code : 32351602 IC

Name of the Paper : Ring Theory and Linear Algebra- II

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each of the questions.

1. (*a*) Let $f(x) = x^3 + 2x + 4$ and $g(x) = 3x + 2$ in $\mathbf{Z}_5[x]$. Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.

(*b*) State and prove Einstein's criterion of irreducibility.

(*c*) If D is a principal ideal domain, prove that every strictly increasing chain of ideals $I_1 \subset I_2 \subset I_3$ .... must be finite in length. Hence, prove that every non-zero and a non-unit element of D has an irreducible factor. 6.5, 6.5, 6.5

2. (*a*) Prove that the ideal $< x >$ in $\mathbf{Q}[x]$ is maximal.

(*b*) (*i*) Prove that $8x^3 - 6x + 1$ is irreducible over **Q**.

(*ii*) Prove that the product of two primitive polynomials is primitive.

(*c*) Prove that in a principal ideal domain, an element is irreducible if and only if it is prime. 6,6,6

P.T.O.

3. (*a*) Suppose that V is a finite dimensional vector space with the ordered basis $\beta = \{x_1, x_2, ..., x_n\}$. Let $f_i (1 \le i \le n)$ be the *i*th coordinate function with respect to β be defined such that $f_i(x_j) = \delta_{ij}$ where δ_{ij} is the Kronecker delta. Let $\beta^* = \{f_1, f_2, ..., f_n\}$. Then prove that β^* is an ordered basis for V^*, and, for any $f \in V^*$, we have $f = \sum_{i=1}^{n} f(x_i) f_i$.

(*b*) For $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \in M_{2 \times 2}(\mathbf{R})$, determine the eigen values of A and eigen space corresponding to each eigen value of A. Also, if possible, find a basis for $\mathbf{R}^2$ consisting of eigen vectors of A.

(*c*) Prove that the characteristic polynomial of any diagonalizable linear operator splits. Is the converse true ? Justify. 6.5, 6.5, 6.5

4. (*a*) Let T be a linear operator on $\mathbf{R}^3$ such that :

$T(a, b, c) = (a + b + c, a + b + c, a + b + c)$.

Let $W = \{(t, t, t) \mid t \in \mathbf{R}\}$ be a subspace of $\mathbf{R}^3$. Show that :

(*i*) W is a T-invariant subspace of $\mathbf{R}^3$.

(*ii*) The characteristic polynomial of T_W divides the characteristic polynomial of T.

(*b*) Let D be the differentiation operator on $P(\mathbf{R})$, the space of polynomials over $\mathbf{R}$. Prove that there exists no polynomial $g(t)$ for which $g(D) = T_0$. Hence, show that D has no minimal polynomial.

(c) Let T be a linear operator on a finite dimensional vector space V and let $p(t)$ be the minimal polynomial of T. Prove that a scalar λ is an eigen value of T if and only if $p(\lambda) = 0$. 6, 6, 6

5. (a) Let V be an inner product space. Prove that :

(i) $\|x \pm y\|^2 = \|x\|^2 \pm 2R\langle x, y\rangle + \|y\|^2$ for all $x, y \in V$ where $R\langle x, y\rangle$ denotes the real part of the complex number $\langle x, y\rangle$.

(ii) $\big|\|x\| - \|y\|\big| \leq \|x - y\|$ for all $x, y \in V$.

(b) Suppose that $S = \{v_1, v_2, \ldots \ldots, v_k\}$ is an orthonormal set in an n-dimensional inner product space V. Show that :

(i) S can be extended to an orthonormal basis $\{v_1, v_2, \ldots \ldots, v_k, v_{k+1}, \ldots, v_n\}$ for V.

(ii) If W = span (S) then $S_1 = \{v_{k+1}, \ldots, v_n\}$ is an orthonormal basis for $W^\perp$.

(iii) If W is any subspace of V, then dim V = dim W + dim $W^\perp$.

(c) Find the orthogonal projection of the given vector on the given subspace W of the inner product space :

$V = \mathbf{R}^3$, $u = (2, 1, 3)$ and

$W = \{(x, y, z) : x + 3y - 2z = 0\}$. 6.5,6.5,6.5

6. (a) State and prove Bessel's inequality.

(b) For the inner product space V over F and linear transformation $g : V \to F$, find a vector y such that $g(x) = \langle x, y\rangle$ for all $x \in V$ where :

$$V = P_2(R) \text{ with } \langle f, h\rangle = \int_0^1 f(t)h(t)\,dt \text{, and}$$

$$g(f) = f(0) + f'(1).$$

(c) Let V be a complex inner product space and let T be a linear operator on V.

Define $T_1 = \frac{1}{2}(T + T^*)$ and $T_2 = \frac{1}{2i}(T - T^*)$

(i) Prove that T_1 and T_2 are self adjoint and that $T = T_1 + iT_2$.

(ii) Suppose also that $T = U_1 + iU_2$ where U_1 and U_2 are self-adjoint. Prove that $U_1 = T_1$ and $U_2 = T_2$.

(iii) Prove that T is normal if and only if $T_1T_2 = T_2T_1$.

6,6,6

This question paper contains 4+2 printed pages]   23116

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2710**

Unique Paper Code : **32357607**

IC

Name of the Paper : **Probability Theory & Statistics**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-3**

Semester : **VI**

Duration : **3 Hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

In all there are *six* questions.

Question No. **1** is compulsory and it contains *seven* parts of 3 marks each, out of which any *five* parts are to be attempted.

In Question Nos. **2** to **6**, attempt any *two* parts from *three* parts.

Each part carries **6** marks.

Use of scientific calculator is allowed.

1. (*i*) If C_1, C_2 and C_3 are events in C, then prove that

$$p_1 \geq p_2 \geq p_3$$

where $p_1 = P(C_1) + P(C_2) + P(C_3)$,

$p_2 = P(C_1 \cap C_2) + P(C_2 \cap C_3) + P(C_1 \cap C_3)$, and

$p_3 = P(C_1 \cap C_2 \cap C_3)$.

P.T.O.

(*ii*) Given the cumulative distribution function

$$F(x) = 0 \text{ if } x < -1$$

$$= (x + 2)/4, \text{ if } -1 \leq x < 1, \text{ and}$$

$$= 1 \text{ if } 1 \leq x,$$

Compute :

(*i*) $P(-1/2 < X \leq 1/2)$;

(*ii*) $P(X = 1)$.

(*iii*) Let pmf $p(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If $p(0) = \frac{1}{4}$, find $E(X^2)$.

(*iv*) If the random variable X has a binomial distribution with the parameters n and θ, then compute the variance, σ^2, of X.

(*v*) Let $F(x, y)$ be the distribution function of X and Y. For all real constants $a < b$, $c < d$, show that

$$P(a < X \leq b, c < Y \leq d) = F(b, d) - F(b, c) - F(a, d) + F(a, c)$$

(*vi*) Let $f_{1/2}^{(x_1/x_2)} = \begin{cases} \frac{c_1 x_1}{x_2^2}, 0 < x_1 < x_2,\ 0 < x_2 < 1 \\ 0, \quad \text{elsewhere} \end{cases}$

be the conditional pdf of X_1 given $X_2 = x_2$.

Also let $f_2(x_2) = \begin{cases} c_2 x_2^4,\ 0 < x_2 < 1 \\ 0, \quad \text{elsewhere} \end{cases}$

be the marginal pdf of X_2.

Determine C_1 & C_2 and hence the joint pdf of X_1 and X_2.

(*vii*) Prove that $P_{i,j}^{n+m} = \Sigma_{k=0}^{\infty} P_{i,k}^{n} P_{k,j}^{m}$, for all n, m and all i, j.

2. (*a*) Let $\{C_n\}$ be a decreasing sequence of events, then show that

$$\lim_{n\to\infty} P(C_n) = P(\lim_{n\to\infty} C_n) = P(\cap_{n=1}^{\infty} C_n)$$

(*b*) In a lot of 50 light bulbs, there are 2 bad bulbs. An inspector examines five bulbs, which are selected at random and without replacement.

(*i*) Find the probability of at least one defective bulb among the five.

(*ii*) How many bulbs should be examined so that the probability of finding at least one bad bulb exceeds 1/2 ?

(c) Find the cumulative distribution function for the following pdf :

$$f(x) = \begin{cases} 1/3 & 0 < x < 1 \\ 1/3 & 2 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Also find the median.

3. (a) Let X have the mgf

$$M(t) = e^{t^2/2}, -\infty < t < \infty$$

Find $E(X^{2k})$ and $E(X^{2k-1})$, for $k = 1, 2, 3, \ldots$

(b) Show by stating all the conditions that the Binomial distribution can be approximated to the Poisson distribution.

(c) Let X have the exponential pdf, $f(x) = \theta^{-1} \exp\{-x/\theta\}$, $0 < x < \infty$, zero elsewhere. Find the moment generating function of X, and hence, the mean, and the variance of X.

