Special Theory of Relativity

(Undergraduate Level)

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What is Special theory of Relativity

In 1905 Einstein Observed that the lows of Physics are same in all non accelerating frame of references and the speed of light in vacuum is independence of motion of all observer.

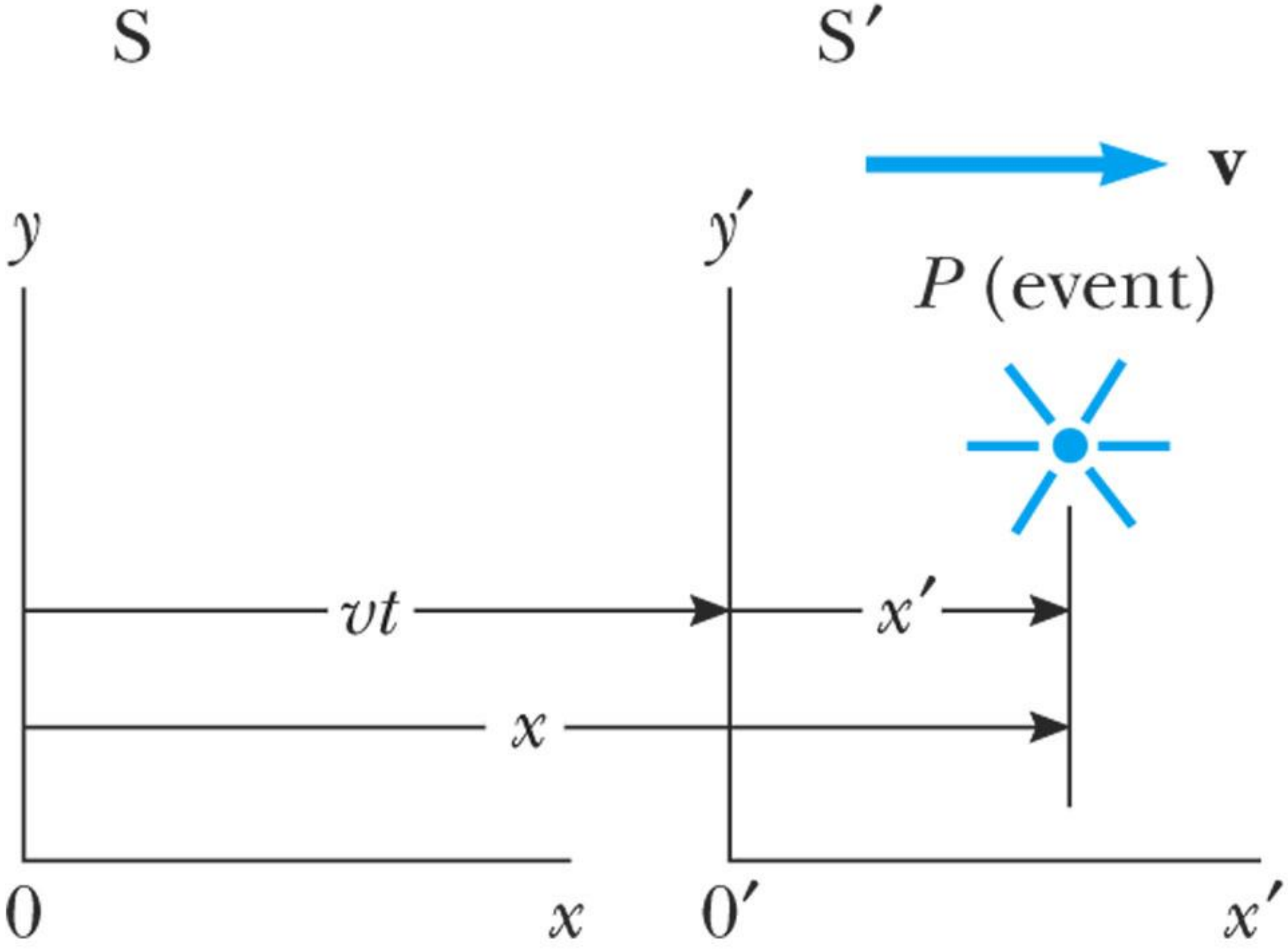
Inertial Frame of Reference

- A reference frame is called an **inertial frame** if Newton's laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, is observed to move in rectilinear motion at constant velocity.

