This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6630**	**HC**
Unique paper code	: **32351201**	
Name of the paper	: **Real Analysis**	
Name of course	: **B.Sc. (Hons.) Mathematics**	
Semester	: **II**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **three** parts from each question.*

All questions are compulsory.

1. (a) Prove that a lower bound v of a nonempty set S in **R** is the Infimum of S if and only if for every $\epsilon > 0$, there exists an $s_\epsilon \in S$ such that $s_\epsilon < v + \epsilon$.

(b) Let S be a nonempty bounded above set in **R**. Let $a > 0$ and $aS = \{as : s \in S\}$, then prove that Sup (aS) = a Sup S.

(c) If x and y are positive real numbers with $x < y$, then prove that there exists a rational number $r \in Q$ such that $x < r < y$.

(d) Show that Sup $\left\{1 - \frac{1}{n} : n \in \mathbf{N}\right\} = 1$. 5,5,5

P. T. O.

2. (a) Define limit point of a set in **R**. Prove that a point $c \in \mathbf{R}$ is a limit point of a set S if and only if every neighbourhood of c contains infinitely many points of S.

(b) Let (x_n) be a sequence of real numbers such that $\lim_{n\to\infty} x_n = x > 0$, then show that there exists a natural number K such that

$$\frac{x}{2} < x_n < 2x \quad \forall\, n \geq K.$$

(c) Use the definition of limit to prove:

i. $\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right) = 0$

ii. $\lim_{n\to\infty} \left(\frac{3n+2}{n+1}\right) = 3.$

(d) Let (x_n) be a sequence of positive real numbers such that $L = \lim_{n\to\infty} \left(\frac{x_{n+1}}{x_n}\right)$ exists. Show that if $L < 1$, then (x_n) converges and $\lim_{n\to\infty} x_n = 0$. (5, 5, 5)

3. (a) Let (x_n) and (y_n) be sequences of real numbers such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then show that $\lim_{n\to\infty} x_n y_n = xy$.

(b) State Squeeze Theorem and hence prove that

$$\lim_{n\to\infty} (a^n + b^n)^{1/n} = b,$$

where $0 < a < b$.

(c) State and prove Monotone Convergence Theorem.

(d) Let (x_n) be a sequence of real numbers defined by

$$x_1 = 8, \; x_{n+1} = \frac{x_n}{2} + 2 \quad \text{for } n \in \mathbf{N}.$$

Show that (x_n) is convergent and find its limit. (5, 5, 5)

4. (a) Show that the following sequences are divergent:

(i) $((-1)^n)$

(ii) $\left(\sin\left(\frac{n\pi}{3}\right)\right)$.

(b) Define a Cauchy sequence and show that every Cauchy sequence of real numbers is bounded.

(c) Prove that the sequence (x_n), where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}, \qquad n \in \mathbb{N}$$

is not a Cauchy sequence.

(d) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be infinite series of positive real numbers such that $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = 0$. Show that if $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. (5, 5, 5)

5. (a) State and prove *n*-th Root Test to test the convergence of an infinite series.

(b) Test for convergence any *two* of the following series:

i. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

ii. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$

iii. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

(c) Define Conditional Convergence. Show that the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)}$$

is conditionally convergent.

(d) Test the following series for Absolute convergence:

$$\sum_{n=1}^{\infty}(-1)^{n}\frac{(n!)^2}{(2n)!}.$$

(5, 5, 5)

This question paper contains 5 printed pages.

Your Roll No. ....

HC

No. of Paper	: **6631**
ique Paper Code	: **32351202**
me of the Paper	: **Differential Equations**
me of the Course	: **B.Sc. (Hons.) Mathematics – I**
mester	: **II**
ration	: **3 hours**
aximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the Sections are compulsory.

Use of non-programmable scientific calculator is allowed.

Section I

1. Attempt any *three* parts of the following: (5+5+5)

a. Solve the differential equation:

$$\left(2xe^{y}y^{4}+2xy^{3}+y\right)dx+\left(x^{2}e^{y}y^{4}-x^{2}y^{2}-3x\right)dy=0.$$

b. Find the general solution of the differential equation:

$$yy''+\left(y'\right)^{2}=yy'.$$

c. Solve the differential equation:

$$\frac{dy}{dx}=\frac{x-y-1}{x+y+3}.$$

d. Solve the initial value problem:

$$\frac{dy}{dx}=2xy^{2}+3x^{2}y^{2},\ y(1)=-1.$$

2. Attempt any *two* parts of the following: (5+5)

a. A water tank has the shape obtained by revolving the parabola $x^2 = by$ around the y axis. The water depth is 4 ft at 12 noon, when a circular plug in the bottom of the tank is removed. At 1 pm, the depth of the water is 1 ft. Find the

P. T. O.

depth $y(t)$ of water remaining after t hours. Also, find when the tank will be empty. If the initial radius of the top surface of the water is 2 ft, what is the radius of the circular hole in the bottom?

b. A certain piece of dubious information about phenyl ethyl amine in the drinking water began to spread one day in the city with a population of 100,000. Within a week 10,000 people heard this rumour. Assume that the rate of increase of the number who have heard the rumour is proportional to the number who have not heard it. How long will it be until half the population of the city has heard the rumour?

c. Consider a body that moves horizontally through a medium whose resistance is proportional to the square of the velocity υ, so that $d\upsilon / dt = -k\upsilon^2$. Show that :

$$\upsilon(t) = \frac{\upsilon_0}{1 + \upsilon_0 kt}$$

and that

$$x(t) = x_0 + \frac{1}{k} \ln(1 + \upsilon_0 kt).$$

Section II

3. Attempt any *two* parts of the following: (8+8)

a. The following differential equation describes the level of pollution in the lake:

$$\frac{dC}{dt} = \frac{F}{V}(C_{in} - C)$$

where V is the volume, F is the flow (in and out), C is th concentration of pollution at time t and C_{in} is th concentration of pollution entering the lake. Le $V = 28 \times 10^6 m^3, F = 4 \times 10^6 m^3 / month$. If only fresh wat enters the lake,

i. How long would it take for the lake with pollution concentration $10^7\ parts/m^3$ to drop below the safety threshold $\left(4 \times 10^6\ parts/m^3\right)$?

ii. How long will it take to reduce the pollution level to 5% of its current level?

b. In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is:

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X .$$

i. Show that the only non-zero equilibrium population is :

$$X_e = K\left(1 - \frac{h}{r}\right).$$

ii. At what critical harvesting rate can extinction occur?

c. In a simple battle model, suppose that soldiers from the red army are visible to the blue army, but soldiers from the blue army are hidden. Thus, all the red army can do is fire randomly into an area and hope they hit something. The blue army uses aimed fire.

i. Write down appropriate word equations describing the rate of change of the number of soldiers in each of the armies.

ii. By making appropriate assumptions, obtain two coupled differential equations describing this system.

iii. Write down a formula for the probability of a single bullet fired from a single red soldier wounding a blue soldier in terms of the total area A and the area exposed by a single blue soldier A_b .

iv. Hence write the rate of wounding of both armies in terms of the probability and the firing rate.

Section III

Attempt any *three* parts of the following: (6+6+6)

a. Find the general solution of the differential equation:

$$x^3 y''' + 6x^2 y'' + 4xy' = 0$$

b. Using the method of undetermined coefficients, solve the differential equation :

$$y''' - 2y'' + y' = 1 + xe^x, y(0) = y'(0) = y''(0) = 1 .$$

c. Using the method of variation of parameters, solve the differential equation :

$$y'' + 3y' + 2y = 4e^x .$$

d. Show that $y_1 = 1 \, and \, y_2 = \sqrt{x}$ are solutions of :

$$yy'' + (y')^2 = 0,$$

but the sum $y = y_1 + y_2$ is not a solution. Explain why.

Section IV

5. Attempt any *two* parts of the following: (8+8)

a. Consider a disease where all those infected remain contagious for life. A model describing this is given by the differential equations

$$\frac{dS}{dt} = -\beta SI, \frac{dI}{dt} = \beta SI$$

where β is a positive constant.

i. Use the chain rule to find a relation between S and I.

ii. Obtain and sketch the phase-plane curves. Determine the direction of travel along the trajectories.

iii. Using this model, is it possible for all the susceptible to be infected?

b. The predator-prey equations with additional deaths by DDT are:

$$\frac{dX}{dt} = \beta_1 X - c_1 XY - p_1 X, \frac{dY}{dt} = -\alpha_2 Y + c_2 XY - p_2 Y$$

where all parameters are positive constants.

i. Find all the equilibrium points.

ii. What effect does the DDT have on the non-zero equilibrium populations compared with the case when there is no DDT?

iii. Show that the predator fraction of the total average prey population is given by:

$$f = \frac{1}{1 + \left(\frac{c_1(\alpha_2 + p_2)}{c_2(\beta_1 - p_1)} \right)}$$

What happens to this proportion f as the DDT kill rates, p_1 and p_2, increase?

c. The pair of differential equations

$$\frac{dP}{dt} = rP - \gamma PT, \frac{dT}{dt} = qP,$$

where r, γ *and* q are positive constants, is a model for a population of microorganisms P, which produces toxins T which kill the microorganisms.

i. Given that initially there are no toxins and p_0 microorganisms, obtain an expression relating the population density and the amount of toxins.

ii. Hence, give a sketch of a typical phase-plane trajectory.

iii. Using phase-plane trajectory, describe what happens to the microorganisms over time.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6801 HC

Unique Paper Code : 42341202

Name of the Paper : Database Management Systems

Name of the Course : B.Sc. (Prog.) / B.Sc. Mathematical Science

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **Section A** is compulsory.

3. Attempt any **5** questions from **Section B.**

Section A

1. (a) Give two reasons for including class/subclass relationships and specialization in a data model? (2)

   (b) "Primary key is a minimal super key". Justify. (2)

P.T.O.

(c) Draw the symbol used in ER diagram for following :

(i) Weak entity

(ii) Key Attribute

(iii) Total Participation of an entity in a relationsh

(3

(d) What is the use of cascade in following SQL statemen

DROP TABLE DEPENDENT CASCADE. (1

(e) Mention the cardinality ratio in the following :

(i) Actors perform in movies

(ii) An Instructor teaches at most one course

(iii) Many musicians perform in an concert

(iv) A painter paints many paintings (4

(f) Given a relational schema R (A,B,C,D,E) with Function dependencies (FDs)

F = {A→B, B→C, AC→D, AC→BC}. Identify :

(i) a functional dependency in F that is consequence of the augmentation rule

(ii) a new functional dependency that can b obtained from FDs in F using the transitivity ru

(3

(g) A STUDENT table has following two attributes Rollno and Course. Write an SQL statement to insert a new attribute Grade to the STUDENT table. (1)

(h) What is meta data? (1)

(i) What are canned transactions? (1)

(j) What are the responsibilities of DBA and database designers? (2+2)

(k) What is a transaction? List any two properties of a transaction. (1+2)

Section B

(a) What is a ternary relationship? Illustrate with an example. (1+3)

(b) Illustrate the following constraints on specialization using a diagram in each case.

(i) Disjoint, total

(ii) Overlapping, partial (2+2)

(c) We can convert any weak entity set to a strong entity set by simply adding appropriate attributes. Why, then, do we have weak entity sets? (2)

3. (a) Draw the diagram of three-schema architecture (
DBMS.
(2

(b) Identify multivalued and composite attributes from th
following complex attribute, If any

{Address_phone({ Phone(Area_code, Phone_number)

Address (Street_address (Number, Street, Apartmen
number), City, State, Zip)) }
(

(c) How does a category differ from a regular share
subclass?
(4

4. (a) Consider the two tables T1 and T2 as shown below

T1

P	Q	R
10	a	5
15	b	8
25	a	6

T2

A	B	C
10	b	6
25	c	3
10	b	5

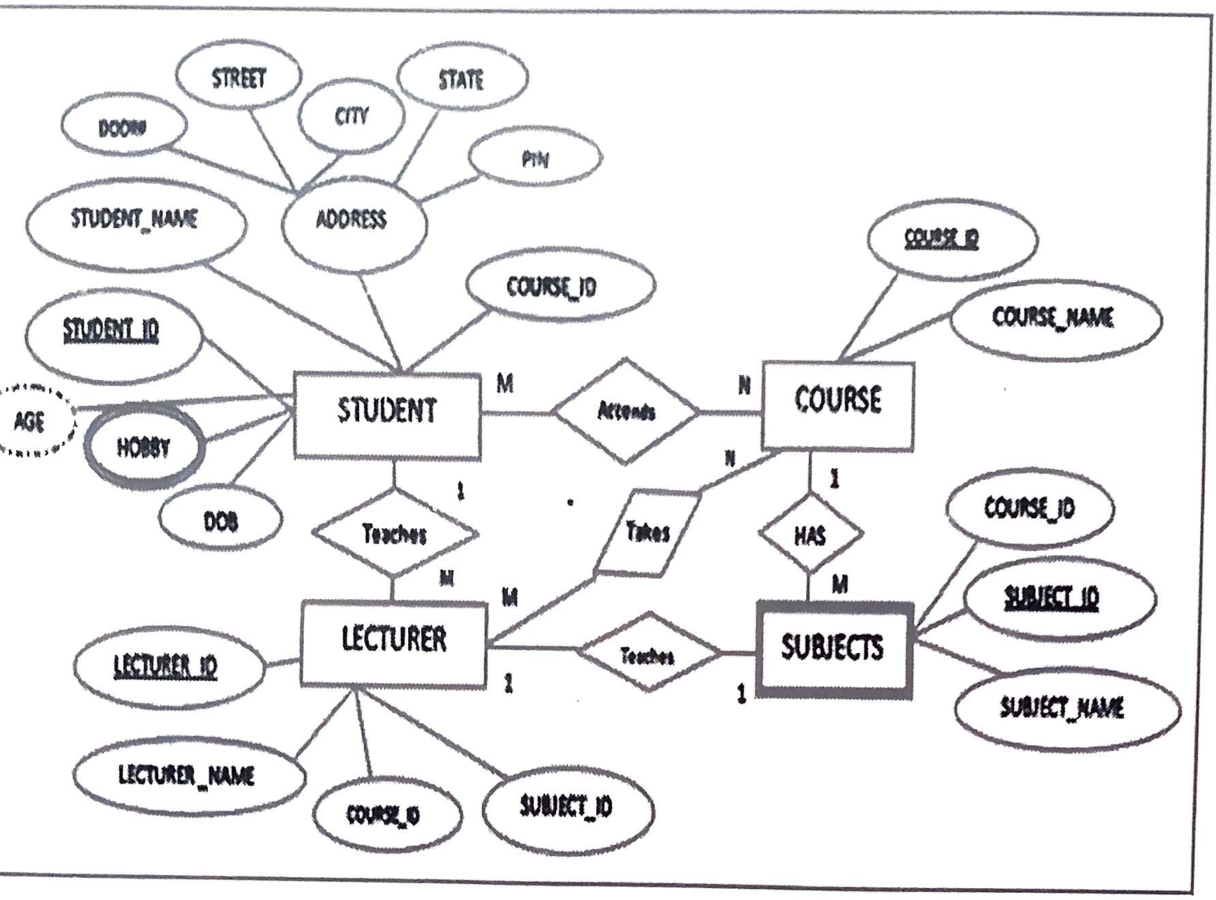
Show the results of the following operations :

(i) T1 Union T2

(ii) T1 Join $_{T1.Q=T2.B}$ T2

(iii) T1 Minus T2 (3)

(b) Consider the following ER Diagram



Map it into relations taking into account different entity types, relationships and attributes. (7)

(a) For a relation R(A,B,C,D) with given dependency set F = {A→BC, BC→D}

(i) Find the primary key for the relation R

(ii) Identify the normal form in which given relation exists as of now. Justify your answer.

P.T.O.

(iii) Normalize R till 3NF, if not already in 3NF.

(1+2+

(b) For the given table in its current state, write down i cardinality and identify two candidate keys.

EMP-SSN	EMP-Name	Date of Birth	Telephone
S1	Smith John	11/03/1987	9999999999
S2	May Helen	12/04/1980	9999999988
S3	Brit Paul	12/04/1980	8899999999
S4	Annie W	01/10/1976	9999999777
S5	Brit Paul	01/10/1977	9999999999

(1+

(c) How are spurious tuples generated? (

Refer the following COMPANY database for the Q and Q 7.