4. (i) Let X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = 15x_1^2x_2 \text{ if } 0 < x_1 < x_2 < 1$$

$$= 0 \text{ elsewhere}$$

Find the marginal pdf of X_1 and X_2 and compute $P(X_1 + X_2 \leq 1)$.

(*ii*) Suppose the joint mgf, $M(t_1, t_2)$, exists for the random variables X_1 and X_2. Then show that X_1 and X_2 are independent if and only if $M(t_1, t_2) = M(t_1, 0)\, M(0, t_2)$; that is, the joint mgf is identically equal to the product of the marginal mgfs.

(*iii*) Let X_1, X_2 be two random variables with joint $p(x_1, x_2) = \frac{1}{2^{x_1 + x_2}}$ for $1 \le x_i < \infty$, $i = 1, 2$, where x_1 and x_2 are integers, zero elsewhere. Determine the joint mgf of X_1, X_2 and show that X_1 and X_2 are independent random variables.

5. (*i*) Let X_1, X_2 be two random variables with joint pdf

$$f(x_1, x_2) = 4x_1x_2, \text{ if } 0 < x_1 < 1,\ 0 < x_2 < 1,$$
$$= 0 \text{ elsewhere}$$

(*a*) Is $E(X_1, X_2) = E(X_1)\, E(X_2)$?

(*b*) Find $E(3X_2 - 2X_1^2 + 6X_1X_2)$.

(*ii*) Suppose (X, Y) have a joint distribution with the variances of X and Y finite and positive. Denote the means and variances of X and Y by μ_1, μ_2 and σ_1^2, σ_2^2 respectively, and let ρ be the correlation coefficient between X and Y. If $E(Y \mid X = x)$ is linear in x, then

$$E(Y \mid X = x) = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1).$$

(*iii*) Let the random variables X and Y have the joint density function

$$f(x, y) = 1, \text{ if } -x < y < x, \ 0 < x < 1$$
$$= 0, \text{ elsewhere}$$

Show that, on the set of positive probability density, the graph of $E(Y \mid x)$ is a straight line, whereas that of $E(X \mid y)$ is not a straight line.

6\. (*a*) (*i*) If X is a random variable with mean μ and variance σ^2, then prove that for any $k > 0$ $P\{|X - \mu| \geq k\} \leq \frac{\sigma^2}{\kappa^2}$.

(*ii*) Find the smallest value of k in above inequality for which the probability that a random variable will take a value between $(\mu - k\sigma)$ and $(\mu + k\sigma)$ is at least 0.99.

(*b*) State the Central limit theorem. Let X_i, $i = 1, 2, \ldots, 10$ be independent random variables, each having uniformly distributed over (0, 1). Estimate $P\{\Sigma_1^{10} X_i > 7\}$.

(*c*) An urn always contains 2 balls. Ball colors are red and blue. At each stage a ball is randomly chosen and then replaced by a new ball, which with probability 0.8 is the same color, and with probability 0.2 is the opposite color, as the ball it replaces. Define an appropriate Markov chain and if initially both balls are red, find the probability that the fifth ball selected is red.

2,700

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2712**

Unique Paper Code : **32357609** **IC**

Name of the Paper : **Bio-Mathematics**

Name of the Course : **B.Sc. (H) Mathematics : DSE-3**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Use of Scientific Calculator is allowed.

1. (*a*) When the drug theophylline is administered for asthma, a concentration below 5 mg/litre has little effect and undesirable side-effects appear if concentration exceeds 20 mg/litre. For a body that weights W kg the concentration when Mmg is present is 2M/W mg/litre. If $\tau = 6$ hours, measures the rapidity at which concentration falls. Find the concentration at time t hours after the initial dose of D mg. If D = 500 mg and W = 70 kg show that the second dose is necessary after about 6 hours to prevent the concentration from being ineffective. 6

P.T.O.

(*b*) Consider the following chemical reaction $A + X \rightarrow 2Y$, with rate constant k. If the reactant A is held at a constant concentration 'a', use the law of mass action to derive the equations for the concentration of X and Y. Suppose the initial concentration of X and Y are X_0 and Y_0 respectively. Solve the system of equations to obtain $X(t)$ and $Y(t)$. 6

(*c*) Describe the Hodgkin-Huxley model governing the nerve impulse transmission. 6

2\. (*a*) Discuss the nature of critical point and give the equation of trajectories for the given system : 6½

$$\dot{x} = -2x + y$$

$$\dot{y} = 4x - 5y$$

(*b*) What is a limit cycle ? Give an example. State Limit cycle criterion and Poincare-Bendixson theorem. 6½

(*c*) Discuss the behaviour of trajectories in the phase plane of $\ddot{x} + 3\dot{x} + 2x = 0$. 6½

3\. (*a*) Provide a full phase plane analysis for the mathematical model of heart beat equations given by

$$\epsilon \frac{dx}{dt} = -(x^3 - Tx + b), \quad T > 0$$

$$\frac{db}{dt} = x - x_0$$

where x is the muscle fibre length, b is the chemical control, $\epsilon > 0$ and (x_0, b_0) is a rest state. 6½

(b) Solve the following ordinary differential equation

$$\frac{dy}{dt} = -\gamma y + u$$

to obtain the period T of the periodic state y, which characterizes the pacemaker. Assume that $0 \le y \le 1$, such that when $y = 1$, the pacemaker fires and when $y = 0$, it jumps back. Also assume that $0 < \gamma < \frac{1}{4}$. 6½

(c) Sketch the nullclines for the system

$$\frac{du}{dt} = u(1 - u^2) - \omega$$

$$\frac{d\omega}{dt} = u.$$

Use Poincare-Bendixon theorem to show that it has limit cycle. 6½

4. (a) Show that the nonlinear conservative system

$$\dot{x} = y$$

$$\dot{y} = \mu \sin x - x, \quad \mu \ge 0$$

has one equilibrium point for $0 \le \mu < 1$ and three for $\mu > 1$. Also discuss the nature of equilibrium points. 6

(*b*) Show that the iteration scheme

$$x_{n+1} = 1 - \mu x_n(1 - x_n)$$

has a stable fixed point $x_0 = 1$ for $\mu < 1$ and that $\mu = 1$ is a bifurcation point where the fixed point $x_0 = 1/\mu$ appears. Show the period doubling occurs as soon as μ exceeds 3. 6

(*c*) Discuss the stability of the limit cycle when $P_0, P_1, \ldots$ are the points on a Poincare line and P_n is at a distance s_n from P_0 and such that

$$s_{n+1} = f(s_n)$$

taking P_0 as the fixed point. 6

5. (*a*) Derive the formula for the Jukes-Cantor distance (d_{JC}) given that all the diagonal entries of Jukes-Cantor matrix M^t are $\frac{1}{4} + \frac{3}{4}\left(1 - \frac{4}{3}\alpha\right)^t$, where α is the mutation rate. Compute the Jukes-Cantor distance $d_{JC}(S_0, S_1)$ to 4 decimal digits, from the 40 base frequency table : 6½

$S_1 \backslash S_0$	A	G	C	T
A	7	0	1	1
G	1	9	2	0
C	0	2	7	2
T	1	0	1	6

(*b*) If D and *d* denotes the alleles for tall and dwarf plant and if W and *w* denotes the alleles for round and wrinkled seed, then for the cross D*d*W*w* × *dd*W*w* compute the following probabilities : 6½

(*i*) The probability of a tall plant with wrinkled seeds.

(*ii*) The probability of a tall plant with round seeds.

(*c*) Write the steps for Neighbor Joining Algorithm. From the distance table compute R_1, R_2, R_3, R_4 and then form a table of values for M for the taxa S_1, S_2, S_3 and S_4 :

	S1	S2	S3	S4
S1		.83	.28	.41
S2			.72	.97
S3				.48

6½

6. (*a*) In mice, an allele A for agouti-or gray-brown, grizzled fur is dominant over the allele *a*, which determines a non-agouti color. Suppose A*a* × A*a* producees 4 offsprings, then compute the probabilities of exactly 0, 1, 2, 3 and 4 of the 4 progeny have agouti fur. 6

(*b*) Let $M(\alpha_1)$ be the Jukes-Cantor matrix with parameter α_1 and $M(\alpha_2)$ be the Jukes-Cantor matrix with parameter α_2. Compute $M(\alpha_1)\ M(\alpha_2)$ to show it has the form $M(\alpha_3)$. Give a formula for α_3 in terms of α_1 and α_2. 6

(*c*) Define bifurcating tree and unrooted tree and give an example of each. Draw all 3 topologically distinct unrooted bifurcating trees that could describe the relationship between 4 taxa. 6

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2842

Unique Paper Code : 32357610 IC

Name of the Paper : Number Theory

Name of the Course : B.Sc. (H) Mathematics : DSE-4

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions selecting *eight* parts from section I, *five* parts each from Sections II and III

Section I

(Attempt any *eight* parts)

1. (*a*) If p is a prime satisfying $n < p < 2n$, show that the binomial coefficient $\binom{2n}{n} \equiv 0 \pmod{p}$.

(*b*) Find the value of $\left(-\frac{72}{131}\right)$.