Newtonian Principle of Relativity

- If Newton's laws are valid in one inertial reference frame, then they are also valid in another inertial reference frame moving at a uniform velocity relative to the first system.

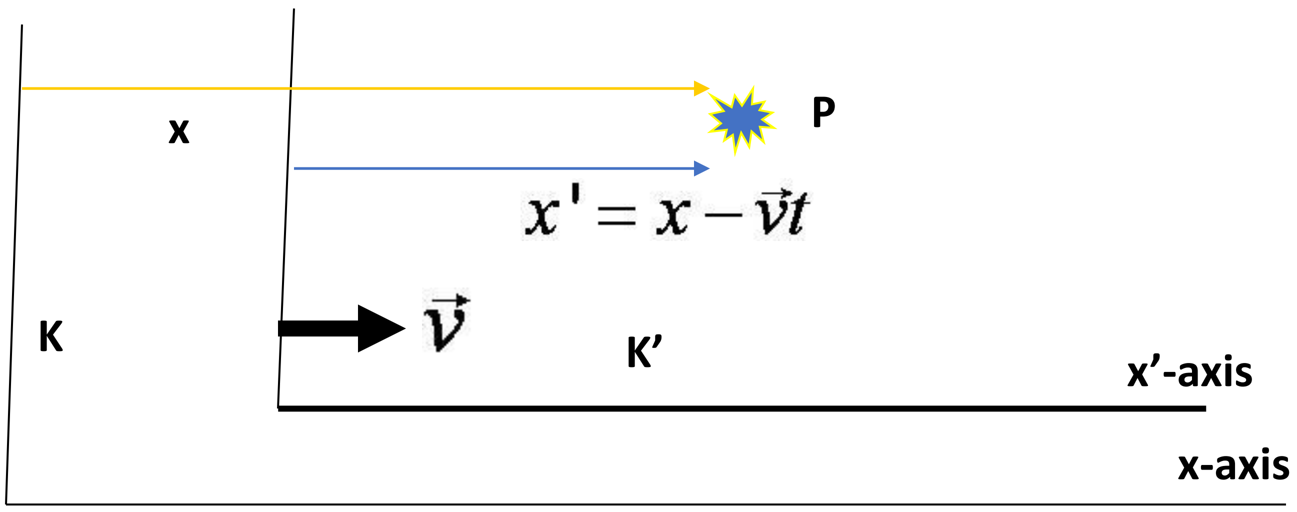
- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance/relativity. So the laws of mechanics are independent on the state of movement in a straight line at constant velocity**


S
S′
v
y
y′
P (event)
vt
x′
x
0
x
0′
x′

The Galilean Transformation

For a point P

- In system K: P = (x, y, z, t)
- In system K': P = (x', y', z', t')



Conditions of the Galilean Transformation

- Parallel axes
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Inverse Relations

Step 1. Replace $\vec{v}$ by $-\vec{v}$

Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed."

$$x = x' + \vec{v}t$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

Need for Ether

- The wave nature of light suggested that there existed a propagation medium called the *luminiferous ether* or just **ether**.
  - Ether had to have such a low density that the planets could move through it without loss of energy
  - It also had to have an enormous elasticity/toughness to support the high velocity of light waves
  - According to classical physics ideas, the ether frame would be a preferred frame, the only one in which Maxwell's equation would be valid as derived