EMPLOYEE (Name, <u>Ssn</u>, Bdate, Address, Gend Salary, SuperSsn, Dno)

DEPARTMENT(Dname, <u>Dnumber</u>, MgrSsn, MgrStartDa

DEPENDENT(<u>Essn</u>, <u>DependentName,</u> Gender, Bda Relationship)

PROJECT(Pname, <u>Pnumber</u>, Plocation, Dnum)

WORKS_ON(<u>Essn</u>, <u>Pno</u>, <u>Hours</u>)

6. (a) Answer the following queries using relational algeb

(i) Retrieve the names and addresses of all t employees who work for 'Administratio department.

(ii) Retrieve the names of the employees who have no dependents.

(iii) Find the names of the employees who work on all the projects controlled by department number 10.

(iv) Retrieve the social security numbers of employees who either work in department No. 2 or directly supervise an employee who works in department No. 2. (4×2)

(b) Illustrate the division operator of relational algebra by taking a suitable example. (2)

Answer the following queries using SQL.

(i) Give a 10 percent raise in salary of all empolyees.

(ii) Update the address of employees living in 'Mumbai' to 'Delhi'.

(iii) For each project on which more than two employees work, retrieve the project number and the number of employees who work on that project.

(iv) Retrieve the names of all employees who do not have supervisors identified by SuperSsn.

(v) For each department, retrieve the department numb number of employees in the department and th average salary. (5×

8. Differentiate the following :

(i) Entity and Referential Integrity

(ii) Procedural and Nonprocedural DML

(iii) Logical and Physical Data Independence

(iv) Database State and Database Schema

(v) Total and Partial Dependency (5×

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6803 HC

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry

Name of the Course : B.Sc. Mathematical Sciences / B.Sc. (Prog.)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory.

4. Marks of each part are indicated.

1. (a) Let f be a function defined by

$$f(x)=\begin{cases}\dfrac{e^{1/x}-1}{e^{1/x}+1}, & x\neq 0\\ 0, & x=0\end{cases}.$$

P.T.O.

Show that f is discontinuous at $x = 0$. State the kind discontinuity. (6

(b) Use $(\in, \delta)$ definition to show that (6

$$\lim_{x \to 4} x^2 = 16.$$

(c) Show that the function f defined by $f(x) = \sqrt{x}$ i uniformly continuous in $[0,1]$. (6

2. (a) Discuss the differentiability of the function f defined b $f(x) = |x| + |x - 1|$ at $x = 0,1$. (6

(b) State Lagrange's Mean Value Theorem. Verify th theorem for the function $f(x) = x(x - 1)(x - 2)$ $\left[0, \frac{1}{2}\right]$. (6

(c) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0.$$ (

3. (a) Find the open intervals on which $f(x) = 3x^4 - 4x^3$ concave up and concave down. Also determine the poi of inflexion, if any. (

(b) Find the position and nature of multiple points of the curve given by $x^4 + y^3 - 2x^3 + 3y^2 = 0$. Also, find tangent(s) at the origin, if any. (6)

(c) Trace the curve $x = a\cos^3\theta,\ y = a\sin^3\theta;\ 0 \le \theta \le 2\pi$. (6)

(a) Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$. Hence evaluate $\int_0^{\pi/2} \sin^5 x\,dx$. (7)

(b) Sketch the curve $x^2y^2 = x^2 - a^2$. (7)

(c) Sketch the polar curve $r = 2 + 3\cos\theta,\ 0 \le \theta \le 2\pi$. (7)

(a) Find the volume of the solid that results when the region enclosed by $y = x^2$, $y = x^3$ is revolved about the x axis. (6)

(b) Describe the graph of the equation

$$x^2 + 9y^2 + 2x - 18y + 1 = 0.$$ (6)

(c) Find an equation for the parabola that has vertex at (1,1) and directrix $y = -2$. Also, write the reflection property of parabola. (6)

6. (a) Sketch the curve $xy = 1$. (6½)

(b) (i) Find $\nabla.(\nabla \times F)$ if $F(x, y, z) = e^{xz}i + 3xe^{y}j - e^{yz}k$

(ii) Prove $div(F+G) = divF + divG$ where $F = F(x,y,z)$ and $G = G(x,y,z)$ are vector valued functions. (6½)

(c) Sketch the graph of $z^2 = x^2 + \frac{y^2}{4}$. (6½)

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7594** **HC**

Unique Paper Code : 32355202

Name of the Course : **Generic Elective : Mathematics**

Name of the Paper : Linear Algebra

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Do any **two** parts from each question.

1. (a) (i) If x and y are vectors in $\mathbb{R}^n$, then prove that

$$\|x+y\| \le \|x\| + \|y\|$$

Also, verify it for the vectors x = [-1,4,2,0,–3] and y = [2, 1, –4, –1,0] in $\mathbb{R}^5$.

P.T.O.

(ii) If x and y are non-zero vectors in $\mathbb{R}^n$ such that $x.y > 0$, then prove that angle between x and y is acute. 6

(b) (i) Define the projection vector of a vector b onto a vector a (where a is a non-zero vector).

(ii) For vectors a = [2, -3, 4] and b = [-6, 2, 7], find $proj_a b$ and verify that b- $proj_a b$ is orthogonal to a. 6

(c) Solve the following system of equations, using Gauss Jordan Reduction method : 6

$$4x - 2y - 6z = 18$$

$$x + y - 3z = 15$$

$$2x - 4z = 16$$

2. (a) Determine whether the vector v = [2,2, –3] is in the row space of the matrix. 6.5

$$A = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 3 & 5 \\ 6 & 1 & 9 \end{bmatrix}$$

(b) Diagonalize the matrix 6.5

$$A = \begin{bmatrix} -2 & 4 & -6 \\ 3 & -3 & 6 \\ 3 & -4 & 7 \end{bmatrix}$$

(c) (i) Find the eigen space E_λ corresponding to eigen value $\lambda = 6$ for the matrix

$$A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$$

(ii) Prove that the subset

$$W = \left\{ \left[a, b, \frac{a}{2} - 2b \right] : a, b \in \mathbb{R} \right\}$$ of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$. 6.5

3. (a) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 5 & -4 & 3 \end{bmatrix}$$

Using rank of A, determine whether the homogeneous system Ax = 0 has a nontrivial solution or not. 6

(b) Use Simplified Span Method to find a simplified general form for all the vectors in span (S), where S = {[1,−1,1], [2,−3,3], [0,1,−1]} is a subset of $\mathbb{R}^3$. 6

(c) Let B = {[2,3,0,−1], [−1,1,1,−1]} and

S = {[1,4,1,−2], [−1,1,1,−1],[3,2,−1,0], [2,3,0,-1]}.

(i) Show that B is a maximal independent subset of S.

(ii) Calculate dim(span(S)).

(iii) Does span (S) = $\mathbb{R}^4$? Why or why not ?

6

5.

4. (a) (i) Show that the mapping $f:\mathbb{R}^2 \to \mathbb{R}^2$ given by f([x,y]) = [x+ky,y] is a linear operator.

(ii) Let V be a vector space, and let x ≠ 0 be a fixed vector in V. Prove that the translation function f : V → V given by f(v) = v + x is not linear. 6.5

(b) Let S = {[1,2], [0,1]} and T = {[1,1], [2,3]} be bases for $\mathbb{R}^2$.

Let v = [1,5] and w = [5,4].

(i) Find the coordinate vectors of v and w with respect to the basis T.

(ii) What is the transition matrix $P_{s \leftarrow t}$ from the basis T to the basis S.

(iii) Find the coordinate vectors of v and w with respect to S using $P_{s \leftarrow t}$.

6.5

(c) Suppose $L : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear operator and L([1,0,0]) = [−3,2,4], L([0,1,0]) = [5, −1, 3] and L[(0,0,1)] = [−4,0,−2]. Find L([x,y,z]), for any [x,y,z] $\in \mathbb{R}^3$ and also find L([6,2,−7]).

6.5

5. (a) For the linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$ given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

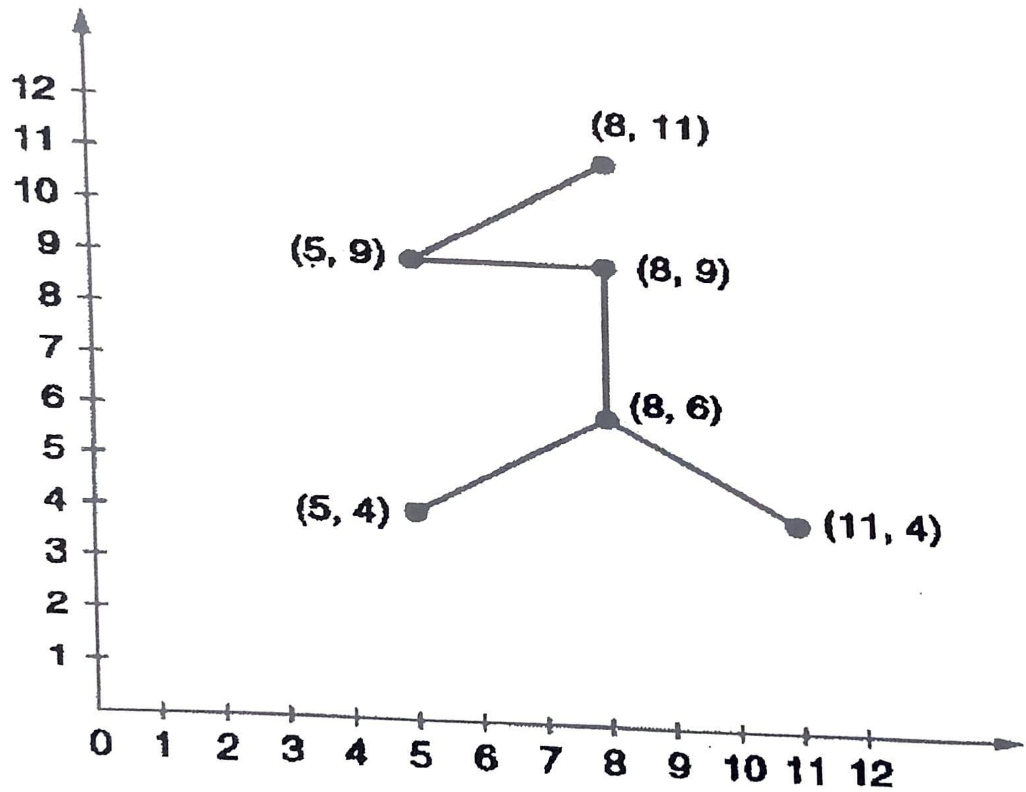
Find a basis for ker(L) and a basis for range (L). Also, verify the Dimension Theorem.

6

P.T.O.

(b) For the adjoining graphic, use homogeneous coordinates to find the new vertices after scaling about (8,4) with scale factors of 2 in the x-direction and $\frac{1}{3}$ in the y-direction. Also, sketch the final figure that would result from this movement.

6



(c) For $W = \text{span}\{[1,-2,-1], [3,-1,0]\}$ in $\mathbb{R}^3$ and $v = [-1, 3, 2]$, decompose v into w_1+w_2, where $w_1 \in W$ and $w_2 \in W^\perp$. Is the decomposition unique ? 6

6. (a) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by :

$$L(v) = Av, \text{where } A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Is L an isomorphism ? Justify your answer.

6.5

(b) Find a basis for the orthogonal complement $W^\perp$ of the subspace $W = \{[x, y, z] \in \mathbb{R}^3 : 2x - 2y + z = 0\}$ of $\mathbb{R}^3$ and verify that $\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^3)$.

6.5

(c) Find Least Squares Solution to the inconsistent linear system Ax = b, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \\ 4 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 12 \\ 15 \\ 4 \end{bmatrix}$$ 6.5

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **8188**	**HC**
Unique paper code	: **62351201**	
Name of the paper	: **Algebra**	
Name of course	: **B.A. (Prog.) Mathematics**	
Semester	: **II**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **two** parts from each question.*

1. (a) Let $\mathbb{R}^3 = \{(a_1, a_2, a_3): a_1, a_2, a_3 \in \mathbb{R}\}$. Show that $\mathbb{R}^3$ is a vector space over the field $\mathbb{R}$ of real numbers with pointwise addition and multiplication. (6)

(b) Prove that the intersection of two subspaces of a vector space $V(F)$ is a subspace of $V(F)$. Show by an example that $W_1 \cup W_2$ for any two subspaces W_1 and W_2 of $V(F)$, may not be a subspace of $V(F)$. (6)

(c) Define basis of a vector space and show that the vectors $(1, 2, 1)$, $(2, 1, 0)$ and $(1, -1, 2)$ form a basis of $\mathbb{R}^3(\mathbb{R})$. (6)

2. (a) Find the values of λ and μ for which the system of equations:

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 3y + \lambda z = \mu$$

has (i) a unique solution, (ii) no solution, (iii) infinitely ma
solutions.

(6.

(b) Reduce the following matrix to normal form and hen
find its rank:

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

(6.

(c) Find the characteristic roots and the corresponding eige
vectors of the following matrix:

$$\begin{bmatrix} 2 & 1 & 0 \\ 9 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(6.

3. (a) Prove that:

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}.$$

(b) Sum the series:

$\sin\theta + x\sin 2\theta + x^2\sin 3\theta + \ldots\ldots\ldots$ up to n terms

where x is a real number and $x \neq \pm 1$.

(c) State De Moivre's Theorem for rational indices and
it to solve the equation:

$$z^7 + z = 0.$$ (6)

4. (a) If one root of the equation

$$x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$$

is $2+\sqrt{3}$, find the other roots. (6.5)

(b) Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$, given that two of its roots are in the ratio $3:5$. (6.5)

(c) Form the cubic equation whose roots α, β, γ satisfy the relations:

$$\alpha + \beta + \gamma = 3, \alpha^2 + \beta^2 + \gamma^2 = 5, \alpha^3 + \beta^3 + \gamma^3 = 11.$$ (6.5)

5. (a) Show that the set $S = \{1, 5, 7, 11\}$ is an abelian group with respect to multiplication modulo 12. (6)

(b) Show that a group G is abelian if and only if $(a.b)^2 = a^2.b^2$ for all $a, b \in G$. (6)

(c) If:

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \text{ and}$$

$$h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix},$$

show that $h^{-1} f h = g$. (6)

6. (a) If H and K are subgroups of a group G, show that $H \cap K$ is a subgroup of G. Is the union of two subgroups also a subgroup? Justify. (6.5)

(b) Describe the group of symmetries of a square. (6.5)

(c) Show that the set of all matrices of the form $\begin{pmatrix} 0 & x \\ 0 & y \end{pmatrix}$ s.

$x, y \in \mathbb{R}$ is a non-commutative ring with respect to matri

addition and matrix multiplication. (6.5)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : **8189** HC

Unique paper code : **62351201**

Name of the paper : **Algebra**

Name of course : **B.A. (Prog.) Mathematics**

Semester : **II**

Duration : **3 hours**

Maximum marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

1. (a) Prove that the set V of all polynomials over $\mathbb{R}$ defined as:

$$V = \{f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$
$$a_i \in \mathbb{R}, n \geq 0\}$$

form a vector space over $\mathbb{R}$ with respect to addition and scalar multiplication of polynomial function. (6)

(b) Define subspace of a vector space over the field F. Show that the set

$$W = \{(a_1, a_2, a_3): a_1 + a_2 + a_3 = 0, a_1, a_2, a_3 \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}^3$. (6)

(c) Show that the vectors $(1, 2, 1)$, $(1, 0, -1)$ and $(0, -3, 2)$ form a basis for $\mathbb{R}^3$.

(6)

2. (a) Obtain complete solution of the system of equations:

$$2x - y + 3z - 5w = -7$$
$$-7y + 3z - 7w = -13$$
$$-3x + 4y + 2z = 0$$

(6.5)

(b) Reduce the following matrix to normal form and hence find out its rank:

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix}$$

(6.5)

(c) State Cayley Hamilton Theorem and use it to find the inverse of the following matrix:

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(6.5)

3. (a) State and prove De Moivre's Theorem for integral indices.