(*c*) Use of theory of congruence to verify that $89 \mid 2^{44} - 1$.

(*d*) Show that $\varphi(3n) = 3\varphi(n)$ if and only if $3 \mid n$.

(e) Given 2 = – 3 (18) + 2 (28) find all the solutions of the linear Diophantine equation $18x + 28y = 20$.

(f) Define σ (n) for a positive integer n. Compute σ (108) using multiplicative property.

(g) Decrypt IABEHEHZR using the linear cipher $C \equiv P - 7 \pmod{26}$.

(h) Test if 2 is a primitive root of 13.

(i) Find out the number of zeroes in which 500! terminates.

(j) What is a conjecture ? Explain it with Goldbach conjecture.

(k) Find the remainder when 2 (26!) is divided by 29.

(l) Verify the Gauss theorem on φ function with $n = 18$. 8×2½

Section II

(Attempt any 5 parts)

2. Let $n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ be the prime factorization of the integer $n > 1$. If f is a multiplicative function that is not identically zero, prove that :

$$\sum_{d|n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) \ldots\ldots (1 - f(p_r)).$$

3. State and prove Chinese Remainder Theorem.

4. Find the solutions of the system of congruences :

$3x + 4y \equiv 5 \pmod{13}$, $2x + 5y \equiv 7 \pmod{13}$

5. Determine all solutions in the positive integers of the following Diophantine equations : $18x + 5y = 48$.

6. If $\gcd(a, 133) = \gcd(b, 133) = 1$, then show that $133 \mid (a^{18} - b^{18})$.

7. Using Wilson's theorem, prove that for any odd prime p :

$$1^2 . 3^2 . 5^2 .... (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}.$$

8. Given $n \geq 1$, let $\sigma_s(n)$ denote the sum of the sth powers of the positive divisors of n; that is $\sigma_s(n) = \sum_{d|n} d^s$. Show that σ_s is a multiplicative function. 5×5½

Section III

(Attempt all *five* parts)

9. Use the information that 3 is a primitive root of 17 to obtain the eight primitive roots of 17.

10. Solve $x^2 \equiv 14 \pmod{5^2}$.

11. A long string of cipher text resulting from a Hill cipher $C_1 \equiv aP_1 + bP_2 \pmod{26}$, $C_2 \equiv cP_1 + dP_2 \pmod{26}$ revealed that the most frequently occurring two-letter blocks were HO and PP, in that order. Find the values of a, b, c and d.

P.T.O.

12. Prove that if p is a prime and $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, $a_n \not\equiv 0 \pmod{p}$ is a polynomial of degree $n \geq 1$ with integral coefficients, then the congruence $f(x) \equiv 0 \pmod{p}$ has at most n incongruent solutions modulo p.

13. Let $n > 1$ and gcd $(a, n) = 1$. If $a_1, a_2, \ldots\ldots, a_{\varphi(n)}$ are the positive integers less than n and relatively prime to n, then show that $aa_1, aa_2, \ldots\ldots, aa_{\varphi(n)}$ are congruent modulo n to a_1, a_2, ...$a_{\varphi(n)}$, in some order. 5×5½

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2843**

Unique Paper Code : **32357611** **IC**

Name of the Paper : **Linear Programming and Theory of Games**

Name of the Course : **B.Sc. (Hons.) Mathematics-DSE-4**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attmpet any *two* parts from each question.

All questions carry equal marks.

1. (*a*) Find all the basic feasible solutions of the following equations :

$$x_1 + x_2 + 2x_3 = 9$$

$$3x_1 + 2x_2 + 5x_3 = 22$$

(*b*) Let $(x_B, 0)$ be a basic feasible solution with objective function value z_B for LPP :

Maximize $z = cx$

subject to

$$Ax = b$$

$$x \geq 0.$$

P.T.O.

By entering an a_j with $z_j - c_j < 0$ and removing a b_r subject to

$$\frac{x_{Br}}{y_{rj}} = \text{Min}\left[\frac{x_{Bi}}{y_{ij}} : y_{ij} > 0\right]$$

Show that we can get a new feasible solution with improved value of the objective function compared to Z_B.

(c) Using simplex method, solve the following system of equations :

$$8x_1 + 2x_2 = 15$$

$$5x_1 + 3x_2 = 19.$$

2. (a) Solve the following problem by two-phase method :

Maximize $z = -x_1 - 2x_2$

subject to :

$$3x_1 + 4x_2 \leq 12$$

$$2x_1 - x_2 \geq 2$$

$$x_1, x_2 \geq 0.$$

(*b*) Solve the following linear program by the big-M method :

Minimize $z = 3x_1 - 3x_2 + x_3$

subject to

$$x_1 + 2x_2 - x_3 \geq 5$$

$$-3x_1 - x_2 + x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0.$$

(*c*) Consider the following problem :

Maximize $z = -3x_1 - 2x_2$

subject to

$$-x_1 + x_2 \leq 1$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1 \geq 0$$

$$x_2 \geq 3/2$$

Solve the problem graphically.

3. (*a*) Write the dual problem for the Linear Programming problem :

Maximize $z = 3x_1 + 5x_2 + 7x_3$

subject to

$$x_1 + x_2 + 3x_3 = 10$$

$$4x_1 - x_2 + 2x_3 \geq 15$$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

(*b*) Apply principle of duality to solve the linear programming problem :

Minimize $z = 2x_1 + 2x_2$

subject to

$$2x_1 + 4x_2 \geq 1$$

$$x_1 + 2x_2 \geq 1$$

$$2x_1 + x_2 \geq 1$$

$$x_1, x_2 \geq 0.$$

(*c*) Use graphical method to solve the dual of the following problem :

Min $z = 2x_1 + x_2 + 3x_3 + 6x_4$

subject to

$$x_1 + x_2 + 3x_3 + 2x_4 \geq 3$$

$$2x_1 + x_2 + x_3 + 3x_4 \geq 2$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

Further, use the complementary slackness theorem to find an optimal solution to the given problem from optimal solution of the dual problem.

4. (*a*) Solve the following transportation problem :

	D	E	F	Capacity
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Requirement	75	20	50	

(*b*) Solve the following assignment problem :

	V	W	X	Y	Z
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

(*c*) (*i*) Define in "Two-Person Zero-Sum" game :

(I) Saddle point

(II) Mixed Strategy.

(*ii*) Use maximin and minimax principle to solve the game :

$$\begin{bmatrix} 3 & -1 & 5 \\ 6 & 4 & 0 \\ 10 & 8 & 6 \end{bmatrix}$$

5. (*a*) Solve the following 4 × 2 game graphically :

$$\text{Player A} \overset{\textit{Player B}}{\begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix}}$$

(*b*) Use dominance to solve the game :

$$\begin{bmatrix} 1 & 3 & 2 & 7 & 4 \\ 3 & 4 & 1 & 5 & 6 \\ 6 & 5 & 7 & 6 & 5 \\ 2 & 0 & 6 & 3 & 1 \end{bmatrix}$$

(*c*) Convert the following game problem, involving "two person; zero-sum game" into a linear programming problem for player A and player B :

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & -2 & 6 \end{bmatrix}}$$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **2317 IC**

Unique Paper Code : 42357618

Name of the Course : **B.Sc. (Prog.)**
B. Sc. Mathematical Science : DSE - 2B

Name of the Paper : Numerical Methods

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) All the **six** questions are compulsory.

(c) Attempt any **two** parts from each question.

(c) Use of non-programmable scientific calculator is allowed.

1. (a) Define Global Truncation error with examples. If the number 754632 rounded off to four significant digits then calculate the absolute error. 6

(b) Perform three iterations of Newton-Raphson method to obtain the root of the equation

$f(x) = x^3 - 2x - 5 = 0$

with initial approximation $x_0 = 3$. 6

P.T.O.

(c) Perform three iterations of secant method obtain the root of the equation
$f(x) = 3x + \sin x - e^x = 0$
with initial approximation $x_0 = 0.3$ and $x_1 = 0.4$. 6

2. (a) Perform four iterations of bisection method obtain the root of the equation
$f(x) = xe^x - 1 = 0$
in the interval with initial [0, 1]. 6

(b) Define rate of convergence. Determine the rate of convergence for the Regula-Falsi method. 6

(c) Perform two iterations of Newton's method to solve the non-linear system of equations
$f(x, y) = x^2 y + y^3 = 10$
$g(x, y) = xy^2 - x^2 = 3$
with initial approximation (0.8 , 2.2). 6

3. (a) Find the inverse of the following matrix using the Gauss Jordan method : 6

$$A = \begin{bmatrix} 6 & 2 & 2 \\ 6 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$

(b) Starting with initial vector (x, y, z) = (0, 0, 0), perform three iterations of Gauss Seidel method to solve the following system of equations. 6
$6x + 15y + 2z = 72,$
$x + y + 54z = 110,$
$27x + 6y - z = 85$

(c) If $f(x) = \frac{1}{x}$ then evaluate the n^{th} order Newton Divided difference. 6

$f[x_0, x_1, x_2, ..., x_n]$

4\. (a) Prove that : 6.5

(i) $\mu^2 = 1 + \frac{\delta^2}{4}$ (ii) $\nabla = -\frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$

(b) Find the Lagrange interpolation polynomial for the following data set :

x	-1	1	4	7
f(x)	-2	0	63	342

Also estimate the value of 'f' at x = 2.5. 6.5

(c) Construct the interpolating polynomial using the Gregory- Newton backward difference interpolation for the given data set : 6.5

x	0	0.2	0.4	0.6
f(x)	0.5403	0.3624	0.1700	-0.0292

Hence estimate the value of f (x) at x = 0.15.