An Absolute Reference System

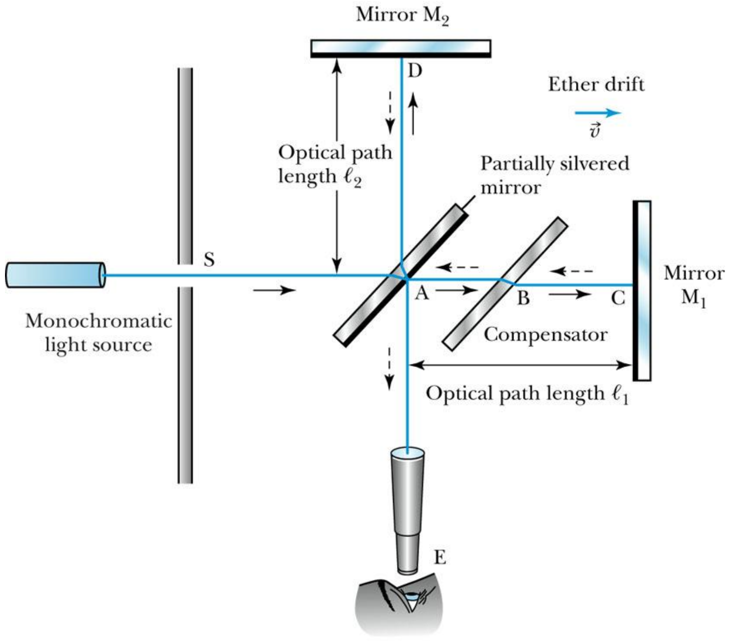
- Ether was proposed as an absolute reference system in which the speed of light was this constant and in all frames moving with respect to that frame, there needed to be modifications of Maxwell’s laws.
- The Michelson-Morley experiment was an attempt to figure out Earth’s relatives movement through (with respect to) the ether so that Maxwell’s equations could be corrected for this effect.

The Michelson Interferometer

1. AC is parallel to the motion of the Earth inducing an “ether wind”

2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions

3. After reflection the beams recombine at A slightly out of phase due to the “ether wind” as viewed by telescope E.



As the direction of the ether wind is unknown, the apparatus has to be turned around by 90 degrees to see a shift (starting form many different initial settings)

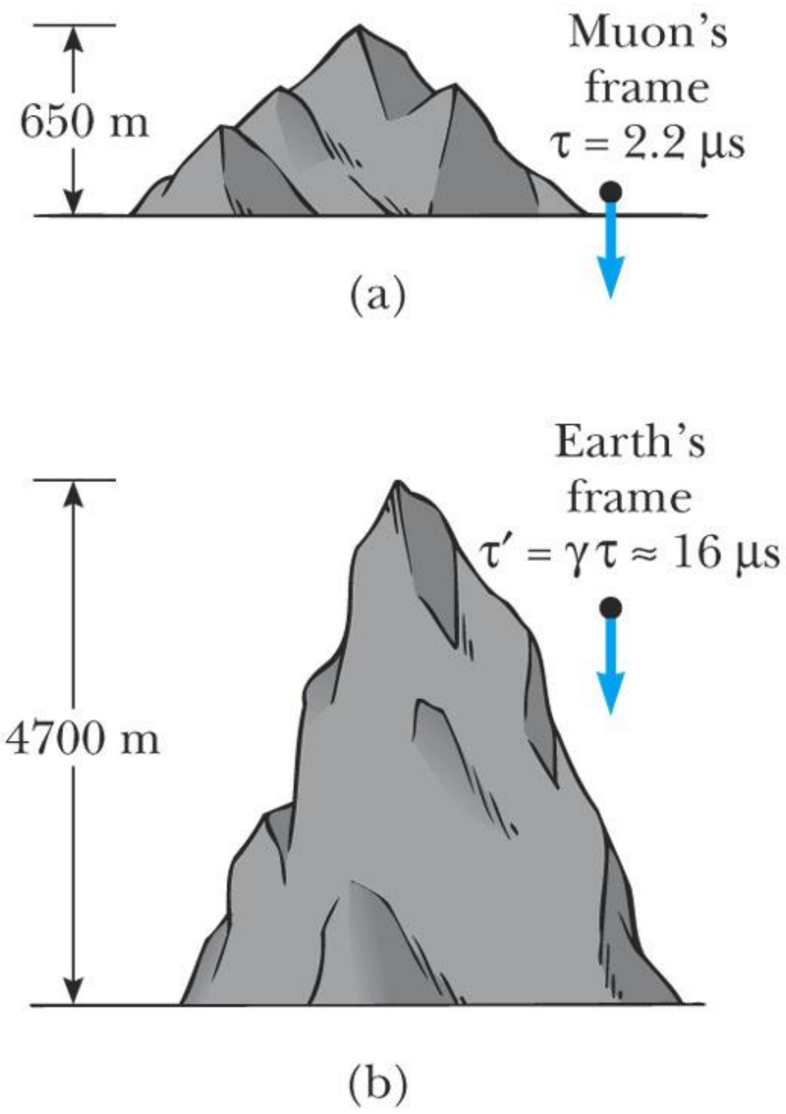
The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1 - v^2 / c^2}$$

…thus making the path lengths equal to account for the zero phase shift.