(6)

(b) Show that if n is any integer, then:

$$\left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n = 2^{n+1} \cos(n\pi / 3).$$ (6)

(c) Sum the series:

$$\cos\theta\sec\theta+\cos 3\theta\sec^2\theta+\cos 5\theta\sec^3\theta+\ldots\ldots\ldots$$

ıp to n terms, where θ is not an odd multiple of $\pi/2$. (6)

. (a) Find the biquadratic equation with rational coefficients ıaving $\sqrt{3}-\sqrt{5}$ as one of its roots. (6.5)

b) Solve the equation $x^3-13x^2+15x+189=0$, given that ›ne root exceeds another by 2. (6.5)

c) If α,β,γ are the roots (not all zero) of the equation:

$$x^3-px^2+qx-r=0,$$

ind the value of :

(i) $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$

(ii) $\sum_{\alpha,\beta,\gamma}\alpha/\beta$ (6.5)

(a) Show that the set $S=\{2,4,6,8\}$ is an abelian group ıith respect to multiplication modulo 10. (6)

) If a and b are elements of a group G such that $ab=ba$, rove that $(ab)^n=a^nb^n$ for every $n\in\mathbb{N}$. (6)

) Find the order of the following permutations in S_4:

(i) $\begin{pmatrix}1&2&3&4\\1&2&4&3\end{pmatrix}$

(ii) $\begin{pmatrix}1&2&3&4\\4&1&2&3\end{pmatrix}$. (6)

6. (a) Let $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$ be the group under matrix addition. Let $H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G : a + b + c + d = 0 \right\}$. Prove that H is a subgroup of G.

(6.5)

(b) Examine which of the following are groups. Justify your answer in each case.

(i) The set $\mathbb{Z}$ of integers with respect to $*$, where $a * b = a + b + 1 \ \forall \ a, b \in \mathbb{Z}$.

(ii) The set $\mathbb{N}$ of natural numbers with respect to addition.

(6.5)

(c) Show that the set $S = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.

(6.5)

This question paper contains 4 printed pages.

Your Roll No. ....

o. of Paper	: **6632**	**HC**
que paper code	: **32351401**	
ne of the paper	: **Partial Differential Equations**	
ne of course	: **B.Sc. (Hons.) Mathematics**	
ester	: **IV**	
ation	: **3 hours**	
ximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

***All** Sections are compulsory.*
Marks of each part are indicated.

SECTION – I

*Attempt any **two** parts out of the following.*

(a) Obtain the general solution of the equation

$$(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z).$$

Find the integral surfaces of this equation passing through (i) the x-axis, (ii) the y-axis, (iii) the z-axis. $\left(7\frac{1}{2}\right)$

(b) Solve the following initial value system:

$$u_t + uu_x = e^{-x}v \;, v_t - av_x = 0$$

with $u(x,0) = x$ and $v(x,0) = e^x$. $\left(7\frac{1}{2}\right)$

P. T. O.

(c) Apply the method of separation of variab
$u(x,y) = f(x)g(y)$ to solve

$$x^2 u_{xy} + 9y^2 u = 0 \qquad u(x,0) = \exp\left(\frac{1}{x}\right)$$ $\left(7\frac{1}{2}\right.$

SECTION – II

*Attempt any **one** part out of the following.*

2. (a) Show that the equation of motion of a one-dimensional wa
equation is $u_{tt} = c^2 u_{xx}$ where $c^2 = \tau/\rho$.

(b) Derive the Laplace equation of motion.

*Attempt any **two** parts out of the following.*

3. (a) Find the characteristics, characteristic coordinates and red
the equation given below to the canonical form:

$$u_{xx} + (\sec h^4 x) u_{yy} = 0.$$

(b) Transform the following equations to the form:
$v_{\xi\eta} = cv$, c =constant, $u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$
by introducing the new variable $v = ue^{-(a\xi + b\eta)}$, where
are undetermined coefficients.

(c) Determine the general solution of the equation given bel 5.

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0$$

SECTION – III

*Attempt any **three** parts out of the following.*

4. (a) Determine the solution of the equation:

$$L[u] \equiv u_{xy} + au_x + bu_y + cu = f(x,y)$$

where a, b, c and f are differentiable functions of (x, y) in some domain D^*, under the boundary conditions that u and u_x are prescribed along the curve C in the xy plane. (7)

(b) Solve the Goursat problem

$$xy^3 u_{xx} - x^3 y u_{yy} - y^3 u_x + x^3 u_y = 0,\ x \neq 0,$$
$$u(x, y) = f(x) \quad on \quad y^2 + x^2 = 16 \quad for \quad 0 \leq x \leq 4,$$
$$u(x, y) = g(y) \quad on \quad x = 0 \quad for \quad 0 \leq y \leq 4,$$

where $f(0) = g(4)$. (7)

(c) Find the solution of the initial boundary value problem:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x, 0) = \sin x, \qquad u_t(x, 0) = x^2$$ (7)

(d) Solve the Cauchy problem for the non-homogeneous wave equation

$$u_{tt} = c^2 u_{xx} + e^x,$$

with the initial conditions:

$$u(x, 0) = 5, \qquad u_t(x, 0) = x^2$$ (7)

SECTION – IV

*Attempt any **three** parts out of the following.*

(a) Solve using the method of separation of variables:

$$u_t = k u_{xx},\quad 0 < x < l,\ t > 0.$$
$$u(0, t) = 0,\ t \geq 0$$
$$u(l, t) = 1,\ t \geq 0$$
$$u(x, 0) = \sin \frac{\pi x}{2l},\quad 0 \leq x \leq l.$$ (7)

P. T. O.

(b) Using the method of separation of variables, discuss problem of vibrating string:

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < l \;,\; t > 0,$$
$$u(x,0) = f(x), \qquad u_t(x,0) = g(x), \quad 0 \le x \le l,$$
$$u(0,t) = u(l,t) = 0, \qquad t > 0.$$

(c) Determine the solution of Initial Boundary Value proble

$$u_{tt} = c^2 u_{xx} + x^2, \quad 0 < x < 1 \;,\; t > 0,$$
$$u(x,0) = x, \qquad u_t(x,0) = 0, \quad 0 \le x \le 1,$$
$$u(0,t) = 0, \qquad u(l,t) = 1, \qquad t > 0.$$

(d) Find the solution of the plucked string of length l eq tion given by:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = 0,$$
$$u_t(x,0) = \begin{cases} \dfrac{v_0 x}{a}, & 0 \le x \le a \\[2ex] \dfrac{v_0 (l - x)}{(l - a)}, & a \le x \le l \end{cases}$$
$$u(0,t) = u(l,t) = 0, \qquad t > 0.$$

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : 6633 **HC**

Unique paper code : 32351402

Name of the paper : **Reimann Integration and Series of Functions**

Name of course : **B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 3 hours

Maximum marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.
All questions are compulsory.

Q.1 (a) Prove that a bounded function f on [a, b] is integrable if and only if for each $\varepsilon > 0$, $\exists$ a partition P of [a, b] such that $U(f, P) - L(f, P) < \varepsilon$. (6)

(b) Show that if f is integrable on [a, b], then $|f|$ is integrable on [a, b], and:

$$\left|\int_a^b f\right| \leq \int_a^b |f|.$$

Hence, show that :

$$\left|\int_{-2\pi}^{2\pi} x^2 Sin^8(e^x)dx\right| \leq \frac{16\pi^3}{3} \quad (6)$$

(c) Suppose f and g are continuous functions on [a, b], and $g(x) \geq 0 \ \forall x \in [a, b]$. Prove that, $\exists \ x \in [a, b]$ such that:

$$\int_a^b f(t)g(t)\,dt = f(x)\int_a^b g(t)\,dt. \quad (6)$$

Q.2 (a) (i) The Dirichlet function $f: [0, 1] \to \mathbf{R}$ is defined by :

$$f(x) = \begin{cases} 1, & x \in [0,1] \cap Q \\ 0 & x \in [0,1] \setminus Q \end{cases}$$

Is f Riemann Integrable? Justify.

(ii) Show that a decreasing function on [a, b] is integrable.

(b) (i) State Fundamental Theorem of Calculus-I.

(ii) Let f be an integrable function on [a, b]. Let P be partition of [a,b] and P* be a refinement of P such that P* = P ∪ {u}. Show that L(f, P) ≤ L(f, P*).

(c) (i) For a bounded function f on [a, b], define the Riemann Sum associated with a partition P. Hence, give Riemann's definition of integrability.

(ii) Calculate $\lim_{h\to 0}\frac{1}{h}\int_{3}^{3+h} e^{t^2}\,dt$

3. (a) Show that:

$$\beta(x,y)=\int_0^1 t^x(1-t)^y\,dt \text{ converges } \Leftrightarrow x>0, y>0 .$$

(b) (i) Define Improper Integral of Type-II. When does it converge? When is it said to diverge ?

(ii) Determine if $\int_{-\infty}^{\infty} e^x\,dx$ is a convergent integral. Justify.

(c) (i) Determine if the following integrals are Improper Integrals. If so, what kind ? Justify.

$$\int_0^{\pi/2}\frac{Sinx}{\sqrt[3]{x}}\,dx, \qquad \int_0^1 x\ln(x)\,dx$$

(ii) Prove that $\int_{\pi}^{\infty}\frac{Sinx}{x}\,dx$ converges absolutely.

4. (a) State and prove Cauchy's criterion for uniform convergence for sequences of functions.

(b) (i) Is the sequence $\{f_n\}$ where :

$$f_n = \frac{1}{n}\sin(nx+n).$$

uniformly convergent on **R** ? Justify. (2)

(ii) Suppose a sequence $\{f_n\}$ converges uniformly to f on a set A, and further suppose that each f_n is bounded on A. Show that the limit function f is bounded on A. (4)

(c) Show that if a>0, then the sequence $\{n^2x^2e^{-nx}\}$ converges uniformly on the interval $[a,\infty)$, but that it does not converge uniformly on the interval $[0, \infty)$. (6)

5. (a) If f_n is continuous on $D \subseteq \mathbf{R}$, for each $n \in \mathrm{N}$ and if Σf_n converges to f uniformly on D, then f is continuous on D. (6)

(b) Show that the series of functions $\Sigma \dfrac{x^n}{(1+x^n)}$, $x \geq 0$, converges uniformly on $[0, a]$ for $0 < a < 1$, but is not uniformly convergent on [0, 1). (6)

(c) A sequence $\{f_n\}$ converges uniformly to f on A_0, if for each $\varepsilon > 0$ there is a natural number $K(\varepsilon)$, depending only on ε, such that if $n \geq K(\varepsilon)$, then:

$$|f_n(x) - f(x)| < \varepsilon \quad \text{for all } x \in A_0.$$

Hence state a necessary and sufficient condition for a sequence $\{f_n\}$ to fail to converge uniformly on A_0 to f.

Apply this condition on sequence $f_n(x) = \dfrac{x}{n}$, for $x \in \mathbf{R}$ to examine for uniform convergence. (6)

6. (a) (i) Suppose that $f(x)=\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R>0$. Then show that $\int_0^x f(t)dt=\sum_{n=0}^{\infty}\frac{a_n}{n+1}x^{n+1}$ for $|x|<R$ (5)

(ii) State when is a power series differentiable term by term. (1.5)

(b) (i) Apply Abel's Theorem to show :

$$\log_e 2 = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\ldots.$$ (3.5)

(ii) Given $\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ for $|x|<1$. Evaluate:

$$\sum_{n=1}^{\infty}\frac{n}{5^n}, \quad \sum_{n=1}^{\infty}\frac{n^2(-1)^n}{3^n}.$$ (3)

(c) (i) App/ly Ratio Test for series to show that radius of convergence R of the power series $\sum a_n x^n$ is given by $\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$, whenever the limit exists. (3.5)

(ii) Find radius of convergence for :

$$\sum_{n=2}^{\infty}(\ln(n))^{-1}x^n, \quad \sum_{n=0}^{\infty}\frac{(n!)^3 x^n}{(3n)!}$$ (3)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6841**

Unique Paper Code : **32353401/42353404** **HC**

Name of the Paper : **Computer Algebra Systems and Related Softwares**

Name of the Course : **B.Sc. (H) Mathematics/B.Sc. Math Sc./ B.Sc. (Prog.)**

Semester : **IV**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *four* questions in all.

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(*i*) The rank of a matrix A in MATLAB is given by the command.....................

(*ii*) In R, the.....................function produces stem and leaf plot of an array.

(*iii*) The command for $\log_{10} 5$ in Mathematics is.....................

(*iv*) ....is the command to write the matrix $\begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix}$ in Maxima.

P.T.O.

(v) The built-in constant e is represented by.....................in Maple.

2. Write the output for the following : 5×1=5

(i) i = 1;

While [i <= 10, i = i + 1; Print[i]; i + +]

(ii) A = {{1, 0, 2}, {2, 3, 0}, {1, 2, 1}};

A^2

(iii) prod(sqrt(i), i, 1, 4);

(iv) f(x): = x^3 + sin (x);

diff (f(x), x);

(v) A = [1, 2, 3; 4, 5, 6; 7, 8, 9];

A(2, :) + A(3, :)

3. Attempt any EIGHT parts from the following : 8×2=16

(i) Define *mesh*() function in MATLAB/Octave with an example.

(ii) Write the commands for the following in Maple :

(a) Binomial coefficient $\binom{7}{2}$

(b) Prime factorization of 654382

(iii) Define and differentiate a function $f(x) = x^4 + 3 \sin x - 2$ in Maple.

(iv) Write a command in Maxima to plot the graph of the function $h(x, y) = x^4y + \cos(x, y)$, for $1 \le x, y \le 2$.

(v) Write any *two* differences between Mathematica and Maxima.

(*vi*) Write the commands for the following in Maxima :

(*a*) $\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{3\pi}{2}\right)$

(*b*) Previous prime number of 2008

(*vii*) Define *pnorm*() and *qnorm*() functions in R. If *pnorm* (–1.645) = 0.04998491 then what is the value of *qnorm*(0.04998491) ?

(*viii*) Write a command in MATLAB/Octave to find :

(*a*) Eigenvalues and eigenvectors of a matrix A.

(*b*) Lower and upper triangular parts of matrix A with a permutation matrix P.

(*ix*) For A = [2, 0, 3; 5, 8, –1; 6, 7, 1]; write the output for the following :

(*a*) A([1, 3], [2, 1])

(*b*) A([1, 2], :) = A([2, 1], :)

(*x*) Write commands in R to simulate a random sample of 15 items from a normally distributed data that has mean 30 and standard deviation 9.

4. Attempt any *four* parts from the following : 4×6=24

(*i*) Write the commands in Maxima for the following :

(*a*) Find M^2 for $M = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$.

(*b*) Find x if $x^2 + x = 1$.

(*c*) Compute $7^{20} mod\ 21$.

(*d*) Find prime factorization of 281.

(ii) Let $f(x) = \dfrac{x^3 \cos(x)}{x^2 + 1}$. Write the commands in Maple to find $f'(x)$, $f''(x)$, $f'(-1)$ and $f''(0)$.

(iii) Explain the following commands in Mathematica with example :

(a) For loop

(b) Do loop

(c) Print

(d) Module

(iv) Write a program to solve the following system of equations in MATLAB/Octave :

$$\begin{aligned} 3u + v - t &= 10 \\ u + 4v - 7w + 2t &= 15 \\ -v + w - 6t &= -4 \\ 7u - 2v + w + t &= 8 \end{aligned}$$

(v) Write the commands in R for the following :

(a) Put the following values into a variable 'score'

30	45	63	72	21
21	45	22	88	61
10	36	20	46	55
21	11	07	54	19

(b) Create a box plot of score.

(c) Create a stem and leaf plot of score.

(d) Create a normal probability plot of score.

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6805 **HC**

Unique Paper Code : 42344403

Name of the Paper : Computer System Architecture

Name of the Course : **B.Sc. (Prog.) / Mathematical Science**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. Attempt any **5** of Question Nos. **2** to **8.**
4. Parts of a question must be answered together.

1. (a) How many 128 words x 8 bits per word RAM chips are needed to provide a memory capacity of 4096 words x 16 bits per word? (2)

(b) Differentiate between Combinational Circuit an
Sequential Circuit. (2

(c) Convert the binary number 11011000111 in octal an
hexadecimal. (2

(d) How many address lines and input-output data lines ar
needed for 2K words x 16 bits per word? (2)

(e) Write micro-operations for the following memory
reference instructions :

(i) ADD to AC

(ii) LDA: Load to AC (2×2=4)

(f) Convert the following numbers to the indicated base:

(i) $(110110100)_2$ to $(..............)_{10}$

(ii) $(7562)_{10}$ to $(..........)_8$ (2×2=4)

(g) Give characteristic tables of SR and JK flip-flop. (3)

(h) Simplify the following expressions using Boolean algebra.