5\. (a) Approximate $f'(-1), f'(1)$ and $f'(3)$ using appropriate two points difference formula for following data :

x	−1	0	1	2	3
f(x)	1/3	1	3	9	27

Also estimate f'(1) using Richardson extrapolation with step size h = 2 . 6.5

(b) Find approximate value of the integral $I = \int_0^{\pi} x^2 \sin(2x)\,dx$ using Simpson's rule. Divide interval [0, π] into four equal sub interval. 6.5

(c) Obtain first extrapolated value of the integral $I = \int_0^{2} e^x\,dx$ using Romberg integration and compare with exact value. 6.5

6. (a) Use fourth order Runge-Kutta method to solve initial value problem

$$\frac{dy}{dt} = yt^2 - 1.1y, \quad y(0) = 1$$

over the interval [0,1] with step size h = 1.

(b) Use Heun's method without iteration to solve initial value problem 6.5

$$\frac{dy}{dt} = (1 + 2t)\sqrt{y}, \quad y(0) = 1,$$

over the interval [0,2] with step size h = 0.5.

(c) Use finite difference method to solve boundary value problem (BVP) 6.5

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = t^2 - y, \quad 0 \le x \le 1,$$

with y (0) = 10, y(1) = 2 and n = 4 sub intervals.

2,600

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : **3376** **IC**

Unique Paper Code : **62357602**

Name of the Paper : **Numerical Analysis**

Name of the Course : **B.A. (Prog.) Maths.: DSE-1B**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any ***two*** *parts from each question. Use of simple calculator is allowed.*

1. (a) Perform three iterations of Newton-Raphson method to obtain root of the equation:

$$f(x) = \cos x - xe^x = 0$$

with initial approximation $x_0 = 1$. 6

(b) Define Truncation Error. Evaluate the sum:

$$\sqrt{3} + \sqrt{5} + \sqrt{7}$$

to four significant digits and find its absolute and relative errors. 6

(c) Perform three iterations of Regula Falsi method to obtain the root of the equation:

$$f(x) = x^3 - 2x^2 - 5 = 0(6)$$

in the interval $[2,3]$. 6

2. (a) Perform three iterations of secant method to obtain the square root of 3 with initial approximation:

$$x_0 = 1,\ x_1 = 2.$$ 6

(b) Perform four iterations of bisection method to find the root of the equation:

$$x^3 - 2x^2 - 0.04x + 0.08 = 0$$

in the interval $[0,1]$. 6

(c) Perform two iterations of Newton's method to solve the non-linear system of equations:

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

with initial approximation $(x_0, y_0) = (1.5, 0.5)$. 6

3. (a) Solve the following system of linear equations using the Gauss elimination method with partial pivoting:

$$2x + 6y + 10z = 0$$
$$x + 3y + 3z = 2$$
$$3x + 14y + 28z = -8$$ 6.5

(b) Perform three iterations of Gauss-Seidel iteration method for the following system of equations:

$$5x + y - 2z = 2$$
$$3x + 4y - z = -2,$$
$$2x - 3y + 5z = 10,$$

starting with initial solution as $(x, y, z) = (0,0,0)$ .6.5

(c) Solve the following system of equations by using the Gauss-Jordan method:

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -3 & 5 \\ -3 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ -3 \end{pmatrix}$$ 6.5

4. (a) Prove the following relation:

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$$ 6

(b) Given the following data:

x	0.1	0.2
$\sin(x)$	0.09998	0.1986

Find Lagrange interpolating polynomial and approximate the value $\sin(0.15)$. Obtain a bound on the truncation error also. 6

(c) The following data represents the function $f(x) = x^{\frac{1}{3}}$:

x	0	1	8
$f(x)$	0	1	2

Find Newton divided difference interpolating polynomial of degree 2. Also find the approximate value of $f(7)$ and compare with the exact value. 6

5. (a) Find $f''(2.0)$ using quadratic interpolation using the following data:

x	2.0	2.2	2.6
$f(x)$	0.69315	0.78846	0.95551

Obtain an upper bound on error also. 6.5

(b) Find the approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$. by using Newton's Cote open formula (i) mid- point rule (ii) two- point rule. 6.5

(c) Evaluate the integral $I = \int_0^2 \frac{dx}{3+4x}$, using Gauss Quadrature 3- point rule. 6.5

6. (a) Use the Euler method to solve boundary value problem

$$y' = 4e^{0.8t} - 0.5y,\ y(0) = 2$$

on the interval $[0,3]$ with $h = 1$. 6.5

(b) Given the initial value problem:

$$y' = -2ty^2,\quad y(0) = 1$$

estimate $y(0.4)$ using Ralston's method (R. K. Method 2nd Order) with $h = 0.2$. 6.5

(c)Using a second order finite difference method with $h = 1$, find the solution of the Boundary Value Problem $y'' - y = x(x-4),\ 0 \le x \le 4$

with $y(0) = y(4) = 0$ .6.5

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This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : **3404** **IC**

Unique Paper Code : **62357602**

Name of the Paper : **Numerical Analysis**

Name of the Course : **B.A. (Prog.) Maths. : DSE-2B**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any ***two*** *parts from each question. Use of non-programmable simple calculator is allowed.*

1. (a) Define Rounding-off error with examples. If 0.333 is the approximate value of $\frac{1}{3}$, find absolute and relative errors. 6

   (b) Perform two iterations of Newton's method to solve the non-linear system of equations:

$$y\cos(xy)+1=0$$
$$\sin(xy)+x-y=0$$

   with initial approximation $(x_0, y_0)=(1,2)$. 6

(c) Perform four iterations of bisection method to obtain the root of the equation:

$$x^3 - 3x^2 - 0.16x + 0.48 = 0$$

in the interval $[0,1]$. 6

2. (a) Perform three iterations of Regula Falsi method to find the root of the equation:

$$f(x) = x^2 - 5 = 0$$

in the interval $[2,3]$. 6

(b) Perform three iterations of Newton-Raphson method to find the square root of $3/4$ with initial approximation $x_0 = 0.5$. 6

(c) Perform three iterations of secant method to find the root of the equation:

$$x^3 - 9x + 2 = 0$$

in the interval $[0,1]$. 6

3. (a) Solve the following system of linear equations using the Gauss elimination method with partial pivoting:

$$x + 2y + 3z = 1$$

$$2x + 4y + 10z = -2$$

$$3x + 14y + 28z = -8.$$

6.5

(b) Perform three iterations of Gauss-Seidel iteration method for the following system of equations:

$$2x - y = 7,\ -x + 2y - z = 1,\ -y + 2z = 1,$$

starting with initial solution as $(x, y, z) = (0,0,0)$. 6.5

(c) Explain Gauss-Thomas algorithm and solve the tri-diagonal system $AX = b$ by using Gauss-Thomas method.

$$\begin{pmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 41 \\ 25 \\ 105 \end{pmatrix}$$ 6.5

4. (a) Given that:

$$f(-2) = 4,\ f(0) = 2,\ f(2) = 8$$

Find the unique polynomial of degree 2 by Lagrange interpolation. Also find bound on the error. 6

(b) For the following data, calculate the differences and obtain the Gregory-Newton forward difference polynomial:

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Also find the approximate value of $f(0.35)$. 6

(c) (i) Find the maximum value of step size h that can be used to tabulate $f(x) = e^x$ on $[0,1]$ using linear interpolation such that $|error| \le 5 \times 10^{-4}$.

(ii) Let $f(x) = \log_e(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$.

Approximate $f(1.04)$ by using Newton divided difference interpolating polynomial. 6

5.(a) Find the approximate value of $I = \int_0^4 2^x dx$ using (i) Trapezoidal rule, (ii) Simpson's rule. 6.5

(b) Find the approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$ by using Newton Cote's open formula for three-point rule. 6.5

(c) Evaluate $\int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ using composite Trapezoidal rule and Romberg integration with $h = 1$ and $h = 1/2$ only. 6.5

Q6 (a) Apply Euler's modified method to approximate the solution of the following initial-value problem and calculate $y(2)$ by using $h = 1$:

$$y' = 4e^{0.8t} - 0.5y, \quad y(0) = 2$$ 6.5

(b) Employ the classical fourth order RK method to integrate $y' = -2ty^2$, $y(0) = 1$ with $h = 0.2$ on the interval $[0, 0.2]$. 6.5

(c) Apply finite difference method to solve the given problem:

$$\frac{d^2y}{dx^2} = y + x(x-4), \quad 0 \le x \le 4$$

with $h = 1$ and $y(0) = 0$, $y(4) = 0$. 6.5

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **2496-A** **IC**

Unique Paper Code : 32353401

Name of the Course : **B.Sc. (Hons.) Mathematics : SEC**

Name of the Paper : Computer Algebra Systems and related Softwares

Semester : IV

Time : 2 Hours ***Maximum Marks : 38***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) This question paper has **six** questions in all.