  - This, however, was an ad hoc assumption that could not be experimentally tested. It turned out to be “less than half of the story”



Length contracted for the moving muon, it's own life time just 2.2 micro seconds

Life time of the muon delayed for observer on Earth so that it can travel the whole distance as observed from Earth

Great thing about special relativity is that one can always take two viewpoints, moving with the experiment, watching the experiment move past, the observations need to be consistent in both cases

They move with about 98 % the speed of light

Lorentz Transformation Equations

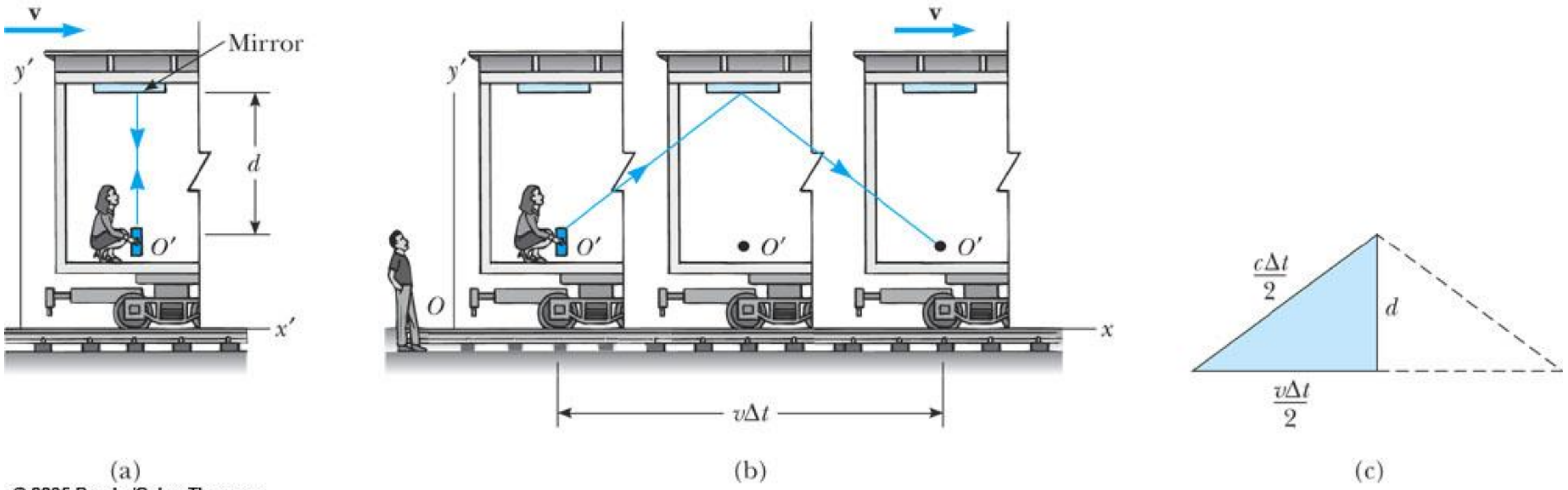
$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \qquad y' = y$$