(i) AB + A(CD+CD′)

(ii) A′BC + AC (2×3=6)

(a) A computer uses a memory unit with 256K words of 32 bits each. A binary instruction code is stored in one word of memory. The instruction has 4 parts: an indirect bit, an operation code, a register code part to specify one of 64 registers and an address part.

(i) How many bits are there in the operation code, the register code part and address part?

(ii) Draw the instruction word format and indicate the number of bits in each part.

(iii) How many bits are there in the data and address inputs of the memory. (3+2+2=7)

(b) Write a short note on Input-Output interface. (3)

(a) Define half adder. Illustrate the same with the help of its truth table and logic diagram. Also write Boolean expressions for carry and sum. (6)

(b) The content of the registers are as follows –

Register A (before operation) 1010

Register B (logic operand) 1100

Perform the following operations on the contents of A using the contents of B-

(i) Selective Complement

(ii) Selective Clear

(4)

4. (a) Write short notes on the following :–

(i) Addressing modes

(ii) Types of ROM (3+3=6)

(b) Explain Direct Memory Access (DMA) with the help of a block diagram. (4)

5. (a) Draw the block diagram of 3x8 decoder using 2x4 decoders. Also explain its working. (4)

(b) Perform the arithmetic operations (+70) + (–80) and (–70) – (–80) in binary using signed 2's complement representation for negative numbers. (3+3=6)

6. (a) Write a program to evaluate the arithmetic statement :

X = (A+B) * (C+D)

using two and three address instructions. Use the symbols ADD, SUB, MUL for arithmetic operations and MOV for the transfer-type operation. Assume that memory operands are in memory addresses A, B, C and D and the result must be stored in memory address X.

(3+3=6)

(b) Represent the following conditional control statement by two register transfer statements with two control functions :

If (P=1) Then ($R_1 \leftarrow R_2$) Else if (Q=1) Then ($R_1 \leftarrow R_2$)

(4)

7. (a) Differentiate between direct and indirect address instructions with the help of an example? How many references to memory are needed for each type of instruction to bring an operand into a processor register? (4+2=6)

(b) Show the value of all bits of a 12-bit register that holds the number equivalent to decimal 215 in –

(i) Binary

(ii) BCD (4)

8. (a) List phases of the instruction cycle. Draw flowchart of the instruction cycle without interrupt. (5)

(b) Simplify the following Boolean function $F(x,y,z) = \Sigma(3,5,6,7)$ using three-variable K-Map. (5)

SET-A

Your Roll No.:.....................

Sl No: 7 Q.P.: 6807A

Unique Paper Code : 42353404

Name of the Paper : Computer Algebra Systems

Name of the Course : **B.Sc (Prog.)**/ B.Sc. Mathematical Science : SEC

Semester : IV

Duration : 2 Hours

HC

Maximum Marks : 38

Instructions for Candidates :

a. Using any one of the **CAS := Mathematica/Maple/Maxima/any other** to answer the questions.

b. Write your Roll No. on the top immediately on receipt of this question paper.

c. This question paper has **four** questions in all.

d. **All** questions are compulsory.

Q3. Write the Output of any **five** from the following: (5×2=10)

i. Sum[i^2,{i,1,10}]
   Product[a[i],{i,1,n}]

ii. Plot[2-x^2,{x,-2,2},Filling→Axis]

iii. Table[If[i≤j,j-3i,E],{i,-2,3},{j,0,3}]//Grid

iv. makelist(makelist(i+j,i,1,5),j,1,5);
    makelist (n^2, n, 1,10,2);

v. Piecewise[{{x,0≤x≤1},{-x,-1<x<0}},1]

vi. Array[Max,{3,3}]//MatrixForm
    Array[Min,{3,3}]

vii. f(x):=4*x+1; g(x):=-x+4; h(x):=9*x-8;
     wxplot2d([f(x),g(x),h(x)],[x,0,2]);

viii. solve (x+y=3, [x,y]); solve (x+4, x); solve (x);

Q4. Attempt any four parts from the following:(4×2.5=10)

Your Roll No.:......................

Sl No. 57 Q.P. 6807A

Unique Paper Code : 42353404

Name of the Paper : Computer Algebra Systems

Name of the Course : B.Sc (Prog.)/ B.Sc. Mathematical Science : SEC

Semester : IV

Duration : 2 Hours

Maximum Marks : 38

Instructions for Candidates :

a. Using any one of the **CAS := Mathematica/Maple/Maxima/any other** to answer the questions.

b. Write your Roll No. on the top immediately on receipt of this question paper.

c. This question paper has **four** questions in all.

d. **All** questions are compulsory.

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ii. Plot[2-x^2,{x,-2,2},Filling→Axis]

iii. Table[If[i≤j,j-3i,E],{i,-2,3},{j,0,3}]//Grid

iv. makelist(makelist(i+j,i,1,5),j,1,5);
    makelist (n^2, n, 1,10,2);

v. Piecewise[{{x,0≤x≤1},{-x,-1<x<0}},1]

vi. Array[Max,{3,3}]//MatrixForm
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vii. f(x):=4*x+1; g(x):=-x+4; h(x):=9*x-8;
     wxplot2d([f(x),g(x),h(x)],[x,0,2]);

viii. solve ([illegible] 3, [x,y]); solve (x+4, x); solve (x);

Q4. Attempt any four parts from the following:(4×2.5=10)

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 8 \\ 16 \\ 2 \end{bmatrix}.$$

(iv) Write commands to find the quotient and remainder when the polynomial $x^4 + 3x^2 - 4$ is divided by $x^2 - 9$.

(v) Write a command to plot the function x^3-9x+5 in the domain [-3,3] and superimpose the critical points as large dots.

(vi) Write a command to find all the zeroes of the function Sin(x).

iii. CAS software gives an additive constant for indefinite integrals.

iv. With the % command, we can call any output.

v. The effect of the 'NSolve' command in Mathematica/Maxima is same as the 'Solve' command followed by the 'N' command.

vi. The block( ) function is used in software Maxima to provide a new variable name space.

vii. Maple is created under the trade name Maplesoft.

viii. The in-built function **N[exper]** in Mathematica/Maxima stands for numerical integration.

Q2. Write a short note on any **four** from the following: $(4\times2.5=10)$

i. What are the rules for defining a function in any CAS?

ii. Differentiate between "*" and "•" in respect of matrices in Maxima/Mathematica.

iii. What is the difference between the commands Factor and FactorInteger.

iv. Explain the commands Minors and LinearSolve.

v. Explain the FindRoot command.

vi. Describe "Computer Algebra Systems".

(iii) Write commands to solve the system of equations Ax=b where

$$A=\begin{bmatrix}2 & 3 & -4\\ 1 & 2 & 3\\ 1 & -1 & 1\end{bmatrix} \text{ and } b=\begin{bmatrix}8\\ 16\\ 2\end{bmatrix}.$$

(iv) Write commands to find the quotient and remainder when the polynomial x^4+3x^2-4 is divided by x^2-9.

(v) Write a command to plot the function x^3-9x+5 in the domain [-3,3] and superimpose the critical points as large dots.

(vi) Write a command to find all the zeroes of the function Sin(x).

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iv. Explain the commands Minors and LinearSolve.

v. Explain the FindRoot command.

vi. Describe "Computer Algebra Systems".

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6807 **HC**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.

1. (a) State and prove Archimedean property of real numbers.

(b) Find Supremum of the following sets :

(i) $\{x \in R \mid 1 < x < 2\}$,

(ii) $\left\{1 - \frac{1}{n}, n \in N\right\}$,

P.T.O.

(iii) $\left\{\frac{1}{n}+\frac{1}{m}, n, m \in N\right\}$.

(c) Show that a countable union of countable sets is countable. Deduce that the set $N \times N$ is countable. (6,6)

2. (a) State the Bolzano Weierstrass theorem for sets. Show that the hypotheses of the theorem cannot be relaxed. Justify your answer.

(b) If (x_n) is a sequence such that $x_n \geq 0 \ \forall \ n$ and (x_n) converges to x then show that $x \geq 0$.

(c) Show that $\lim_{n\to\infty} n^{1/n} = 1$. (6,6)

3. (a) Show that the sequence (a_n), where $a_n = \left(1+\frac{1}{n}\right)^n$ converges and $\lim_{n\to\infty} a_n$ lies between 2 and 3.

(b) State Cauchy's Convergence Criterion for sequences and hence show that the sequence (S_n), where

$S_n = 1+\frac{1}{3}+\frac{1}{5}+\cdots\cdots+\frac{1}{2n-1}$ does not converge.

(c) Let $\sum_{n=1}^{\infty} u_n$ be a positive term series such that $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = L$, then show that $\sum_{n=1}^{\infty} u_n$ converges if $L < 1$ and diverges if $L > 1$. What happens when $L = 1$? (6½,6½)

(a) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}$

(ii) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$

(iii) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots\cdots$

(b) State Cauchy's nth Root Test for an infinite positive term series and hence test the convergence of the series

$\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$, $x > 0$.

(c) Define absolute convergence and conditional convergence for an infinite series. Prove that absolute convergence implies convergence but the converse is not true. (6½,6½)

5. (a) State the Weierstrass M-test for the convergence of a series of functions and hence test the convergence of the series $\sum_{n=1}^{\infty} r^n \mathrm{Cos}(nx)$, $0 < r < 1$ for every real x.

(b) Show that the sequence (f_n) of functions, where $f_n(x) = x^n$, is uniformly convergent on [0, k], $k < 1$ and only pointwise convergent on [0, 1].

(c) Define Exponential function E(x) and Cosine function C(x) in terms of a power series. Find the domain of convergence of the respective power series.

(6½,6½

6. (a) Show that a constant function k is integrable and $\int_a^b k\,dx = k(b-a)$.

(b) Show that if a function f defined on [a, b] is monotonic then it is integrable on [a, b].

(c) Let f be a bounded real function defined on [a, b]. Let P be any partition of [a, b]. Define the upper and lower sum of f over P and show that

$$m\,(b-a) \leq L\,(P, f) \leq U(P, f) \leq M\,(b-a),$$

where M and m are bounds of f over [a, b]. (6,6

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6843**

Unique Paper Code : **42343408** HC

Name of the Paper : **PHP Programming**

Name of the Course : **B.Sc. Mathematical Sciences/**

B.Sc. (Prog.) : SEC

Semester : **IV**

Duration : **2 Hours** Maximum Marks : **25**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt any *three* questions from Questions **2–5**.

1. (*a*) Write a function to add two numbers passed as arguments to it. 2

(*b*) What is the difference between the following : 2

(*i*) "==" and "==="

(*ii*) "&&" and "and" operators

(*c*) What is the output of the following code : 2

```
<?php
    $str = "Hello! My name is Cameron Fox. Coffee?"
    $find = array('/is/','/coffee/');
    $replace = array('/was/','/tea/');
echopreg_replace ($find, $replace, $str);
  ?>
```

(*d*) Explain POST method and list *two* advantages. 2

(*e*) Write PHP statements for the following : 2

(*i*) to send a query to the database.

(*ii*) to retrieve the number of rows affected by an INSERT, DELETE or UPDATE query.

2. (*a*) Write a PHP program to read an array and sort it using built in functions. 3

(*b*) What is the difference between "call by value" and "call by reference"? 2

3. (*a*) How are the variables and constants defined in a PHP program ? Give *one* example for each. 3

(*b*) Explain the use of **foreach** with example. 2

4. Write PHP code for reading two values from an HTML form and display it. 5

5. Explain the following functions with a suitable example for each : 5

(*a*) trim()

(*b*) ucfirst()

(*c*) printf()

(*d*) strnatcmp()

(*e*) explode()

6. (*a*) Create a table Student with the following attributes : 3

(*i*) Name

(*ii*) Roll No.

(*iii*) Marks

(*iv*) Address

Assume suitable datatypes for the attributes.

(*b*) Insert two rows in the Student table using a single insert command with appropriate values. 2

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7726**

Unique Paper Code : **32355444** **HC**

Name of the Paper : **Elements of Analysis**

Name of the Course : **Generic Elective : Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Define a countable set. Show that the set $N \times N$ is countable. 7.5

(*b*) Let the function f be defined by $f(x) = \dfrac{(2x^2 + 3x + 1)}{(2x - 1)}$ for $2 \le x \le 3$. Find a constant M such that $|f(x)| \le M$ for all x satisfying $2 \le x \le 3$. 7.5

P.T.O.

(c) State Completeness property of R. Find the supremum and infimum of the following sets :

(i) $A = \{x \in R : x \leq 5\}$.

(ii) $B = \left\{\frac{6n+3}{n} : n \in N\right\}$. 7.5

2. (a) State Monotone Convergence theorem and use it to show that the sequence $\langle x_n \rangle$, where

$$x_n = \left(1 + \frac{1}{n}\right)^n, \text{ for every } n \in N$$

is convergent and its limit lies between 2 and 3. 7.5

(b) If $\lim_{n \to \infty} x_n = x$, then show that $\lim_{n \to \infty} |x_n| = |x|$, where $x \neq 0$. Is the converse true ? Justify. 7.5

(c) State the Squeeze Theorem and use it to prove that $\lim_{n \to \infty} a^{1/n} = 1$, where $a > 0$. 7.5

3. (a) State Cauchy's convergence criterion for series. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. 6.5

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges, if $p > 1$. 6.5

(c) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$.

(ii) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}}$. 6.5

4. (a) State Root Test. Test the convergence of the following series : 6

(i) $\sum_{n=1}^{\infty} \frac{n^n}{e^n}$

(ii) $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

(b) State Leibnitz Test. Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \log(n)}{n}$. 6

(c) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms, then show that $\sum_{n=1}^{\infty} (a_n)^2$ also converges. 6

5. (a) Determine radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$. 5

(*b*) Derive the power series expansion for e^x. 5

(*c*) Define cosine and sine function as sum of power series and prove that :

$$C^2(x) + S^2(x) = 1, \forall x \in R,$$

where $S(x)$ and $C(x)$ denote the sine and cosine functions respectively. 5

6. (*a*) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$. Also check the convergence at the end points of the interval. 5

(*b*) Show that if $|x| < 1$, then

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}.$$ 5

(*c*) State Cauchy-Hadamard theorem and verify it for the power series $\sum_{n=2}^{\infty} \frac{x^n}{\log n}$. 5

This question paper contains 4 printed pages.

Your Roll No. ....

o. of Paper : **8229** **HC**

ique paper code : **62354443**

me of the paper : **Analysis**

me of course : **B.A. (Prog.) Mathematics**

nester : **IV**

ration : **3 hours**

ximum marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

*This question paper has **six** questions in all.*

*Attempt any **two** parts from each question.*

All questions are compulsory.

Marks are indicated against each part of the questions.

a) If **A** and **B** are non-empty bounded above subsets of $\mathbb{R}$ and $C=\{x+y \mid x\in A, y\in B\}$ then show that:

$$\text{Sup}(C) = \text{Sup}(A) + \text{Sup}(B)$$ **(6.5)**

) Define neighborhood of a point, an open set and a closed set. Give an example of each of the following:

(i) A non-empty set which is a neighborhood of each of its points with the exception of one point.

P. T. O.

(ii) A non-empty open set which is not an interval.

(iii) A non-empty closed set which is not an interval.

(iv) A non-empty set which is neither an open set nor a closed set.

(6.5)

(c) Define limit point of a set. Find the limit points of **Z**, the set of integers and **Q**, the set of rational numbers.

(6.5)

2.(a) Prove that the intersection of a finite number of open sets is an open set. Is the intersection of infinite family of open sets an open set? Justify.

(6.5)

(b) Show that every continuous function on a closed interval is bounded.

(6.5)

(c) Show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $]0, \infty[$.

(6.5)

3.(a) When is a sequence $< a_n >$ said to converge to a number a? Show that the sequence $a_n = n^{1/n}, \forall n$ converges to 1.

(6.5)

(b) Prove that every monotonically increasing and bounded above sequence converges.