(c) **All** questions are compulsory.

Unit - 1 (CAS)

Note : The answers should be written in only **one** of the CAS : Maxima/Mathematica/Maple or any other.

P.T.O.

1. Fill in the blanks : 1×5=5

   (a) .... command is used to find the product of two matrices m, n.

   (b) The function................. is used to find the n^{th} prime.

   (c) .... command is used to find the value of exponential constant up to 20 digits.

   (d) The symbol .... is used as delayed operator.

   (e) .... command is used to find the transpose of a matrix.

2. Attempt any **six** parts from the following :

   1.5×6

   (a) Write the command to evaluate the expression $2x^2+x=1$.

   (b) Write the command to plot the functions Sin(x) and Cos(x) in the range $-10<x<10$.

   (c) Write the command to evaluate (i) 7^{22} mod23 (ii) $\log_{10}(5.65)$.

   (d) Write the command to create a 6×6 sparse matrix with non-zero entries :

   (1,2) = 3; (4,3) = 3; (4,5) = 7; (6,1) = 4

   (e) Write the command to evaluate $\int_{1/4}^{1/2} \frac{1}{x^2} dx$.

(f) Write the command to evaluate $\sum_{i=1}^{n-1}\left(\frac{1+2i}{n}\right)^2$.

(g) Write the command to create the matrix

$$A=\begin{bmatrix} 7 & -1 & 4 & 3 \\ -1 & 3 & -2 & 5 \\ 0 & 8 & 0 & 7 \end{bmatrix}.$$

Further, write the commands to obtain its second column and the determinant.

(h) Write the command to obtain a 2×4 matrix with random entries within the range of 2 to 10.

3. Attempt any **two** parts from the following : 4×2

(a) For the matrix,

$$A=\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & -1 \\ 2 & 5 & 3 \end{bmatrix},$$

write commands for :

(i) diagonalization of the given matrix.

(ii) finding its inverse.

(b) Write the command to print first 10 prime numbers.

(c) Write a program to find the gcd of two integers a and b using Euclidean Algorithm and hence find the gcd of 120 and 75.

Unit-II (Software R)

4. Write **True** or **false** for the following : 1×4

(a) The data object combining text and numbers is of type 'text'.

(b) If 'name' is a 10 items vector then name[2:7] shows its second and seventh item.

(c) The length of the following vector is 5 :

days = {2, 4, 5, 5, 4, NA}.

(d) plotpie command is used to draw a pie chart.

5. Attempt any **four** parts from the following : 1.5×4

(a) (i)Write command to read data from the file "hybrid .csv".

(ii) Using scan function, enter the following data :

Subject = {Eng, Sociology, Science, History}.

(b) For a 3 × 3 matrix

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 7 & 1 & 4 \\ 8 & 3 & 5 \end{bmatrix},$$

write the command to give column and row headings.

(c) For the list, m={5, 8, 3, 8, 7, 2} , write the output for the following :

(i) order(m), (ii) rank(m).

(d) Write the command to convert the following data in integers :

M= {3.5, 1.2, 4.3, 7.1, 8.7}.

(e) For the following data vectors

Length={7, 8, 9, 11.5},

Height={4, 9.5, 3.9, 2.5};

write the command to construct the dataframe 'dimension'.

(f) For the following data object 'fw'

abund	flow
1	7
25	12
15	8
12	19
7	14

write the command to view the first four entries of column 'flow'.

6. Attempt any **two** parts from the following : 3×2

(a) For the vector, Data_mp ={ 3, 2, 1, 5, 5, 3, 5, 8, 7, 6, 9, 1, 9, 5, 8}; write the command to :

(i) find the cumulative sum.

(ii) find the 20%,50%,40% quantiles.

(iii) create the stem and leaf plot for the above vector.

(b) For the following two dimensional data,

data 1	data 2	data 3
23	25	34
23	45	12
21	32	21
21	47	43

write the command to :

(i) display the first and third rows.

(ii) determine the structure of the data object.

(iii) For the above data, draw a bar chart with appropriate labels.

(c) Write the commands in R for the following :

(i) Put the following values into a variable d :

3, 5, 7, 3, 2, 6, 8, 5, 6, 9, 4, 5, 7, 3, 4.

(ii) Find mean of d.

(iii) Find the largest value in d.

(iv) Find variance of d.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2301**

IC

Unique Paper Code : **42353404**

Name of the Paper : **Computer Algebra Systems**

Name of the Course : **B.Sc. (Prog.)/B.Sc. Math. Sciences : SEC**

Semester : **IV**

Duration : **2 Hours**

Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Using any one of the CAS := Mathematica/Maple/Maxima/Matlab to answer the questions.

This question paper has *four* questions in all.

All questions are compulsory.

1. True/False (Give satisfactory Explanation/Example) : $8\times1=8$

   (*i*) Does the suffix ".nb" stand for "notebook" ?

   (*ii*) In Mathematica, every built-in function name begins with a small letter.

   (*iii*) Do the commands D[f[x]] and f'[x] provide the same output ?

P.T.O.

(*iv*) The syntax NSolve[$-1+3x+x^2=0$,x,15] is correct.

(*v*) The syntax Makelist[n^2,n,1,10,2]; is well defined.

(*vi*) The output of Factor [x^2–2] is $(x-\sqrt{2})(x+\sqrt{2})$.

(*vii*) x := RandomInteger[10]; {x,x,x} will give the same value of x in output.

(*viii*) Can we plot $y=4x+1$, $y=-x+4$ and $y=9x-8$, for $0 \le x \le 2$ in a single graph ?

2. Attempt any *four* parts from the following : $4\times2\frac{1}{2}=10$

(*i*) What is the significance of simpsum command in the simplification to sums in maxima ?

(*ii*) Explain Reduce and Solve command.

(*iii*) Explain the use of 'Manipulate' command.

(*iv*) What is the use of command Direction → 1 in Limit command ? Can we change that value 1 with any other integer ?

(*v*) Define Matrix Form and Min command with suitable example.

(*vi*) Explain the role of Aspect Ratio and Plot Style of Plot options with syntax.

3. Write the Output of any *five* from the following : 5×2=10

(*i*) Plot [Sin[x],{x, 0, 2Pi}, Ticks → {{0, Pi, 2Pi}, {0, 0.5, 1}},

AxesLabel → {x, y}, PlotLabel → Sin[x]].

(*ii*) Solve :

([2*x+y–3*z=10,x+4*y+2*z=12,–x+y+z=0],[x,y,z]);

(*iiii*) $M = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, N = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 1 & 6 \end{bmatrix}$

M*N

(*iv*) A=DiagonalMatrix[{a,b,c},1}]// MatrixForm

B=Table[i+j, {i,4}, {j,4}] // MatrixForm

A+B

(*v*) $M = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$M_{[[2]]} = M_{[[2]]} + M_{[[1]]}$

M // MatrixForm

Transpose[M] // MatrixForm

(*vi*) Plot[{x,x^2},{x,0,4},PlotRange→{0,5},PlotStyle→

{Black,Directive[Thick,Dotted,Black]}]

(*vii*) f(x):= x+ sin(x);

'diff('f(x), x)=diff(f(x), x);

at ('diff ('f(x), x), x=0)= at (diff (f(x), x) , x=0);

(*viii*) wxplot2d(x^2 ,[x, 0, 4],[box,false]);

wxplot3d ([Cos(t), Sin(t), a], [t, 0, 2*%pi], [a, -1,1]);

4. Provide the Syntax of any *four* from the following : 4×2½=10

(*i*) Write the syntax for the plotting of unit sphere in any software.

(*ii*) Give the syntax for finding the Ist derivative and Indefinite integral of the function $f(x)=x^2+\cos x$ using any software.

(*iii*) Write the commands for the solution of the following equations without using solve command.

$x-2y=5$ and $4x-3y=4$.

(*iv*) Write the command for $\lim_{x\to 0}\frac{\cos x}{x}$ and $\lim_{x\to\infty}\frac{\sin x}{x}$.

(*v*) Provide the syntax of piecewise command with the help of example.

(*vi*) Write the syntax for the addition operation for any *two* matrices of 3*3 order in the form of matrices.

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2306**

Unique Paper Code : **42353604** **IC**

Name of the Paper : **Transportation and Network Flow Problems**

Name of the Course : **B.Sc. Programme/B.Sc. Math. Sciences : SEC**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **55**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *four* questions in all.

All questions are compulsory.