$$z' = z$$

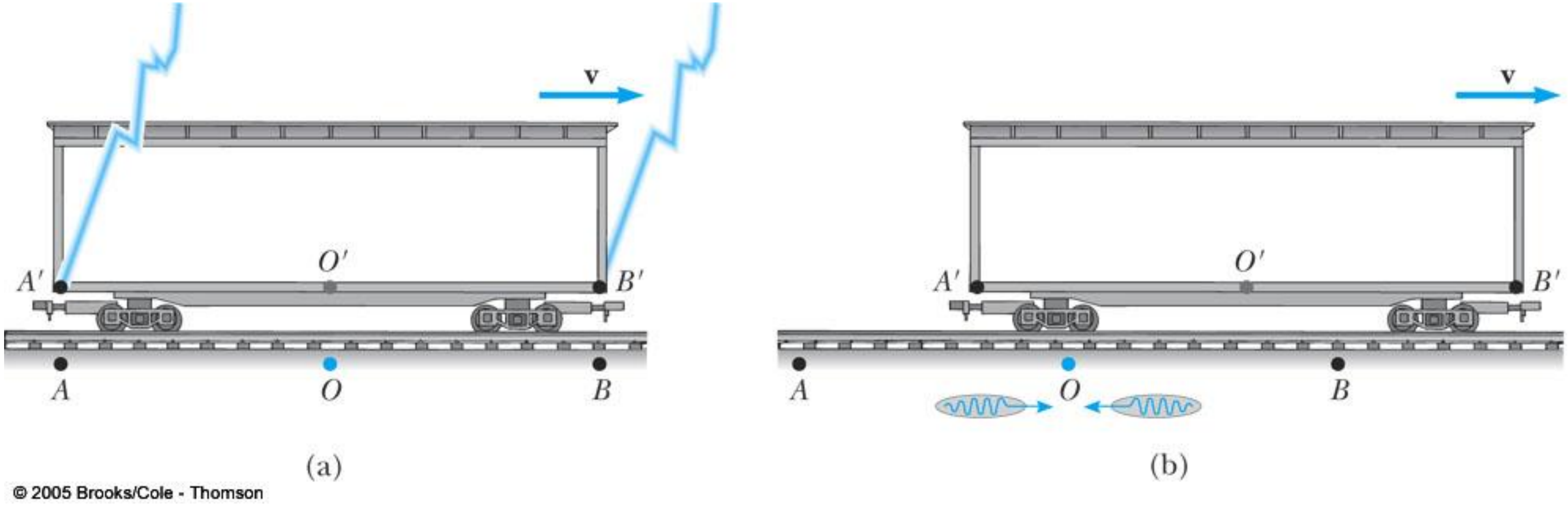
So there is four-dimensional space time !!!

Time = spatial distance divided by c, arrow of time is determined by second law of thermodynamics



Mary has a light clock. A suitable clock is just any periodic process, the time it takes for one cycle of the process is the period, its inverse is the frequency.

Tom watching Mary go by figures that her time is delayed (dilated) due to her moving in a straight line with a constant high velocity with respect to him.



No simultaneity if not also at the same position, just a consequence of the Lorentz transformations.

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for u'_x, u'_y, and u'_z can be obtained by switching primed and unprimed and changing v to $-v$:

$$u'_x = \frac{u_x - v}{1 - (v/c^2)u_x}$$

$$u'_y = \frac{u_y}{\gamma\left[1 - (v/c^2)u_x\right]}$$

$$u'_z = \frac{u_z}{\gamma\left[1 - (v/c^2)u_x\right]}$$

Einstein’s Two Postulates

With the belief that Maxwell’s equations (and with it all of the known physics of the time) must be valid in all inertial frames, Einstein proposes the following postulates:

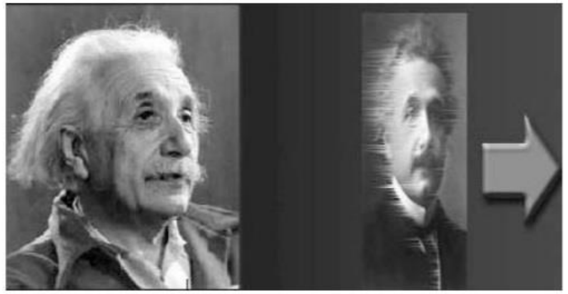
1) **The principle of (special) relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