(6.5)

(c) Show that $\lim_{n\to\infty}\left(\frac{n}{2^n}\right) = 0$ (6.5)

4.(a) Define a Cauchy sequence. Prove that a sequence of real numbers is convergent if and only if it is Cauchy. (6)

(b) Show that the sequence $< a_n >$ defined by

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, \qquad \forall n$$

does not converge. **(6)**

(c) Show that the sequence $< a_n >$ defined by

$$a_1 = \frac{3}{2}, \; a_{n+1} = 2 - \frac{1}{a_n}, \qquad \forall n \geq 1$$

is bounded and monotonic. Also find $\lim_{n\to\infty}(a_n)$. **(6)**

a) State and prove Cauchy's general principle for the convergence of an infinite series. **(6)**

b) Test for convergence the following series :

i. $\sum_{n=1}^{\infty}(\sqrt{n^4+1} - \sqrt{n^4-1})$

ii. $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ **(6)**

c) Test for convergence the series:

$$\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots,$$

for all positive values of x. **(6)**

a) Prove that an absolutely convergent series is convergent. Give an example to show that the converse is not always true. **(6)**

) Test the convergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n \cos n\alpha}{n\sqrt{n}}$, α being real

(ii) $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^{1/n}$ (6)

(c) Prove that every monotonic function f on $[a, b]$ is Riemann integrable on $[a, b]$. (6)

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **6635** **HC**

Unique Paper Code : 32351601

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Complex Analysis

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) All questions are compulsory.

(c) Question **NO.1** has been divided in **5** parts and each part is of **3** marks.

(d) Each question from **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each question.

(e) Please, read the instructions carefully.

1. (a) Consider the complex function $f(z) = z^2$: Find images of x-axis (real axis) and y-axis (imaginary axis).

(b) Show that the following functions are nowhere differentiable.

(i) $f(z) = z + \bar{z}$,

(ii) $f(z) = e^y \cos x + ie^y \sin x$

(c) Find all zeros of the function sin z

P.T.O.

(d) Using Cauchy Integral formula, evaluate, when $\gamma = C(0;2) \equiv \{z \in \mathbb{C} : |z| = 2\}$,

$$\int_{\gamma} \frac{z^3+5}{z-i} dz$$

(e) Does there exist a function $f : D \to \mathbb{C}$ such that $f\left(\frac{1}{n}\right) = \frac{n}{n+1}$ for all $n \in \mathbb{N}$, where D is unit disc in the complex plane $\mathbb{C}$.

2. (a) Prove that :

$$\lim_{z \to 0} \frac{\overline{z}^2}{z} = 0$$

Show that

$$\lim_{z \to \infty} \frac{z^2}{(2z-1)^2} = \frac{1}{4}$$

(b) Show that the given functions are continuous everywhere on $\mathbb{C}$.

(i) The conjugation map $f(z) = \overline{z}$.

(ii) The affine map f(z)az + b; where a, b $\in \mathbb{C}$ and $a \neq 0$.

(c) Show that if a function f is continuous throughout a closed and bounded region D, there exists a nonnegative real number M such that $|f(z)| \leq M$ for all points $z \in D$, where equality holds for at least one such z:

3. (a) Prove that f defined by

$$f(z)=\begin{cases}\dfrac{z^5}{|z|^4} & \text{if } z\neq 0\\ 0 & \text{if } z=0\end{cases}$$

satisfies the C-R equations at $z = 0$ but is not differentiable there.

(b) Suppose that f is an analytic function in a domain D (non-empty open connected set). Show that f will be a constant function if the following conditions are satisfied in D:

(i) $f'(z)=0 \quad \forall z\in D$;

(ii) $|f(z)|$ constant in D.

(c) Find numbers $z = x + iy$ such that $e^z = 1 + i$.

4. (a) Without evaluating the integral, show that

$$\left|\int_C \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}$$

where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the ,first quadrant.

(b) If C is positively oriented contour $|z|=1$ and $f(z) = e^z$, evaluate $\int_C \frac{e^z}{z^4}$.

P.T.O.

(c) Let z_1 and z_2 be two distinct complex numbers that lie interior to the simple closed contour C. Show that :

$$\int_C (z-z_1)^{-1}(z-z_2)^{-1}\,dz = 0$$

5. (a) Define an entire function. Show that if a function is entire and bounded in the complex plane, then it is constant throughout the plane.

(b) Give two Laurent series expansions in powers of z for the function $f(z) = \dfrac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid.

(c) Find the Maclaurin series expansion of the function sin z.

6. (a) Find the nature of the singularities of the following functions

(i) $f_1(z) = \dfrac{z - \sin z}{z^4}$ (ii) $f_3(z) = \dfrac{1-e^z}{1+e^z}$

(b) State Cauchy's Residue Theorem. Use it to evaluate the integral $\displaystyle\int_C \frac{5z-2}{z(z-1)}\,dz$ where C is the circle $|z| = 2$, described counterclockwise.

(c) Show that :

$$\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{5-4\cos\theta} = \frac{\pi}{6}$$

2700

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : 6636 **HC**

Unique Paper Code : 32351602

Name of the Course : **B.Sc.(Hons.)Mathematics**

Name of the Paper : Ring Theory and Linear Algebra-II

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each of the question.

1. (a) State and prove Division Algorithm for $F[x]$ where F is a field.

(b) Let p be a prime and $f(x) \in Z_p[x]$ be irreducible over Z_p such that $\deg f(x) = n$, prove that $\frac{Z_p[x]}{<f(x)>}$ is a field with p^n elements.

P.T.O.

(c) In $\mathbb{Z}[\sqrt{-5}]$, prove that $1+3\sqrt{-5}$ is irreducible but not prime.
6, 6, 6

2. (a) Prove that a polynomial of degree n over a field F has atmost n zeros, counting multiplicity. Give an example to show that the result does not hold if F is not a field.

(b) Let F be a field and. $p(x) \in F[x]$ Then prove that $\langle p(x) \rangle$ is a maximal ideal in F[x] if and only if p(x) be irreducible over F.

(c) Prove that the ring of Gaussian integers $\mathbb{Z}(i)$ is a Euclidean domain. 6.5, 6.5, 6.5

3. (a) Let $V = P_1(1+R)$ and for $p(x) \in V$, define,

$f_1, f_2 \in V$ by $f_1(p(x)) = \int_0^1 p(t)\,dt$ and

$f_2(p(x)) = \int_0^2 p(t)\,dt$

Prove that f_1, f_2 is a basis for V* and find a basis for V for which it is the dual basis.
6

(b) Prove that a linear operator T on a finite dimensional Vector space V is diagonalizable if and only if there exists an ordered basis β for V consisting of eigen vectors of T.

(c) Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$ Show that A is diagonalizable and find a 2×2 matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

6, 6, 6

4. (a) Let T be a linear operator on a finite dimensional vector space V and W be a T invariant subspace of V. Prove that the characteristic polynomial of T_W divides the characteristic polynomial of T.

(b) Let T be a linear operator on an dimensional vector space V such that V is a T cyclic subspace of itself. Prove that f(t), the characteristic polynomial of T and p(t) the minimal polynomial of T have the same degree.

(c) Let T be a linear operator $P_2(\mathbb{R})$on such that $T(g(x)) = g'(x) + 2g(x)$. Find minimal polynomial of T. Hence, also determine if T is diagonalizable.

6.5, 6.5, 6.5

5. (a) State and prove Cauchy Schwarz inequality in an inner product space V over F and use it to prove triangle inequality in V.

(b) Apply the Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthonormal basis β for span(S) and compute the Fourier coefficients of the given vector x relative to β.

$$V = \mathbb{R}^3, S = \{(1,1,1),(0,1,1),(0,0,1)\} \text{ and } x = (1,0,1)$$

(c) Let V be a finite dimensional inner product space and let T be a linear operator on V. Show that there exists a unique linear operator $T^*: V \to V$ such that

$$\langle T(x), Y \rangle = \langle x, T^* y \rangle \text{ for all } x, y \in V.$$

6.5, 6.5, 6.5

6. (a) For the set of data $\{(-3,9),(-2,6),(0,2),(1,1)\}$, use the least squares approximation to find the best fit with a linear function. Also compute the error E.

(b) State and prove Schur Theorem.

(c) Let V be an inner product space and let T be a linear operator on V. Then prove that T is an orthogonal projection if and only if T has an adjoint T* and $T^2 = T = T^*$.

6, 6, 6

2700

[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **6810-A** **HC**

Unique Paper Code : 42357618

Name of the Course : **B.Sc.(Mathematical Sciences)/B.Sc.(Programme) DSE-2B**

Name of the Paper : Numerical Methods

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Questions No. **six** are compulsory. Attempt any **two** parts from each question.

(c) Use of non-programmable scientific calculator is allowed.

1. (a) Define the order of convergence of an iterative method. Derive the order of covergence of the Newton Raphson method . 5

(b) Use Bisection method to determine the root of the equation:

$$X^2 - 3 = 0$$

on the interval (1,2) up to four iterations. 5

(c) Find a real root of the equation:

$$x^3 - 5x + 1 = 0$$

on the interval (0,1). Perform four iterations using Secant Method. 5

2. (a) Apply Newton Raphson's method to determine the root of the equation-

$$x^3 + x^2 - 3x - 3 = 0$$

on the interval (1,2). Perform three iterations. 6.5

(b) Perform two iterations of Newton's method to solve the non-linear system of equations

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

Take the initial approximations as 6.5

$x_0 = 1.5$, $y_0 = 0.5$.

(c) Solve the following system of linear equations using Guass Seidel method

$$5x + y + 2z = 10$$

$$-3x + 9y + 4z = -14$$

$$x + 2y - 7z = -33$$

Perform three iterations taking initial approximation $X^{(0)} = (0,0,0)$. 6.5

3. (a) Using Guass Jordan method find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}.$$

Hence solve the following system of equations :

$$3x - y + 2z = 1$$

$$x + y + 2z = 2$$

$$2x - 2y - z = 3$$

(6.5)

P.T.O.

(b) Solve the system of linear equations

$$3x + y + z = 2$$
$$x + 4y + 2z = -5$$
$$x + 2y + 5z = 2$$

using Guass Jacobi method performing three iterations . Take the initial approximation as

$$X^{(0)} = (0,0,0).$$ 6.5

(c) Using the cubic splines fit the following data :

Estimate the value of f ( ) ?

i	x_i	$f(x_i)$
1	3.0	2.5
2	4.5	1.0
3	7.0	2.5

6.5

4. (a) Use the Lagrange's interpolation to find a polynomial that passes

through the points (-2,4) , (0,2) and (2,8). Hence approximate the function at the point x = -1. (6.5)

(b) (i) Construct the divided difference table for the following data :

x :	0.0	0.1	0.2	0.3	0.4	0.5
f(x) :	-1	-1.27	-0.98	-0.63	-0.22	0.25

3

(ii) Show that : 3.5

$$\Delta = \frac{1}{2}\delta^2 + \sqrt{1 + \frac{1}{4}\delta^2}$$

(c) Construct the forward difference table for the following data :

x :	0.1	0.2	0.3	0.4	0.5
f(x) :	1.4	1.56	1.76	2.0	2.28

Hence obtain the Gregory Newton forward difference interpolating polynomial. Interpolate the function at $x = 0.25$. 6.5

5. (a) Evaluate $\int_1^2 \frac{dx}{1+x^2}$ using Simpson's 3/8 rule for n=9 correct to 5 decimal places.

6.5

P.T.O.

(b) Using Euler's method find the approximate value of y when x=0.3

$$\frac{dy}{dx} = x + y^2$$

$y(0)=1$ and $h=0.1$ 6.5

(c) Apply finite difference method to solve the given problem

$$\frac{d^2y}{dx^2} = y + x(x-4), \quad 0 \le x \le 4$$

$y(0)=0$, $y(4)=0$ with $h=1$

6.5

6. (a) Using the central difference formula for $f'(x)$ evaluate $f'(3)$ for the following data

X :	1	2	3	4	5
f(x):	3	5	9	17	33

Further apply Richardson Extrapolation for h = 1 , 2 and evaluate the value the value of

$f'(3)$.Compare the two values.

6.5

(b) Apply mid-point theorem (R.K. Second order) to solve the initial value problem

$$\frac{dy}{dx} = yx^3 - 1.5y$$

from x=0 to 2 where y(0)=1 by using h=1 6.5

(c) Estimate $\int_{0.1}^{1.3} 5xe^{-2x}dx$ by 3-point Gauss quadrature rule. 6.5

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6810B **HC**

Unique Paper Code : 42357618

Name of the Paper : Numerical Methods

Name of the Course : **B.Sc. (Mathematical Sciences) / B.Sc. (Prog.) : DSE-3B**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** question are compulsory
3. Attempt any **two** parts from each question.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Use bisection method to determine the real root of the equation

$$x^3 - 5x + 1 = 0$$

lying in the interval (0,1). Perform three iterations. (6.5)

(b) Perform three iterations of secant method to find the smallest positive root of the equation

$$x^3 + x^2 - 3x - 3 = 0.$$ (6.5)

(c) Differentiate between iterative methods and direc methods. Give the geometric interpretation of Newto Raphson iterative method to find a root of an equatior (6.5

2. (a) Apply Newton Raphson method to find $\sqrt{18}$. Obtai the result correct upto three decimal places. (6.5

(b) Solve the system of linear equations using Guas elimination method by applying partial pivoting :

$$10x - y + 2z = 4$$

$$x + 10y - z = 3$$

$$2x + 3y + 20z = 7$$ (6.5)

(c) Solve the system of linear equations using Guass Jacobi iteration scheme by taking the initial approximation $X^{(0)} = (0,0,0)$

$$4x + y + z = 2$$

$$x + 5y + 2z = -6$$

$$x + 2y + 5z = -4$$

Perform three iterations.

3. (a) For what values of a does the Guass Seidel method converge for the following system of linear equations?

$$\begin{bmatrix} 1 & -a \\ a & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

a being a real number. (6)

(b) Find a polynomial of degree 2 or less using Lagranges interpolation for the following data f(0) = 1, f(1) = 3, f(3) = 55. Is this polynomial unique? (6)

(c) Calculate the nth divided difference for the function f(x) = 1/x based on the points $x_0, x_1, x_2, \ldots\ldots\ldots\ldots, x_n$. (6)

4. (a) Construct the forward difference table for the following data :

x :	5	10	15	20	25	30
f(x) :	9962	9848	9659	9397	9063	8660

Write down the values of $\Delta^2 f(10)$, $\Delta^3 f(5)$. (6.5)

(b) Construct the backward difference table for the following data :

x :	1	2	3	4	5	6
f(x) :	1	8	27	64	125	216

Hence find the Gregory Newton backward difference interpolating polynomial. (6.5)

(c) Find the Newton's divided difference polynomial for the following data.

x :	−2	−1	1	3
f(x) :	−15	−4	0	20

Hence approximate f(0). (6.5)

5. (a) Approximate the integral $\int_4^5 \log x dx$ in steps of $h = 0.2$ by using

(i) Trapezoidal rule

(ii) Simpsons Rule (6)

(b) Using Euler's method to approximate the values of y(0.1), y(0.2), y(0.3) for the initial value problem

$\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. (6)

(c) Solve the differential equation $\frac{dy}{dx} = 1 + xy$ with $y(0) = 1$.

Approximate y(0.2) using Range Kutta method of 4th order. (6)

6. (a) Approximate the integral $I = \int_0^1 \frac{dx}{1+x}$ using 3-point Guassian Quadrature rule. (6)

(b) Approximate f′(3) for the following data

x :	1	2	3	4	5
f(x) :	2	4	8	16	32

using (i) 3-point backward difference formula

(ii) 3-point forward difference formula. (6)

(c) Use the finite difference method to solve the boundary value problem

$$\frac{d^2y}{dx^2} = y + x(x-4), \quad 0 \le x \le 4$$

with $y(0) = 0, \; y(4) = 0.$

This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8743** **HC**

Unique Paper Code : 42353604

Name of the Course : **B.Sc. Programme: Mathematics : SEC**

Name of the Paper : Transportation and Network Flow Problems

Semester : VI

Time : 3 Hours ***Maximum Marks : 55***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) This question paper has **FOUR** questions in all.

(c) **All** questions are compulsory.

1. MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major Distribution centers in Denver and Miami. The capacities of the three plants during the next quarter are 1000, 1500, and 1200 cars. The quarterly demands at the two distribution centers are 2300 and 1400 cars. The transportation cost per car on the different routes rounded to the closest dollar are given in the Table

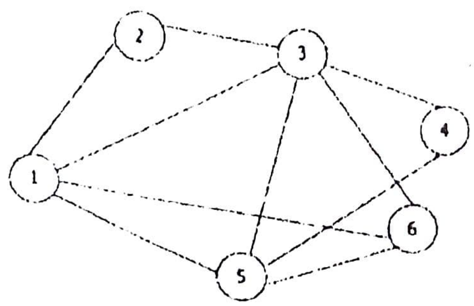
Table : Transportation Cost per Car

	Denver(1)	Miami(2)
Los Angeles (1)	\$80	\$215
Detroit(2)	\$100	\$108
New Orleans (3)	\$102	\$68

Formulate the Transportation Model. 5

2. Attempt any **FIVE** parts from the following

(i) For the network given below, determine
(a) a path (b) a cycle (c) a tree
(d) a spanning tree
(e) the sets *N* and *A* 6



(ii) Compare the initial basic feasible solutions obtained by the Northwest-Corner method AND Least-Cost method for the following transportation problem 3+3=6

		Destination			Supply
Source	1	2	3	4	**30**
	3	3	2	1	**50**
	4	2	5	9	**20**
Demand	**20**	**40**	**30**	**10**	

(iii) Five different machines can do any of the five required jobs and the associated cost matrix is as follows. Find out minimum cost possible through optimal assignment of machine to jobs. 6

		Machine				
		1	2	3	4	5
	A	11	17	8	16	20
	B	9	7	12	6	15
Jobs	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

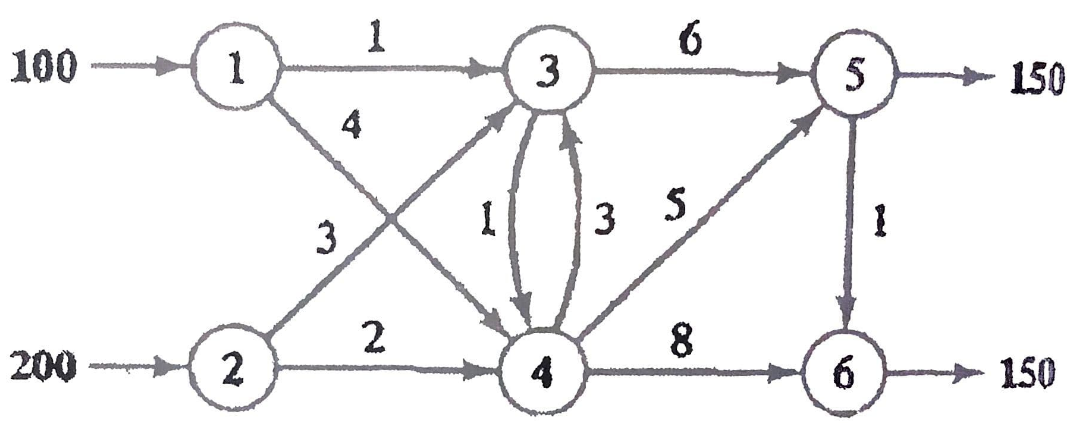
P.T.O.

(iv) The network in the following figure gives the shipping routes from nodes 1 and 2 to nodes 5 and 6 by way of nodes 3 and 4. The unit shipping costs are shown on the respective arcs.

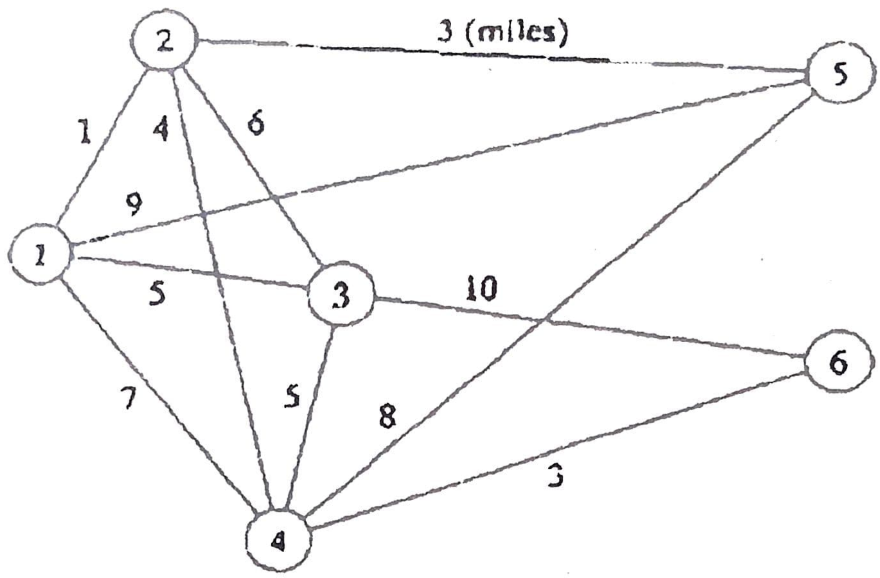
(a) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(b) Only develop the corresponding transshipment model table.

2+4=6



(v) Midwest TV Cable Company is in the process of providing cable services to five new housing development areas. The adjoining figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network **starting at node 5.** 6



(vi) A publisher has a contract with an author to publish a textbook. The activities associated with the production of the textbook are given below. The author is required to submit to the publisher a hard copy and a computer file of the manuscript.

2+1+3=6

Activity	Predecessors	Duration (Week)
A: Manuscript proofreading by editor	---	3
B: Sample pages preparation	---	2

P.T.O.

C: Book cover design	---	4
D: Artwork preparation	---	3
E: Author's approval of edited manuscript and sample pages	A,B	2
F: Book formatting	E	4
G: Author's review of formatted pages	F	2
H: Author's review of artwork	D	1
I: Production of printing plates	G,H	2
J: Book production and binding	C,I	4

(a) Develop the associated network for the project.

(b) Find the minimum time of completion of the project.

(c) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table

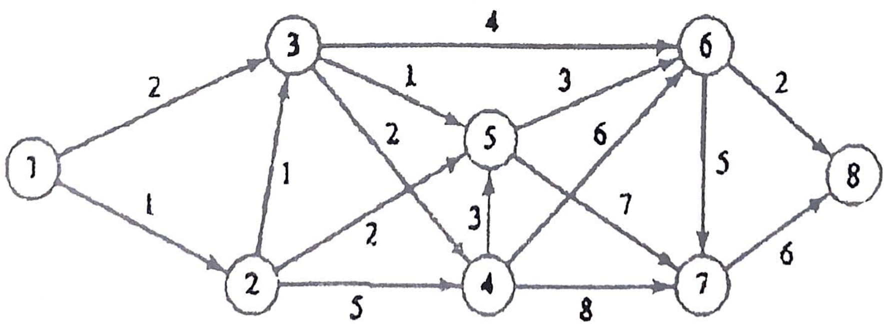
(a) Use the Vogel Approximation Method (VAM) to find a starting solution.

(b) Use this starting solution to find the optimal solution by the method of multipliers. 5+5=10

	X	Y	Z	**Supply**
A	1	2	6	**7**
B	0	4	2	**12**
C	3	1	5	**11**
Demand	**10**	**10**	**10**	

4. Attempt any **ONE** from the following

(i) The network in following figure gives the distances in miles between pairs of cities. Use Dijkstra's algorithm to find the shortest route between
(a) cities 1 and 8
(b) cities 4 and 7 7+3=10



P.T.O.

(ii) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc (2, 4) is directional, so that no traffic is allowed from node 4 to node 2. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between

(a) node 5 to node 2

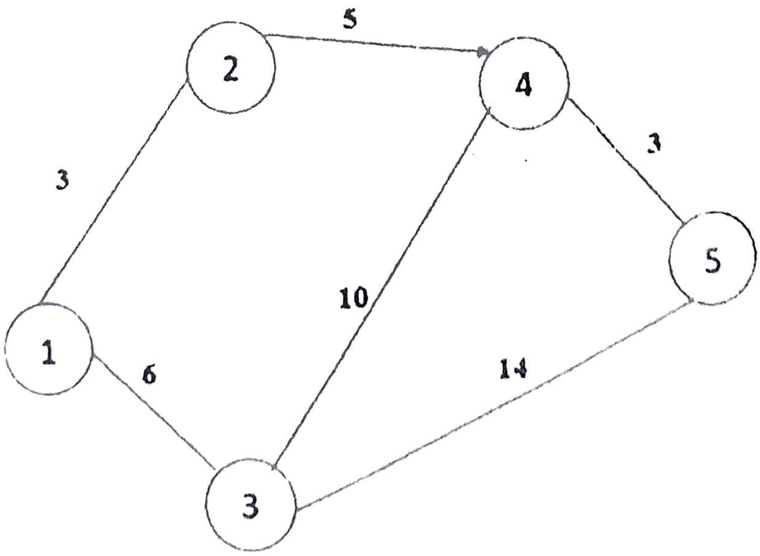
(b) node 1 to node 4

(c) node 2 to node 3

(d) node 3 to node 5

(e) node 1 to node 5

5×2=10



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Your Roll No. : ....

Sl. No. of Q. Paper : **9488** **HC**

Unique Paper Code : 32357607

Name of the Course : **B.Sc.(Hons.)Mathematics : DSE-3**

Name of the Paper : Probability Theory & Statistics

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) In all there are **six** questions.

(c) Question No.**1** is compulsory, and it contains **seven** parts of **3** marks each, out of which any **five** parts are be attemped .

(d) In Question No.**2** to **6**, attempt any **two** parts . Each part carries **6** marks.

(e) Use of scientific calculator is allowed.

P.T.O.

1. (i) The distribution function of a rando[m] variable is

$$F(x)=\begin{cases}0, & x<-2\\ \dfrac{x+4}{8} & \text{if } -2\le x<2\\ x^2 & \text{if } x\ge 2.\end{cases}$$

Obtain $P(X=-2), P(X=2)$ and $P(0<X\le 2)$.

(ii) Find the moment generating function (m[gf]) of a random variable X having t[he] probability density function (pdf):

$$f(x)=\frac{1}{2}e^{-|x|}, \quad -\infty<x<\infty.$$

(iii) A coin is tossed until a head appears. Wh[at] is the expectation of the number of tos[ses] required ?

(iv) If X is a random variable having pmf

$$p(x)=\begin{cases}1/8, & x=-1\\ 6/8, & x=0\\ 1/8, & x=1.\end{cases}$$

Use Chebyshev's inequality to obtain upper bound of $P[|X|\ge 1]$.

(v) If the joint distribution of X and Y is given by

$$F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}), & x>0, y>0\\ 0, & elsewhere,\end{cases}$$

Find $P(1<X\leq 3, 1<Y\leq 3)$.

(vi) If the probability is 0.75 that an applicant for driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try ?

(vii) If the joint mgf of two random variables X_1 and X_2 is given by

$$M(t_1,t_2)=e^{\mu_1 t_1+\mu_2 t_2+\frac{1}{2}(t_1^2\sigma_1^2+t_2^2\sigma_2^2)+\rho t_1 t_2\sigma_1\sigma_2}$$

Where $\mu_1,\mu_2,\sigma_1,\sigma_2,\rho$ are constants, then show that $Cov(X,Y)=\rho\sigma_1\sigma_2$.

(a) (i) Prove that for any random variable,

$$P(X=x)=F_X(x)-F_X(x-)$$

for all $x\in R$, where $F_x(x-)=\lim_{z\uparrow x}F_x(z)$

(ii) There are 5 red chips and 3 blue chips in a bowl. The red chips are numbered 1,2,3,4,5 respectively and the blue chips are numbered 1,2,3 respectively. If 2 chips are to be drawn at random and without replacement, find the probability that these chips will have either the same number or the same colour.

(b) Let X have the pdf

$$f(x) = \frac{x^2}{18}, -3 < x < 3,$$

= zero elsewhere.

(i) Find the cdf of X

(ii) Find $P(|X| < 1)$ and $(X^2 < 9)$

(c) (i) Find the characterstic function of Poisso distribution with parameter λ.

(ii) Find the median of the distribution wi probality mass function

$$p(x) = \begin{cases} \frac{\lfloor 4}{\lfloor x \lfloor 4-x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x} & , x = 0,1,2,3,4 \\ 0 & \text{elsewhere} \end{cases}$$

3. (a) Let X have the uniform continuo distribution with parameters α and β.

Find the mgf, the mean, and the varian of this distribution.

(b) Let a random variable X have the Poisson distribution with parameter λ. Then show that as $\lambda \to \infty$, the moment generating function of $Z = \frac{X - \lambda}{\sqrt{\lambda}}$ approaches the moment generating function of standard normal distribution.

(c) Prove that normal distribution is a symmetrical distribution.

4. (a) Let (X_1, X_2) have the joint pmf as given below :

(x_1, x_2)	(0,0)	(0,1)	(0,2)	(1,1)	(1,2)	(2,2)
$p(x_1, x_2)$	1/12	2/12	1/12	3/12	4/12	1/12

Find $E(X_1), E(X_2), Var(X_1)$ and $Var(X_2)$.
Is $E(X_1X_2) = E(X_1)E(X_2)$?

(b) Let (X_1, X_2) have the joint cdf $F(x_1, x_2)$ and let X_1 and X_2 have the marginal cdfs $F_1(x_1)$ and $F_2(x_2)$ respectively. Prove that X_1 and X_2 are independent if and only if

$$F(x_1, x_2) = F_1(x_1)F_2(x_2)$$

for all $(x_1, x_2) \in R^2$.

P.T.O.

(c) Given the two random variables X and Y that have the joint density

$$f(x,y)=\begin{cases}24xy, & x>0, y>0, x+y<1\\ 0, & \text{elsewhere.}\end{cases}$$

Find the regression equation of X on Y. Also compute P $(0 < X < 1/2 \mid Y = 5/8)$.

5. (a) Let X and Y have the joint pdf

$$f(x,y)=\begin{cases}e^{-y}, & 0<x<y<\infty\\ 0, & \text{elsewhere.}\end{cases}$$

Find the joint mgf of X and Y and hence compute the correlation coefficient .

(b) Let X and Y have the joint pdf

$$f(x,y)=\begin{cases}3x, & 0<y<x<1\\ 0, & \text{elsewhere.}\end{cases}$$

Find the conditional mean and variance of X given Y = y, $0 < y < 1$.

(c) Define bivariate normal distribution for a random vector (X,Y) and find marginal density function of Y.

6. (a) Let $\{X_i\}$ be a sequence of independent, identically distributed random variables, each with mean μ and variance σ^2. Then show that the mgf of $\frac{X_1 + ... + X_n}{\sqrt{n}}$ is $\left(1+\frac{t^2}{2n}\right)^n$, when n is large. Hence prove the central limit theorem.

(b) (i) The lifetime of a special type of battery is a random variable with mean 40 hours and standard deviation 20 hours. A battery is used until it fails at which point it is replaced by a new one. Assuming a stockpile of 20 such batteries, the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained.

$(\text{Use } \Phi(1) = 0.8413)$

(ii) Let the Markov chain consisting of the states 0,1,2,3 have the transition probability matrix

$$\begin{Vmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Vmatrix}$$

Determine which states are transient and which are recurrent.

(c) If the probability density of X is given by

$f(x) = 630\, x^4 (1-x)^4$ for $0 < x < 1$

$= 0$ elsewhere

find the probability that it will take on a value within two standard deviations of the mean and compare this probability with the lower bound provided by Chebyshev's theorem.

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **9488-A** **HC**

Unique Paper Code : 32357610

Name of the Course : **B.Sc.(Hons.) Mathematics : DSE-4**

Name of the Paper : Number Theory

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions selecting **eight** parts from section **I**, **five** parts each section **II** and **III**.

Section - I

(Attempt any **eight** parts) $8\times2\frac{1}{2}$

(a) Show that for any integer a, $a^3\equiv 0, 1$ or $6 \pmod 7$.

(b) Use Gauss Lemma to compute Legendre symbol $\left(\frac{11}{23}\right)$.

(c) Use Quadratic Reciprocity law to compute Legendre symbol $\left(\frac{1234}{4567}\right)$.

P.T.O.

(d) Use Fermat theorem to test if 17 divides $13^{98}-1$.

(e) Find the order of the integer 3 modulo 19.

(f) Prove that if the integer n has r distinct odd prime factors, then $2^r \mid \varphi(n)$.

(g) For the Exponential cryptosystem (Pohling - Hellman system), with the public key (41,7), compute the deciphering exponent.

(h) Encrypt the message I LOVE DOING NUMBER THEORY using the linear cipher $C \equiv 2P+4 \pmod{26}$.

(i) Find the number of primitive root of 2029.

(j) Write a short note on Gauss and his work.

(k) Define a multiplicative function. Compute $\tau(1245)$.