1. Hero Auto has three plants in *Gurugram*, *Haridwar*, and *Satyavedu*, and two major distribution centers in Delhi and Nagpur. The capacities of three plants during the next quarter are 500, 1000, and 700 cars and demands at the two distribution

centers are 1300 and 900 cars. The transportation costs per car on the different routes, rounded to the closest rupees, are given in the following table :

Table : Transportation Cost per Car

	Delhi (1)	Nagpur (2)
Gurugram (1)	Rs. 100	Rs. 235
Haridwar (2)	Rs. 120	Rs. 128
Satyavedu (3)	Rs. 122	Rs. 88

Formulate the Transportation Model. 5

2. Attempt any *five* parts from the following :

(*i*) Compare the initial basic feasible solutions obtained by the Northwest-Corner method and Least-Cost method for the following transportation problem : 3+3=6

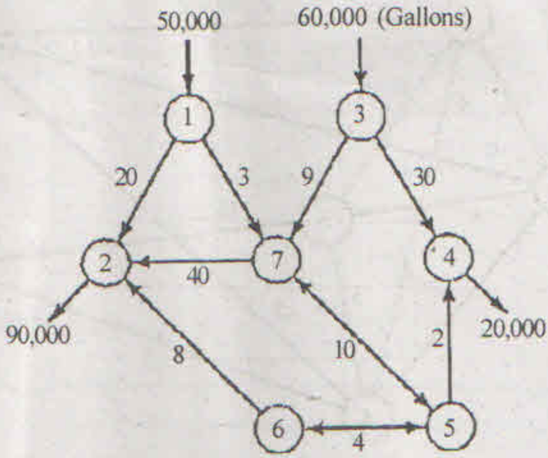
	Destination				**Supply**
Source	11	13	17	14	**250**
	16	18	14	10	**300**
	21	24	13	10	**400**
Demand	**200**	**225**	**275**	**250**	

(*ii*) Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table. Find the assignment of men to jobs that will minimize the total time taken. 6

Men

		1	2	3	4	5
	A	3	8	2	10	3
	B	8	7	2	9	7
Jobs	C	6	4	2	7	5
	D	8	4	2	3	5
	E	9	10	6	9	10

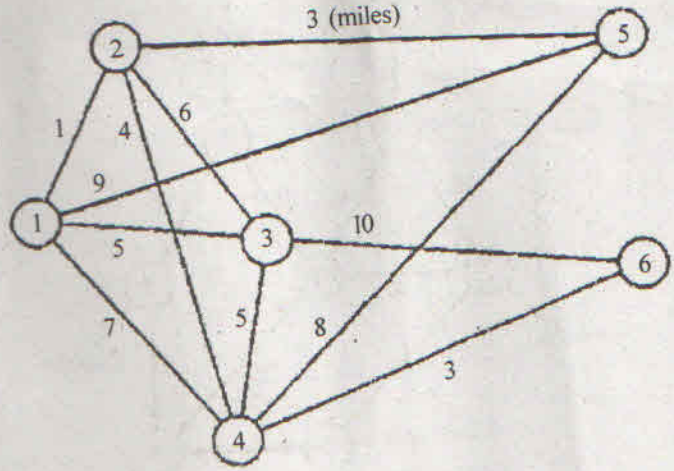
(*iii*) Consider the oil pipeline network shown in the following figure. The different nodes present pumping and receiving stations. Distances in miles between the stations are shown on the network :



(*a*) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(*b*) Only develop the corresponding transshipment model table. 2+4=6

(*iv*) Midwest TV Cable Company is in the process of providing cable services to five new housing development areas. The adjoining figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network starting at node 6. 6



(*v*) Draw the Network defined by the sets N=1,2,3,4,5,6} :

A={(1,2),(2,3),(3,4),(4,5), (5,6),(1,5),(1,3),(1,6),(3,6),(3,5)}

Also determine (*a*) a path (*b*) a cycle (*c*) a tree (*d*) a spanning tree. 6

(*vi*) The activities associated with a certain project are given below : 2+1+3=6

Activity	Predecessors	Duration (Week)
A	–	8
B	–	10
C	–	8
D	A	10
E	A	16
F	D, B	17
G	C	18
H	C	14
I	F, G	9

(*a*) Develop the associated network for the project.

(*b*) Find the minimum time of completion of the project.

(*c*) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table : 5+5=10

(*a*) Use the Vogel Approximation Method (VAM) to find a starting solution.

(*b*) Use this starting solution to find the optimal solution by the method of multipliers :

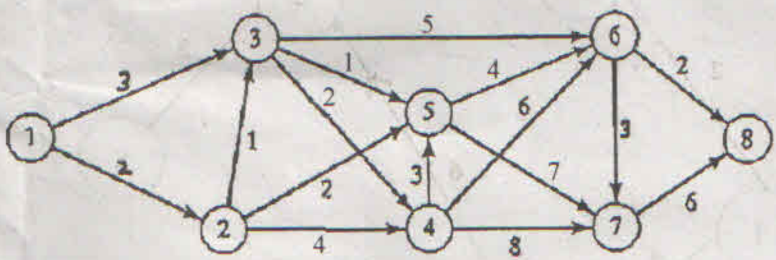
	X	Y	Z	Supply
A	5	1	8	12
B	2	4	0	14
C	3	6	7	4
Demand	9	10	11	

4. Attempt any *one* from the following :

(*i*) The network in the following figure gives the distances in miles between pairs of cities. Use Dijkstra's algorithm to find the shortest route between : 7+3=10

(*a*) cities 1 and 8

(*b*) cities 4 and 7.



(*ii*) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc(3, 4) is directional, so that no traffic is allowed from node 4 to node 3. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between :

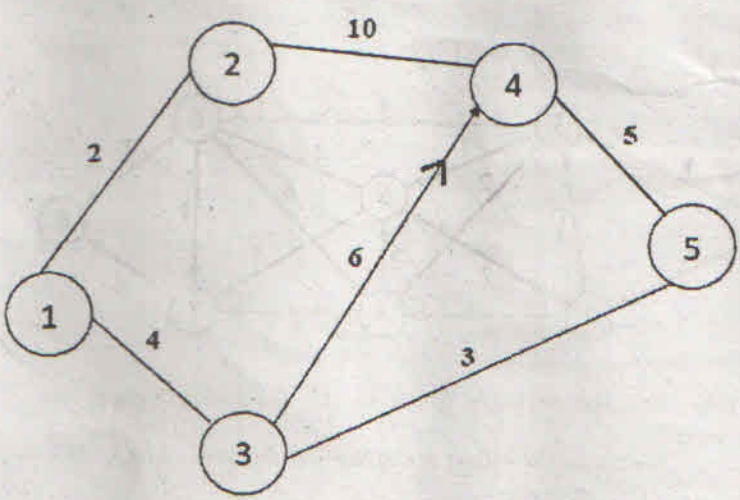
(*a*) node 5 to node 2

(*b*) node 1 to node 4

(*c*) node 2 to node 3

(*d*) node 3 to node 5

(*e*) node 1 to node 5 5×2=10



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Your Roll No...............

Sr. No. of Question Paper : 3292 **IC**

Unique Paper Code : 62353606

Name of the Paper : Transportation and Network Flow Problem

Name of the Course : **B.A. Programe : Mathematics : SEC**

Semester : VI

Duration : 3 Hours Maximum Marks : 55

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **FOUR** questions in all.
3. **All** questions are compulsory.

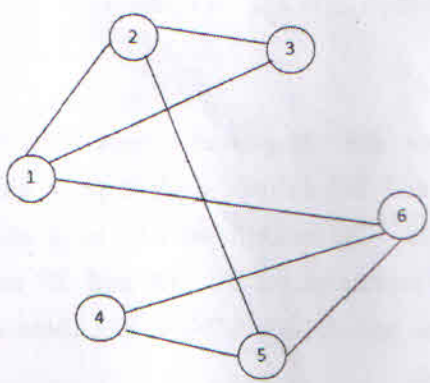
1. Three electric power plants with capacities of 20, 40 and 30 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 30, 35 and 25 million kWh. The price per million kWh at the three cities is given in Table

		City		
		1	2	3
Plant	1	\$600	\$700	\$400
	2	\$320	\$300	\$350
	3	\$500	\$480	\$450

The utility company wishes to determine the most economical plan for the distribution. Formulate the model as a transportation model. (5)

2. Attempt any **FIVE** parts from the following :

(i) For the Network given below, determine (a) a path (b) a cycle (c) a tree (d) a spanning tree (e) the sets N and A (6)



(ii) Compare the initial basic feasible solutions obtained by the Northwest-Corner method **AND** Least-Cost method for the following transportation problem. (3+3=6)

Warehouse

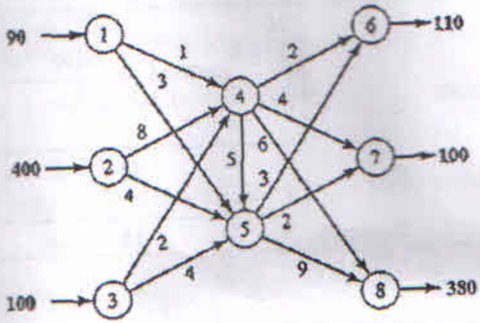
		1	2	3	4	Supply
Factory	1	10	2	20	11	15
	2	12	7	9	20	25
	3	4	14	16	18	10
	Demand	5	15	15	15	

(iii) Solve the following Assignment Problem using Hungarian Method. The Matrix entries represent the processing times in hours. (6)

Operators

		1	2	3	4	5
Jobs	1	9	11	14	11	7
	2	6	15	13	13	10
	3	12	13	6	8	8
	4	11	9	10	12	9
	5	7	12	14	10	14

(iv) The network in figure shows the routes for shipping cars from three plants (nodes 1, 2 and 3) to three dealers (nodes 6 to 8) by way of two distribution centers (nodes 4 and 5). The shipping costs per car (in $100) are shown on the arcs.