2) **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

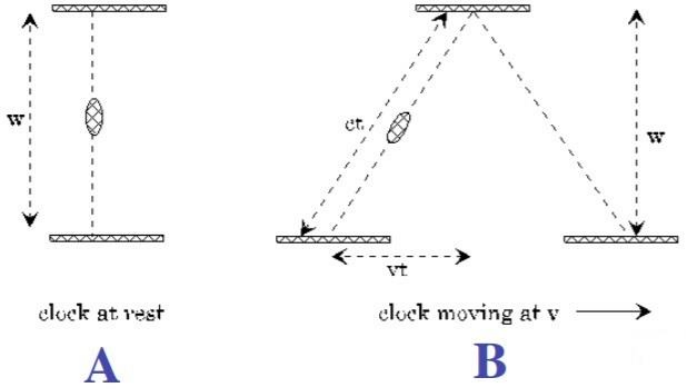
Time dilation

Time Dilation

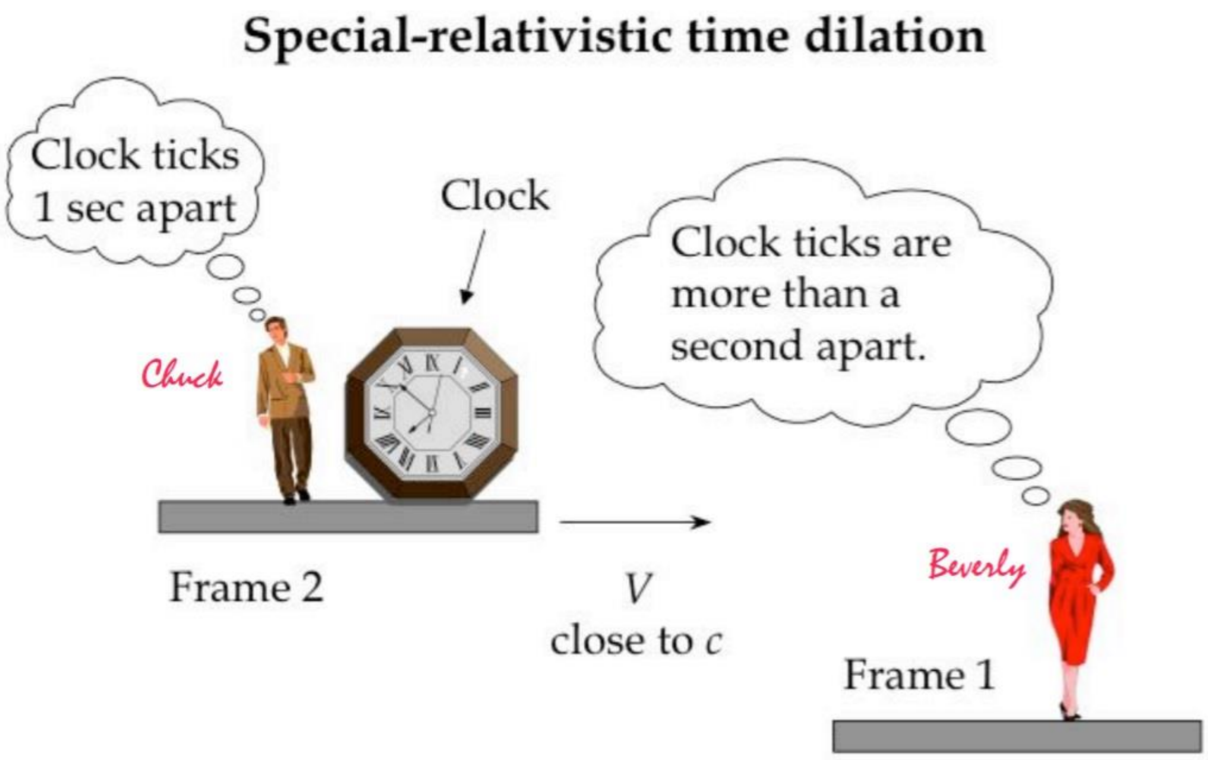
A light pulse goes from the floor to the ceiling and back. Since c=const but the distance is longer in case B (moving frame):
Time intervals seen in moving reference frames appear longer than the same interval seen at rest.



[Image from http://www.mncs.k12.mn.us/physics/relativity]



Time dilation continue..