(l) Prove that $\varphi(2^n-1)$ is a multiple of n for any n>1.

Section - II

(Attempt any **five** parts) $5\times5\frac{1}{2}$

1. Using Euclidean algorithm and theory of linear Diophantine equation, divide 100 into two summands such that one is divisible by 7 and other by 11.

2. State and prove Wilson's theorem.

3. Prove that the congruences x ≡ a (mod n) and x ≡ 3 (mod m) admit a simultaneous solution if and only if gcd (n,m) | (a-b); if a solution exists, confirm that it is unique modulo *lcm* (n,m)

4. Solve x ≡ 5(mod 6), x ≡ 4(mod 11), x ≡ 3(mod 17).

5. Prove the threre are an infinite number of primes of the form 4n + 3.

6. State Fermat's theorem and show, by means of an example, that the converse is not true,

7. Define a complete set of residues modulo p. Prove that if gcd (a, n) =1, then the integers c, c + *a*, c + 2*a*, c + 3*a*, ..., c +(*n*-1) a form a complete set of residues modulo *n* for any c .

Section - III

(Attempt any **five** questions) $5\times5\frac{1}{2}$

8. State and prove Quadratic Reciprocity Law.

9. Encrypt the plaintext message GOLD using RSA algorithm with key (2561,7).

10. Let *p* if be an odd prime and gcd (a,p)=1, then show that *a* is a quadratic residue of *p* if and only if $a^{\frac{p-1}{2}} \equiv 1$ (mod *p*).

11. Solve $3x^2 + 9x + 7 \equiv 0 \pmod{13}$.

12. If the integer a has the order k modulo n and $h>0$, then show that a^h has order $\frac{k}{\gcd(h,k)}$ modulo n.

13. If p is an odd prime and $gcd(a, p)=1$, then show that the congruence $x^2 \equiv a \pmod{p^n}$, where $n \geq 1$ has a solution if and only if $\left(\frac{a}{p}\right)=1$.

14. Let r be a primitive root of the odd prime p. Prove that if $p \equiv 3 \pmod 4$, then $-r$ has order $\frac{p-1}{2}$ modulo p.

15. Prove that if p is prime number and $d \mid p-1$, then congruence $x^d - 1 \equiv 0 \pmod p$ has exactly d solutions.

16. If $p \neq 3$ is an odd prime, then show that

$$\left(\frac{3}{p}\right) = \begin{cases} 1, & \text{if } p \equiv \pm 1 \pmod{12} \\ -1, & \text{if } p \equiv \pm 5 \pmod{12} \end{cases}$$

[This question paper contains 4 printed pages]

Your Roll No.	: ....
Sl. No. of Q. Paper	: **9488-A** **HC**
Unique Paper Code	: 32357610
Name of the Course	: **B.Sc.(Hons.) Mathematics : DSE-4**
Name of the Paper	: Number Theory
Semester	: VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions selecting **eight** parts from section **I**, **five** parts each section **II** and **III**.

Section - I

(Attempt any **eight** parts) $8\times 2\frac{1}{2}$

(a) Show that for any integer a, $a^3 \equiv 0, 1$ or $6 \pmod 7$.

(b) Use Gauss Lemma to compute Legendre symbol $\left(\frac{11}{23}\right)$.

(c) Use Quadratic Reciprocity law to compute Legendre symbol $\left(\frac{1234}{4567}\right)$.

P.T.O.

(d) Use Fermat theorem to test if 17 divides $13^{98}-1$.

(e) Find the order of the integer 3 modulo 19.

(f) Prove that if the integer n has r distinct odd prime factors, then $2^r \mid \varphi(n)$.

(g) For the Exponential cryptosystem (Pohling-Hellman system), with the public key (41,7), compute the deciphering exponent.

(h) Encrypt the message I LOVE DOING NUMBER THEORY using the linear cipher $C \equiv 2P+4 \pmod{26}$.

(i) Find the number of primitive root of 2029.

(j) Write a short note on Gauss and his work.

(k) Define a multiplicative function. Compute $\tau(1245)$.

(l) Prove that $\varphi(2^n-1)$ is a multiple of n for any $n>1$.

Section - II

(Attempt any **five** parts) $5\times5\frac{1}{2}$

1. Using Euclidean algorithm and theory of linear Diophantine equation, divide 100 into two summands such that one is divisible by 7 and other by 11.

2. State and prove Wilson's theorem.

3. Prove that the congruences $x \equiv a \pmod{n}$ and $x \equiv 3 \pmod{m}$ admit a simultaneous solution if and only if gcd (n,m) | (a-b); if a solution exists, confirm that it is unique modulo *lcm* (n,m)

4. Solve $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.

5. Prove the threre are an infinite number of primes of the form 4n + 3.

6. State Fermat's theorem and show, by means of an example, that the converse is not true,

7. Define a complete set of residues modulo p. Prove that if gcd (a, n) =1, then the integers c, $c + a$, $c + 2a$, $c + 3a$, ..., $c + (n-1)a$ form a complete set of residues modulo n for any c.

Section - III

(Attempt any **five** questions) $5 \times 5\frac{1}{2}$

8. State and prove Quadratic Reciprocity Law.

9. Encrypt the plaintext message GOLD using RSA algorithm with key (2561,7).

10. Let p if be an odd prime and gcd (a,p)=1, then show that a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.

11. Solve $3x^2 + 9x + 7 \equiv 0 \pmod{13}$.

12. If the integer a has the order k modulo n and $h>0$, then show that a^h has order $\frac{k}{\gcd(h,k)}$ modulo n.

13. If p is an odd prime and $\gcd(a, p)=1$, then show that the congruence $x^2 \equiv a \pmod{p^n}$, where $n \geq 1$ has a solution if and only if $\left(\frac{a}{p}\right)=1$.

14. Let r be a primitive root of the odd prime p. Prove that if $p \equiv 3 \pmod 4$, then $-r$ has order $\frac{p-1}{2}$ modulo p.

15. Prove that if p is prime number and $d \mid p-1$, then congruence $x^d - 1 \equiv 0 \pmod p$ has exactly d solutions.

16. If $p \neq 3$ is an odd prime, then show that

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1500

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **9489A**

Unique Paper Code : **32357611** **HC**

Name of the Paper : **Linear Programming and Theory of Games**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-4**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions carry equal marks.

1. (a) Consider the following problem :

Minimize $z = cx$

Subject to

$$Ax = b$$

$$x \geq 0$$

P.T.O.

where A is an $m \times n$ matrix with rank m. Let x be the extreme point of the feasible region. Show that x is also a basic feasible solution of the system :

$$Ax = b, \ x \geq 0.$$

(b) Solve the following problem by simplex method :

Maximize $z = x_1 - 2x_2 + x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 12$$

$$2x_1 + x_2 - x_3 \leq 6$$

$$-x_1 + 3x_2 \leq 9$$

$$x_1, x_2, x_3 \geq 0.$$

(c) Use simplex method to obtain the inverse of the following matrix :

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 9 \end{bmatrix}.$$

(a) Solve the following problem by two-phase method :

Maximize $z = 5x_1 - 2x_2 + x_3$

Subject to :

$$2x_1 + 4x_2 + x_3 \leq 6$$

$$2x_1 + x_2 + 3x_3 \geq 2$$

$$x_1, x_2 \geq 0$$

x_3 is unrestricted.

(b) Solve the following problem by the big-M method :

Minimize $z = 3x_1 + 2x_2 + 4x_3 + 8x_4$

Subject to

$$x_1 + 2x_2 + 5x_3 + 6x_4 \geq 8$$

$$-2x_1 + 5x_2 + 3x_3 - 5x_4 \leq 3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

(c) Find all the basic feasible solution of the equations

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2.$$

3. (a) Show that the objective function value for any feasibl solution to the minimization problem is always greate than or equal to the objective function value for ar feasible solution to the maximization problem.

(b) Write the dual of the primal problem given below

Min $z = x_1 + x_2 + x_3$

Subject to

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$x_1, x_2 \geq 0$ x_3 is unrestricted.

(c) Use the graphical method to solve the dual of the following problem :

$$\text{Max} \quad z = x_1 + 2x_2 + 6x_3$$

Subject to

$$3x_1 + x_2 + 2x_3 \leq 6$$

$$2x_1 + 3x_2 + x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0.$$

Further, use the complementary slackness theorem to find an optimal solution to the given problem from optimal solution of the dual problem.

4. (a) Solve the following transportation problem :

	D_1	D_2	D_3	D_4	Availability
S_1	2	3	11	7	6
S_2	1	0	6	1	1
S_3	5	8	15	9	10
Requirement	7	5	3	2	

(*b*) Solve the following assignment problem :

	I	II	III	IV	V
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

(*c*) (*i*) Find minimax and maximin for the following game :

$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}.$$

(*ii*) Define for theory of Games :

(I) Rectangular Game

(II) Pay off matrix

(III) Saddle Point.

(*a*) Solve the following game graphically :

Player B

$$\text{Player A} \begin{bmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{bmatrix}$$

(*b*) Solve by principle of dominance, the following game :

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \\ 1 & 3 & 2 \end{bmatrix} \end{array}$$

(*c*) Give the linear programming formulation for player A and player B for the following game :

Player B

$$\text{Player A} \begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix}.$$

This question paper contains 5 printed pages.

Your Roll No. ....

S. No. of Paper	: **9490**	**HC**
Unique Paper Code	: **32357609**	
Name of the Paper	: **Bio-Mathematics**	
Name of the Course	: **B.Sc. (Hons.) Mathematics : DSE-3**	
Semester	: **VI**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory. Attempt any two parts from each question. All questions carry equal marks. Use of Scientific Calculator is allowed.

1. (a) Discuss the model describing drug concentration and residual concentration at any time nt_0, in which drug decays according to equation $\frac{dc}{dt} = \frac{-c(t)}{\tau}$. When dose is administered regularly at time $t=0, t_0, 2t_0, 3t_0, \ldots$ with assumption that each dose raises the drug concentration by fixed amount c_0. Find the maximum possible concentration and residue as n increases. 6

   (b) Let X and Y represent the population of predator and prey respectively. In absence of predation, X decreases exponentially and Y follows a logistic growth. The rate at which prey is eaten is proportional to the product of densities of X and Y. Determine the rest states in this Volterra-Lotka

Model of predator prey interaction. Also, find the appropriate relation between the constants of proportionality involved. 6

(c) State Law of Mass Action and derive the mathematical model governed by intermediaries X and Y in trimolecular reaction:

$$A \rightarrow X$$
$$B + X \rightarrow Y + D$$
$$2X + Y \rightarrow 3X$$
$$X \rightarrow E$$

where A, B, D, E are initial and final products and all rate constants are equal to 1. 6

2. (a) Discuss the nature of critical point and give the equations of trajectory for the given system

$$\dot{x} = 4x + 5y$$

$$\dot{y} = -5x - 4y$$ $6\frac{1}{2}$

(b) State Existence Theorem-I and show that the conditions of Existence Theorem-I are sufficient but not necessary by an example. $6\frac{1}{2}$

(c) Discuss the trajectories of $\ddot{x} - 4\dot{x} + 40x = 0$ in the phase plane by substituting:

$$x = \rho \cos \Phi, \ y = \rho \sin \Phi$$ $6\frac{1}{2}$

3. (a) Sketch and verify the phase portrait for the local linearized model of the heartbeat given by:

$$\epsilon \frac{dx}{dt} = -a(x - x_0) - (b - b_0)$$

$$\frac{db}{dt} = (x - x_0)$$

where x is the muscle fibre length, b is the electrical control variable and t is the time, (x_0, b_0) is the rest state. $6\frac{1}{2}$

(b) Provide a full phase plane analysis for the heart beat equations

$$\epsilon \frac{dx}{dt} = -(x^3 - Tx + b)$$

$$\frac{db}{dt} = (x - x_0) + (x_0 - x_1)u$$

by appropriately defining u, the control variable associated with the pacemaker, x is the muscle fibre length, b is the electrical control variable and t is the time. (x_0, b_0) is the rest state and x_1 corresponds to the systolic state. $6\frac{1}{2}$

(c) Find the constraints on a, b and γ assuming it has a unique rest state, taking the solutions to the travelling wave equations

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(1 - u)(u - a) - \omega$$

$$\frac{\partial \omega}{\partial t} = bu - \gamma\omega$$

in the form $u = \emptyset(x + ct)$, $\omega = \psi(x + ct)$. $6\frac{1}{2}$

(a) Show that

$$\dot{x} = x(y - 1)$$

$$\dot{y} = \mu - y(x + 1)$$

has an equilibrium point, which is a stable node for $\mu < 1$, that becomes a saddle point as μ passes through the bifurcation point $\mu = 1$. Also show that there is an additional equilibrium point when $\mu > 1$, which is a stable node. 6

(b) Define:

(i) Bifurcation
(ii) Bifurcation point
(iii) Bifurcation diagram.

Make the sketches for Pitchfork bifurcation, Saddle-node bifurcation and Hopf bifurcation. 6

(c) For the iteration scheme

$$x_{n+1} = \mu x_n(1 - x_n), \qquad n \geq 1$$
$$x_0 = \lim_{n\to\infty} x_n$$

show that there are bifurcations at $\mu = 1$ and $\mu = 3$. 6

5. (a) Explain Jukes-Cantor model (JC model). Estimate the proportion of the sites that will have a base A in the ancestral sequence and base T in the descendant sequence after one time step. $6\frac{1}{2}$

(b) From the given distance table, construct a rooted tree showing the relationship between S1, S2, S3 and S4 by UPGMA

	S 1	S 2	S 3	S 4
S 1		.45	.27	.53
S 2			.40	.50
S 3				.62

$6\frac{1}{2}$

(c) In mice, an allele A for agouti- or gray-brown, grizzled fur is dominant over the allele a, which determines a non-agouti color. If an Aa×Aa cross produces 6 offsprings, then:

(i) What is the probability that exactly 4 of 6 offspring have agouti fur?

(ii) What is the probability that more than half of 6 offspring have agouti fur? $6\frac{1}{2}$

(a) Suppose S_0 and S_1 are the ancestral and descendent 40-base sequence. From the given frequency table, find all the 16 conditional probabilities $P(S_1=i \mid S_0=j)$, where $i, j =$ A, G, C, T rounding off to 3 decimal places and form a table of the same.

$S_1 \backslash S_0$	A	G	C	T
A	7	0	1	1
G	1	9	2	0
C	0	2	7	2
T	1	0	1	6

Also write a short note on basic model of molecular evolution. 6

(b) Define phylogenetic tree, bifurcating tree and unrooted tree and give an example of each. 6

(c) Form a Punnett square for Dd×Dd and DdWw×ddWw. 6

This question paper contains 5 printed pages.

Your Roll No. ....

No. of Paper	: 9490	HC
ique Paper Code	: 32357609	
ame of the Paper	: **Bio-Mathematics**	
ame of the Course	: **B.Sc. (Hons.) Mathematics : DSE-3**	
emester	: **VI**	
uration	: **3 hours**	
aximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

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(a) Discuss the model describing drug concentration and residual concentration at any time nt_0, in which drug decays according to equation $\frac{dc}{dt} = \frac{-c(t)}{\tau}$. When dose is administered regularly at time $t=0, t_0, 2t_0, 3t_0, \ldots$ with assumption that each dose raises the drug concentration by fixed amount c_0. Find the maximum possible concentration and residue as n increases. 6

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P. T. O.

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(a) Suppose S_0 and S_1 are the ancestral and descendent 40-base sequence. From the given frequency table, find all the 16 conditional probabilities $P(S_1=i \mid S_0=j)$, where $i, j=$ A, G, C, T rounding off to 3 decimal places and form a table of the same.

$S_1 \backslash S_0$	A	G	C	T
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Also write a short note on basic model of molecular evolution.

6

(b) Define phylogenetic tree, bifurcating tree and unrooted tree and give an example of each. 6

(c) Form a Punnett square for Dd×Dd and DdWw×ddWw.

6

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **9562** **HC**

Unique Paper Code : 62353606

Name of the Course : **B.A. (Programme) : Mathematics- SEC**

Name of the Paper : Transportation and Network Flow Problems

Semester : VI

Time : 3 Hours ***Maximum Marks : 55***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) This question paper has **FOUR** questions in all.

(c) **All** questions are compulsory.