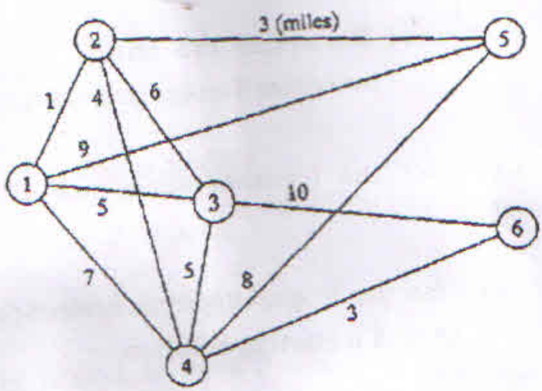


(a) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(b) Only develop the corresponding transshipment model table. (2+4=6)

(v) A Company is in the process of providing cable service to five new housing development areas. (6)

Figure below depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network starting at node 4.



(vi) The activities associated with a certain project given below (2+1+3=6)

Activity	Predecessors	Duration (Week)
A:	--	4
B:	--	3
C:	A,B	2
D:	A,B	5
E:	B	6
F:	C	4
G:	D	3
H:	F,G	7
I:	F,G	4
J:	E,H	2

(a) Develop the associated network for the project.

(b) Find the minimum time of completion of the project.

(c) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table.

(a) Use the Vogel Approximation Method (VAM) to find a starting solution.

(b) Hence find the optimal solution by the method of the multipliers. (5+5=10)

	Destination			Supply
Source	\$0	\$2	\$1	6
	\$2	\$1	\$5	9
	\$2	\$4	\$3	5
Demand	5	5	10	

4. Attempt **ANY ONE** from the following :

(i) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc (4, 5) is directional, so that no traffic is allowed from node 5 to node 4. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between (5×2=10)

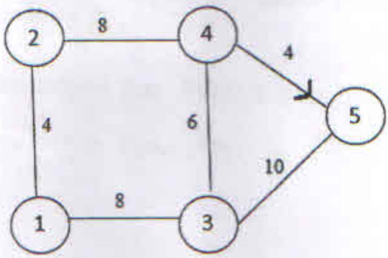
(a) node 5 to node 2

(b) node 1 to node 4

(c) node 2 to node 3

(d) node 3 to node 5

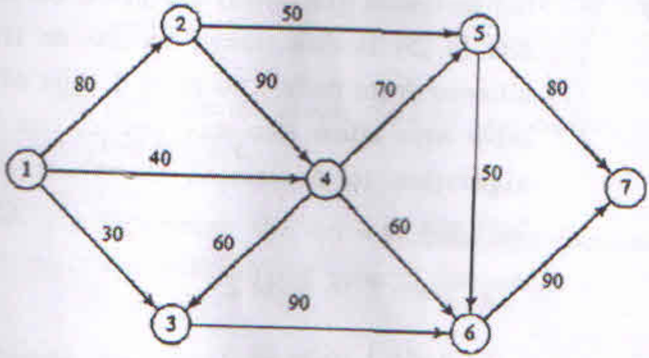
(e) node 1 to node 5



(ii) Use Dijkstra's algorithm to determine the shortest path (6+4=10)

(a) From node 1 to 5

(b) From node 2 to 7


1
2
3
4
5
6
7
80
50
90
70
80
40
50
30
60
60
90
90

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2980**

Unique Paper Code : **32355202** **IC**

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective—Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If x and y are vectors in $\mathbf{R}^3$, then prove that :

$$\|x\| - \|y\| \leq \|x + y\| \leq \|x\| + \|y\|.$$ 6½

(*b*) Let x and y be nonzero vectors in $\mathbf{R}^3$. If $x \cdot y \leq 0$, then prove that :

$$\|x - y\| > \|x\|.$$

Is the converse true ? Justify. 6½

(c) Solve the following system of linear equations using the Gauss-Jordan method :

$$2x_1 + x_2 + 3x_3 = 16$$

$$2x_1 + 12x_3 - 5x_4 = 5$$

$$3x_1 + 2x_2 + x_4 = 16$$ 6½

2. (a) Define the rank of a matrix and determine the rank of $\begin{pmatrix} 1 & 2 & -2 & -11 \\ 2 & 4 & -1 & -10 \\ 3 & 6 & -4 & -25 \end{pmatrix}$. 6

(b) Prove that the matrix $\begin{pmatrix} 7 & 1 & -1 \\ 11 & -3 & 2 \\ 18 & 2 & -4 \end{pmatrix}$ cannot be diagonalized. 6

(c) Let V be a vector space over **R**, then for any vector v in V and every nonzero real number a, prove that $av = 0$ if and only if $v = 0$. 6

3. (*a*) Let $S = \left\{ \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & -5 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} \right\}$ be a subset of 2×2 real matrices. Use the Simplified Span Method to find a simplified form for the vectors in span(S). Is the set S linearly independent ? Justify. $4\frac{1}{2}+2$

(*b*) Define a basis for a vector space. Show that the set :

$$B = \{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$$

is a basis for $\mathbf{R}^3$. $2+4\frac{1}{2}$

(*c*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

has a solution or not. If so, find the solution. $4+2\frac{1}{2}$

4. (*a*) Consider the ordered basis S = {[1, 0, 1], [1, 1, 0], [0, 0, 1]} for $\mathbf{R}^3$. Find another ordered basis T for $\mathbf{R}^3$ such that the transition matrix from T to S is : 6

$$P_{S \leftarrow T} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}.$$

(*b*) Suppose L : $\mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator and L([1, 1]) = [1, –3] and L([–2, 3]) = [–4, 2]. Express L([1, 0]) and L([0, 1]) as linear combinations of the vectors [1, 0] and [0, 1]. 6

(*c*) Let L : $\mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation given by :

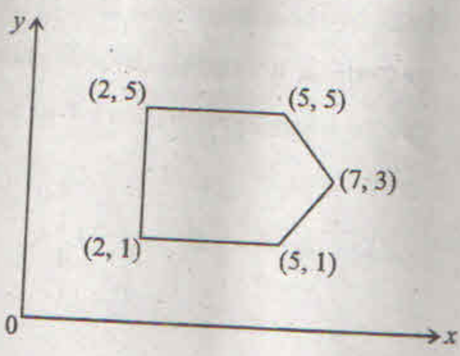
$$L([x, y, z]) = [-2x + 3z, x + 2y - z]$$

Find the matrix for L with respect to the bases :

B = {[1, –3, 2], [–4, 3, –3], [2, –3, 2]} for $\mathbf{R}^3$

and C = {(–2, –1], [5, 3]} for $\mathbf{R}^2$. 6

5. (*a*) For the graphic figure below, use homogeneous coordinates to find the new vertices after performing a scaling about the point (3, 3) with scale factors of 3 in the *x*-direction and 2 in the y-direction. Then sketch the final figure that would result from this movement : 4+2½



(b) Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{pmatrix} 3 & 1 & -3 \\ 2 & 1 & -1 \\ 2 & 3 & 5 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for ker(L) and a basis for range (L), also verify the dimension theorem. 4+2½

(c) Show that a mapping $L : P_2 \to P_2$ given by $L(p(x)) = p(x) + p'(x)$ is an isomorphism, where P_2 is the vector space of all polynomials of degree ≤ 2. 6½

6. (a) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $3x - y + 4z = 0$. Let $v = [2, 2, -3] \in \mathbf{R}^3$. Find $\text{proj}_{W^\perp} v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^\perp$. Is the decomposition unique ? 6

(*b*) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}$ of $\mathbf{R}^3$, find a basis for W and the orthogonal complement $W^\perp$. Also verify that :

$$\dim(W) + \dim(W^\perp) = \dim(\mathbf{R}^3).$$

4+2

(c) If $A = \begin{pmatrix} 1 & -1 \\ 4 & 1 \\ 2 & 3 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}, z = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find vector v satisfying the inequality :

$$\| Av - b \| \leq \| Az - b \|.$$

6

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **3104**

IC

Unique Paper Code : **32355444**

Name of the Paper : **Elements of Analysis**

Name of the Course : **Mathematics : Generic Elective for Honours**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Define supremum and infimum of a bounded set of real numbers. Show that if S is a non-empty subset of **R** which is bounded below then : 7.5

$$\inf S = -\sup(-S).$$

P.T.O.