1. Three electric power plants with capacities of 20, 35 and 20 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 25, 30 and 20 million kWh. The price per million kWh at the three cities is given in Table

		City 1	City 2	City 3
Plant	1	\$500	\$600	\$300
	2	\$220	\$200	\$250
	3	\$400	\$380	\$350

The utility company wishes to determine the most economical plan for the distribution. Formulate the model as a transportation model. 5

2. Attempt any **FIVE** parts from the following

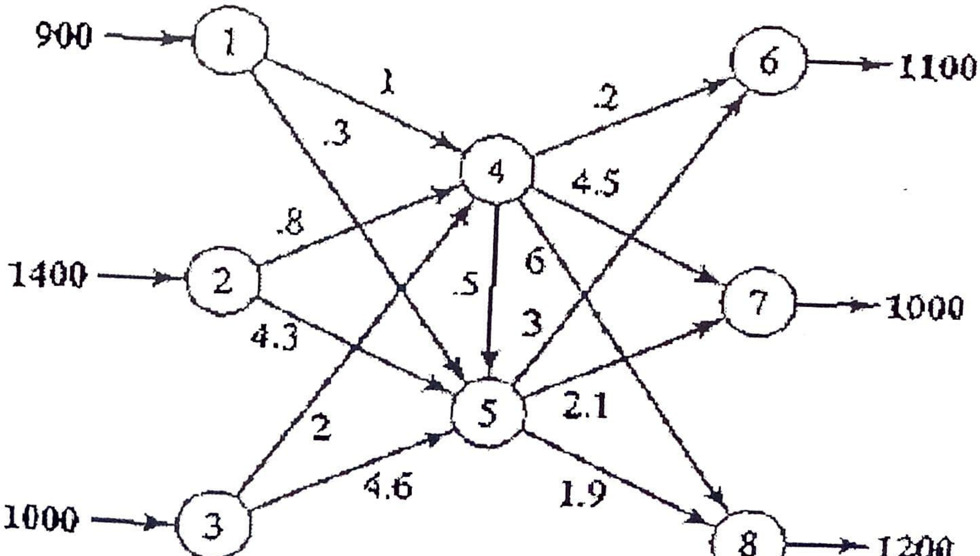
(i) Compare the initial basic feasible solutions obtained by the Northwest-Corner method **AND** Least-Cost method for the following transportation problem 3+3=6

	Destination				**Supply**
Source	10	15	27	14	**200**
	15	10	44	11	**300**
	20	14	16	11	**400**
Demand	**200**	**225**	**275**	**200**	

(ii) Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table. Find the assignment of men to jobs that will minimize the total time taken. 6

		Men				
		A	B	C	D	E
	1	9	11	14	11	7
	2	6	15	13	13	10
Jobs	3	12	13	6	8	8
	4	11	9	10	12	9
	5	7	12	14	10	14

(iii) The network in figure shows the routes for shipping cars from three plants (nodes 1, 2 and 3) to three dealers (nodes 6 to 8) by way of two distribution centers (nodes 4 and 5). The shipping costs per car (in $100) all shown on the arcs.



P.T.O.

(a) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(b) Only develop the corresponding transshipment model table. 2+4=6

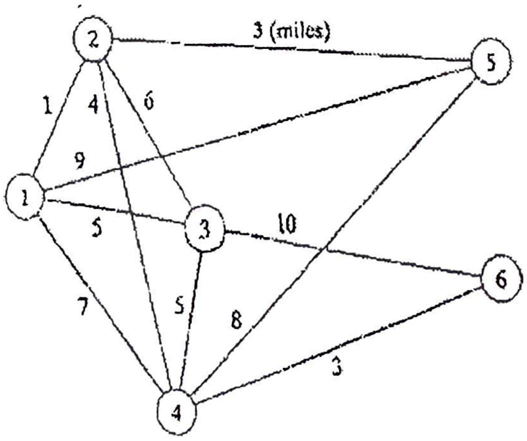
(iv) Draw the Network defined by

$$N = \{1,2,3,4,5,6\}$$

$$A = \{(1,2),(1,3),(1,6),(2,3),(2,5),(4,5),(4,6),(5,6)\}$$

Also determine (a) a path (b) a cycle (c) a tree (d) a spanning tree 6

(v) Midwest TV Cable Company is in the process of providing cable service to five new housing development areas. Figure 1 depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network **starting at node 2.** 6



(vi) The activities associated with a certain project are given below

2+1+3=6

Activity	Predecessors	Duration (Week)
A:	--	2
B:	--	5
C:	A,B	4
D:	A,B	7
E:	B	2
F:	C	3
G:	D	4
H:	F,G	6
I:	F,G	5
J:	E,H	3
K:	I,J	2

(a) Develop the associated network for the project.

(b) Find the minimum time of completion of the project.

(c) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table

		Destination			Supply
Source	\$10	\$4	\$2		8
	\$2	\$3	\$4		5
	\$1	\$2	\$0		6
Demand	7	6	6		

(a) Use the Vogel Approximation Method (VAM) to find a starting solution.

(b) Hence find the optimal solution by the method of the multipliers.

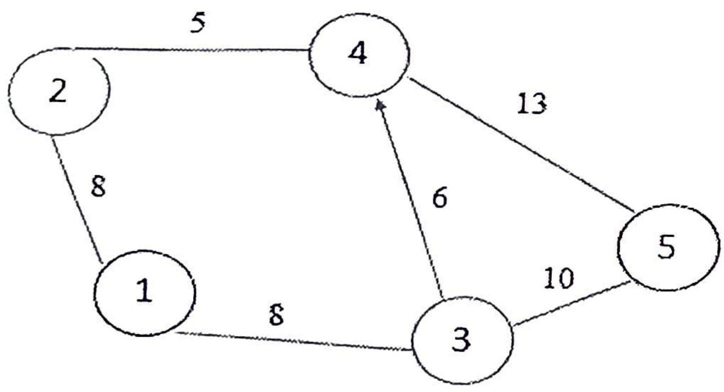
5+5=10

4. Attempt any **ONE** from the following

(i) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc (3, 4) is directional, so that no traffic is allowed from node 4 to node 3. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between

5×2=10

(a) node 5 to node 2
(b) node 1 to node 4
(c) node 2 to node 3
(d) node 3 to node 5
(e) node 1 to node 5

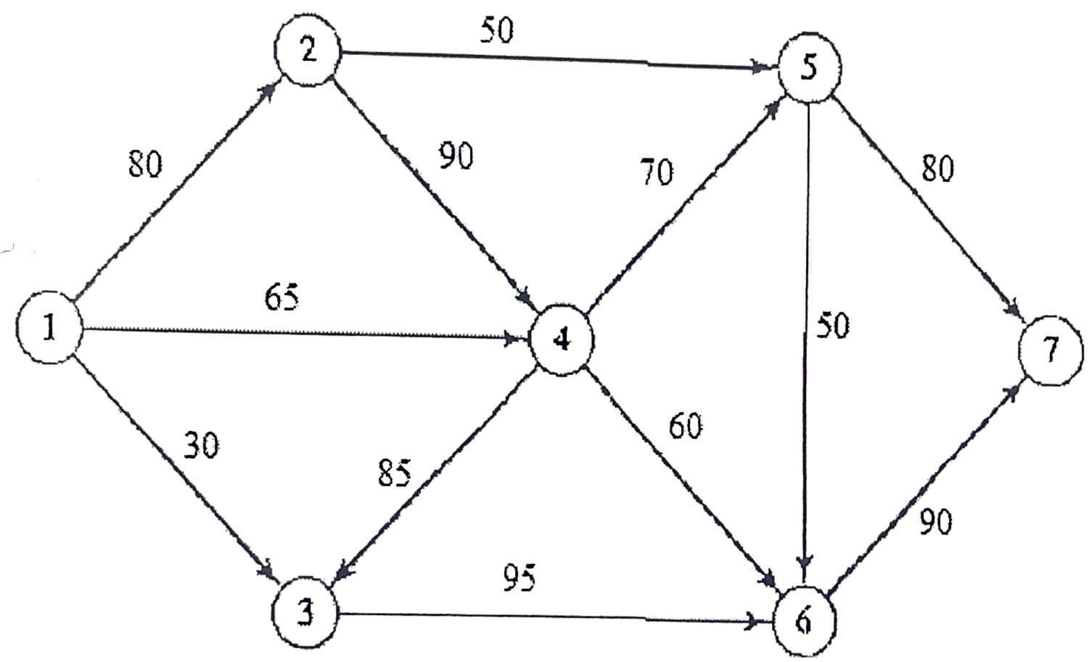


(ii) Use Dijkstra's algorithm to determine the shortest path

(a) From node 1 to 5

P.T.O.

(b) From node 2 to 7 of the network given in the following figure : 6+4=10



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This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **9591**

Unique Paper Code : **62355604** **HC**

Name of the Paper : **General Mathematics-II**

Name of the Course : **Mathematics (Generic Elective)**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions as
per directed question wise.

Section - I

Attempt any *five* questions out of the following :

(*a*) How and when did Cantor die ?

(*b*) Write a short summary on the life of an Indian Mathematician.

(*c*) Discuss the major contributions of Hilbert.

(*d*) Why was Emmy Noether not allowed to join the faculty at Gottingen University ?

(*e*) What were the contributions of Hardy in the field of Mathematics ?

(*f*) Discuss the career of Hausdorff. 3×5

P.T.O.

Section -II

2. Attempt any six questions out of the following :

(*a*) Define functions. Illustrate the location of point of extrema and inflection with the help of the graphs.

(*b*) What is a triangle ? Classify triangles on the basis of its sides and angles.

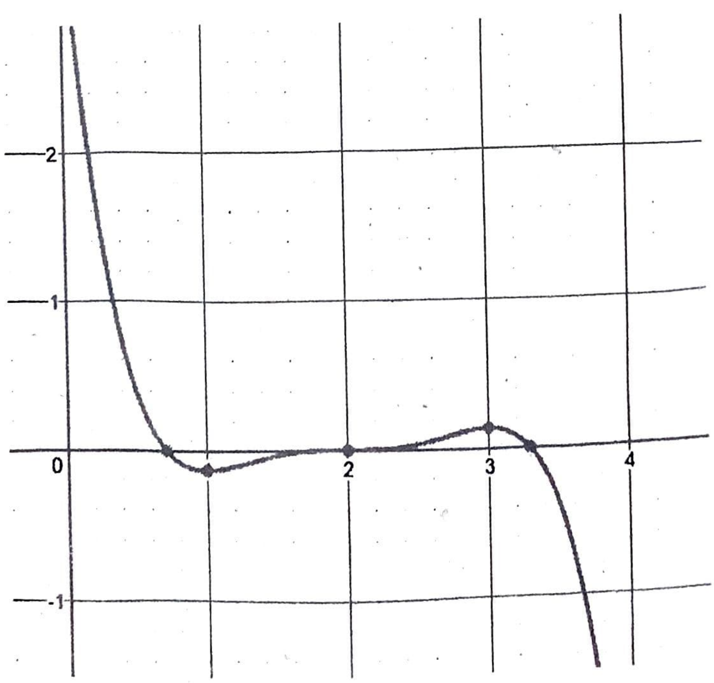
(*c*) What is basic Tilings ? Discuss.

(*d*) What is a perspective ? What role does it play in buildings, monuments and paintings ?

(*e*) What is a Konigsberg Bridge problem ? Explain how this problem led to the discovery of Euler's formula for networks.

(*f*) Discuss the significance of Golden ratio in nature.

(*g*) Find maxima and minima and saddle points of the function in the following graph. Find the intervals in which the function is increasing or decreasing.



(*h*) Verify which of the following are even or odd functions using graphs :

(*i*) Cos x

(*ii*) Sin x

(*iii*) Tan x. 5×6

Section-III

3. Attempt any *five* questions from the following :

(*a*) Use Gauss Elimination method to solve the following system of linear equations ;

$$4x_1 - 2x_2 + x_3 = 20,$$

$$x_1 + x_2 + x_3 = 5,$$

$$9x_1 + 3x_2 + x_3 = 25.$$

(*b*) Without row reduction, find A^{-1} and compute A^{-3}, given $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.

(*c*) Use the Gauss Jordan method to convert the matrix into row reduced echelon form :

$$A = \begin{bmatrix} 4 & 3 & -5 \\ 1 & 1 & 0 \\ 1 & 1 & 5 \end{bmatrix},$$

(*d*) Find the inverse of the coefficient matrix and use it to solve the following system of equations :

$$5x_1 - x_2 = 20,$$

$$-7x_1 + 2x_2 = -31.$$

(e) Find the rank of the following matrix :

$$\begin{bmatrix} 2 & -5 & 3 \\ -1 & -3 & 4 \\ 7 & -12 & 5 \end{bmatrix}$$

(f) Find the trivial or non-trivial solutions of the followin homogeneous system :

$$5x_1 - 2x_3 = 0,$$

$$-15x_1 - 16x_2 - 9x_3 = 0,$$

$$10x_1 + 12x_2 + 7x_3 = 0.$$

(g) Determine whether the vector x = [–3, –6] is a linea combination of $a_1 = [1, 4]$, $a_2 = [-2, 3]$. 5×

[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : 9650 **HC**

Unique Paper Code : 62357602

Name of the Course : **B.A.(Programme) Mathematics : DSE-4**

Name of the Paper : Numerical Analysis

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) All **six** questions are compulsory.

(c) Attempt any **two** parts from each question.

(d) Use of non-programmable scientific calculator is allowed.

1. (a) Perform three iterations of Secant method to obtain a root of the equation 6

$$x^2 - 7 = 0$$

with initial approximation $x_0 = 2,\ x_1 = 3$.

P.T.O.

(b) Perform four iterations of bisection method to obtain the root of the equation

$$x^3 - 3x^2 - 0.09x + 0.27 = 0$$ 6

in the interval $[0,1]$.

(c) If true value is 0.000310698 and approximated value is 0.0049065, then find the absolute and the relative error. Differentiate between round-off error and truncation error. 6

2. (a) Perform two iterations of Newton's method to solve the non-linear system of equations 6

$$x^2 y + y^3 = 10$$

$$xy^2 - x^2 = 3$$

with initial approximation $(x_0, y_0) = (0.8, 2.2)$.

(b) Perform three iterations of Regula Falsi method to find the root of the equation 6

$$f(x) = x^3 - 2 = 0$$

in the interval $[1,2]$.

(c) Perform three iterations of Newton-Raphson method to obtain the cube root of 17 with initial approximation $x_0 = 2$.

6

3. (a) Find the inverse of the following matrix using the Gauss Jordan method :

6.5

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(b) Solve the tridiagonal system $AX = b$ by using Gauss- Thomas algorithm.

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ 0 & 3 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 17 \\ 22 \end{pmatrix}$$

6.5

(c) For the following system of equations: 6.5

$$10x + 4y - 2z = 12$$

$$x - 10y - z = -10$$

$$5x + 2y - 10z = -3$$

(i) Show that Jacobi iteration scheme converges.

(ii) Perform three iterations of Jacobi iteration scheme starting with initial vector $(x, y, z) = (0,0,0)$.

4. (a) For the following data, obtain the Gregory - Newton backward difference polynomial.

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

Also find the approximate value of f at $x = 0.25$. 6

(b) Prove the following :

$$1+\Delta = 1+\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{1}{4}\delta^2}$$

(c) The function $f(x)=\sin(x)$ is defined on $[1,3]$. Find Lagrange interpolating linear polynomial on intervals [1, 2] and [2, 3]. Hence find the approximate value of $f(1.5)$ and $f(2.5)$. 6

5. (a) The following data for the function $f(x)=x^4$ is given (6.5)

X	0.4	0.6	0.8
f(x)	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation.

(b) Find integral of $I = \int_0^2 e^{-x^2} dx$ using Simpson's rule and Trapezoidal rule. (6.5)

(c) Evaluate $\int_0^1 \frac{dx}{1+x}$ using composite Trapezoidal rule and Romberg integration with $h = 1$ and $h = 1/2$ only. 6.5

6. (a) Employ the classical fourth order RK method to integrate $y' = 4e^{0.8t} - 0.5y$, $y(0) = 2$ from $t = 0$ to 1 using a step size of 1. 6.5

(b) Apply Euler's method to approximate the solution of

$$y' = x + y, \quad y(0) = 2,$$ 6.5

and calculate $y(1)$ by using h=0.5.

(c) Apply finite difference method (second order) to solve the given problem : 6.5

$$y'' = y + x, \quad y(0) = 0, \quad y(1) = 0,$$

with $h = 1/4$.