(b) Define a countable set. Show that S is countable if there exists a surjection of **N** to S. 7.5

(c) (i) State and prove Archimedean property of **R**.

(ii) Show that for $a, b \in \mathbf{R}$

$$\frac{|a+b|}{1+|a+b|} \leq \frac{|a|}{1+|a|} + \frac{|b|}{1+|b|}.$$ 3.5+4

2. (a) Let (x_n) be a sequence of positive real numbers such that :

$$l = \lim_{n \to \infty} \frac{x_{n+1}}{x_n}$$

exists. If $l < 1$, then show that (x_n) converges to 0. Hence deduce that :

$$\lim_{n \to \infty} \frac{n}{2^n} = 0.$$ 7.5

(b) State and prove squeeze theorem. Use it to determine the limit of the sequence $\left((n!)^{\frac{1}{n^2}}\right)$. 7.5

(c) (i) Define a convergent sequence and a Cauchy sequence. Show that every convergent sequence is a Cauchy sequence.

(ii) Show that the sequence (3^n) does not converge.

5+2.5

3. (a) State and prove Cauchy convergence criterion for infinite series. 6.5

(b) State root test for an infinite series and test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$

(ii) $\sum_{n=1}^{\infty} 2^n e^{-n}$. 6.5

(c) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n!}{3.5.7 \ldots\ldots (2n+1)}$

(ii) $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$. 3+3.5

4. (*a*) State limit comparison test for a positive term infinite series and hence test the convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$

(*ii*) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n+2}}$. 6

(*b*) Show that every absolutely convergent series in **R** is convergent. Is the converse true ? Justify. 6

(*c*) Test the convergence and absolute convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^3}, \quad \alpha \in \mathbf{R}$

(*ii*) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+2}$. 3+3

5. (*a*) Determine the interval of convergence for the power series :

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2}.$$ 5

(*b*) Derive the power series expansion for $f(x) = e^x$. 5

(*c*) Prove that :

$$\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2} \text{ for } |x| < 1$$

and evaluate :

(*i*) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(*ii*) $\sum_{n=1}^{\infty} \frac{n(-1)^n}{3^n}$. 5

6. (*a*) Find the radius of Convergence of the power series :

$$x - \frac{x^2}{2^2} + \frac{2!}{3^3}x^3 - \frac{3!}{4^4}x^4 + \ldots\ldots .$$ 5

(*b*) Find a power series representation of :

$$f(x) = \frac{1}{2+x}$$

and its domain. 5

(c) State differentiation theorem for power series. Show that :

$$C'(x) = -S(x) \text{ and } S'(x) = C(x), \quad x \in \mathbf{R}$$

where $C(x)$ and $S(x)$ are the power series of Sine and Cosine, respectively. 5

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3262 **IC**

Unique Paper Code : 62353424

Name of the Paper : Computer Algebra Systems

Name of the Course : **B.A. (Prog.) Mathematics : Skill Enhancement Course**

Semester : IV

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Using any one of the CAS :=Mathematica/Maple/ Maxima/any other to answer the questions.
3. This question paper has **four** questions in all.
4. **All** questions are compulsory.

1. Fill in the blanks : (8×1=8)

   (i) The ______ operator is to be used while assigning value 2 to a variable x.

   (ii) The output given on input π is ______ .

P.T.O.

(iii) The command which undoes the effect of Factor command is the ______ command.

(iv) The most recognized CAS ______ was created by Stephen Wolfram.

(v) The command ______ returns the nth derivative of f with respect to x.

(vi) The ______ brackets are used to group terms in algebraic expressions.

(vii) The command ______ is used to find the quotient when one polynomial is divided by another.

(viii) The option ______ causes the left hand limits to be computed by the 'Limit' command.

2. Write a short note on any **four** from the following : (4×2.5=10)

(i) How to find the limit of a function at a point in any CAS?

(ii) How to find maxima and minima of a function in any CAS?

(iii) How to differentiate a function in any CAS?

(iv) How to find eigenvalues and eigenvectors of a given 3×3 matrix in any CAS?

(v) Differentiate between the commands 'Solve' and 'NSolve'.

(vi) Differentiate between the commands 'AxesLabel' and 'PlotLabel'.

3. Write the output of any **five** from the following :

(i) Limit[$Sin[x], x \to$ Infinity]

Limit[$\frac{Sin[x]}{x}$,x→ Infinity]

(ii) Solve [a x + b y = c,d x + e y = f,{x,y}]

Solve[x == 0]//Grid

(iii) x=RandomInteger[ ];

{2 x,2 x}

(iv) Plot[$x^{1/3}$,{x,-8,8}]

Plot[Cos[x],{x,0,Pi},Ticks → {Range[0,Pi,Pi/2],Automatic}]

(v) g[x_]: = x^3 − 9x + 5

Solve[g'[x] == 0,x]

extrema = {x,g[x]}/.%

(vi) 'diff(f(x) * g(x),x,2)=diff(f(x) * g(x),x,2);

'diff(diff(x^6,x),x)=diff(diff(x^6,x),x);

(vii) f[i_,j_]:=i^2+j^3;

(g=Array[f,{3,2}])//MatrixForm

(h=Array[Min,{3,2}])//MatrixForm

g+h//MatrixForm (5×2=10)

4. Provide the Syntax of any **four** from the following :

(i) Write the manipulate command in the plotting of $f(x) = x^2 + \sin x$ using directive and blend commands.

(ii) Write the command to sketch the graph of $f(x)=\frac{1}{x^2}$ and then evaluate the definite integral of f(x) from x=1 to x=3.

(iii) Write the command to enter a matrix with the integers 1 through 5 on the diagonal, 0 below the diagonal, and 5 above the diagonal.

(iv) Write the syntax for finding eigenvalues and eigenvectors of any 3×3 lower triangular matrix.

(v) Write the command to get f′(0) and f″(1), where $f(x)=\frac{x^2}{1+x^3}$.

(vi) Graph the functions $y = x\sin(1/x)$ and $z=\frac{xy}{x^2+y^2}$. (4×2.5=10)

This question paper contains 3 printed pages.

Your Roll No. ....

S. No. of Paper	: **3343**	**IC**
Unique Paper Code	: **62355604**	
Name of the Paper	: **General Mathematics II**	
Name of the Course	: **B.A. (Prog.) Mathematics : GE**	
Semester	: **VI**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions as directed questionwise.*

SECTION I

1. Attempt any *five* questions out of the following:

(a) Discuss the academic life of Hardy.

(b) Write a short summary on Cantor's life.

(c) What was Ramanujan's area of research?

(d) When did Hilbert publish its first book? What was the foundation of that book?

(e) Mention any *two* achievements of Banach.

(f) Comment on the following statement: "Emmy Noether was never given her due academically because she was a woman, a Jew and a pacifist." 3×5

P. T. O.

SECTION II

2. Attempt any **six** questions out of the following:

(a) Define increasing and decreasing functions. Illustrate these functions with the help of examples.

(b) Define Circle and its diameter. What is the difference between tangent and secant of a circle? Also define segment and sector of the circle.

(c) Discuss all possible symmetries of an equilateral triangle.

(d) Show that the sequence of ratio of Fibonacci number to the one preceding it converges to the golden ratio.

(e) State the similarities and differences between Mobius strip and Klein bottle.

(f) Write short notes on any *two* of the following:

(i) Fractals

(ii) Chromatic Number

(iii) Konigsberg Bridge problem.

(g) If $\sin x = -5/13$, then find the other trigonometric angles.

5×6

SECTION III

3. Attempt any *five* questions from the following:

(a) Use Gauss Jordan method to convert the following matrix to reduced row echelon form:

$$\begin{bmatrix} 2 & 1 & 8 \\ 1 & -3 & 9 \\ 8 & 9 & 2 \end{bmatrix}.$$

(b) For the matrices $A = \begin{bmatrix} -4 & 10 & 0 \\ -3 & -5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -7 & 2 \\ 5 & -1 & 0 \end{bmatrix}$, find the rank of A+B.

(c) Solve the following homogeneous system of equations:

$$4x_1 - 8x_2 - 2x_3 = 0$$
$$3x_1 - 5x_2 - 2x_3 = 0$$
$$2x_1 - 8x_2 + x_3 = 0.$$

(d) Express the vector $x = [2,-1,4]$ as a linear combination of the other vectors, if possible:

$$a_1 = [3,6,2],\ a_2 = [2,10,-4].$$

(e) Find the inverse of the coefficient matrix and hence find the solution set of the system:

$$-7x_1 + 5x_2 + 3x_3 = 6$$
$$3x_1 - 2x_2 - 2x_3 = -3$$
$$3x_1 - 2x_2 - x_3 = 2.$$

(f) Use Gauss Elimination method to give the complete solution set of the following system of equations:

$$3x - 4y = 2$$
$$5x + 2y = 12$$
$$-x + 3y = 1.$$

(g) Find A^{-1} using row reduction for the matrix:

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 8 & -4 & 9 \\ -4 & 6 & -9 \end{bmatrix}.$$

5×6

200