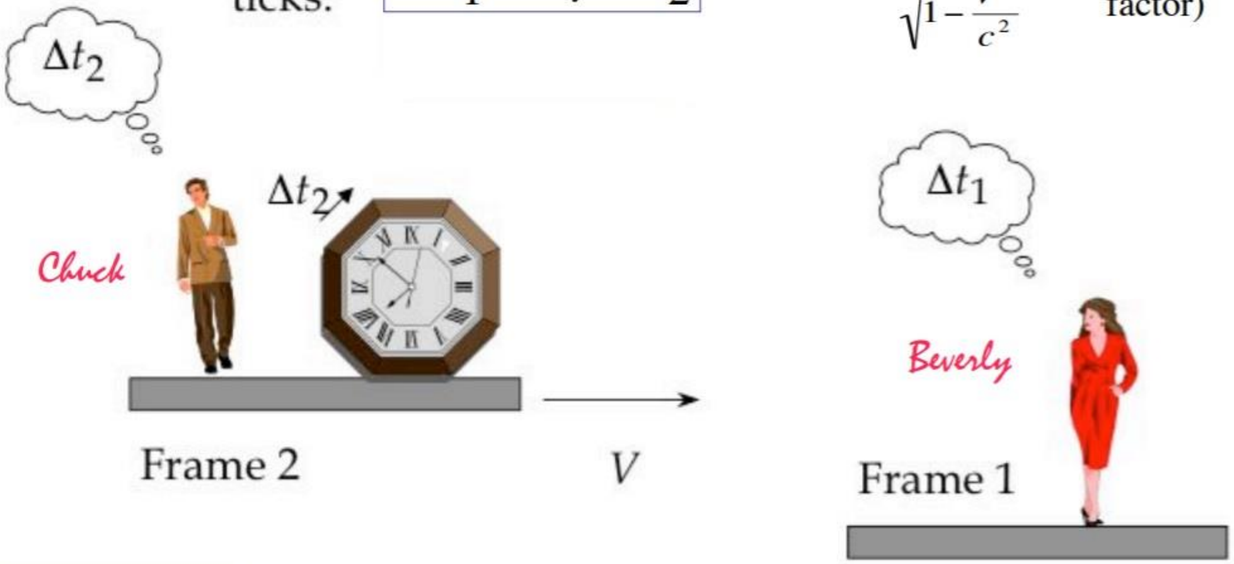


The clock is at rest with respect to the reference frame 2

Time dilation continue..

Special-relativistic time dilation (continued)

Time between ticks: $\boxed{\Delta t_1 = \gamma \Delta t_2}$ where $\gamma = \dfrac{1}{\sqrt{1-\dfrac{V^2}{c^2}}}$ (Lorentz factor)



$\boxed{\Delta t_1 > \Delta t_2}$ The clock is at rest in Frame 2 (Chuck's frame)

Understanding time dilation with example..

Some numerical examples of time dilation

Clock

$\Delta t_2 = 1$ sec between ticks

Her results:

Observer measures the intervals between ticks on a moving clock, using her own clock for comparison.

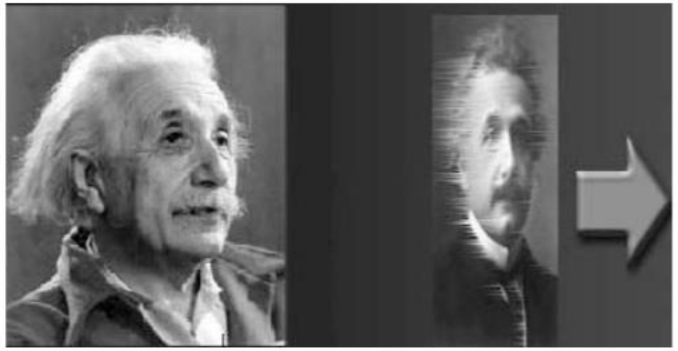
$V = 0$	$\Delta t_1 = 1$ sec
10 km/sec	1 sec
1000 km/sec	1.000006 sec
100000 km/sec	1.061 sec
200000 km/sec	1.34 sec
290000 km/sec	3.94 sec

Δt_1 would always be 1 sec if Galileo's relativity applied.

Length Contraction

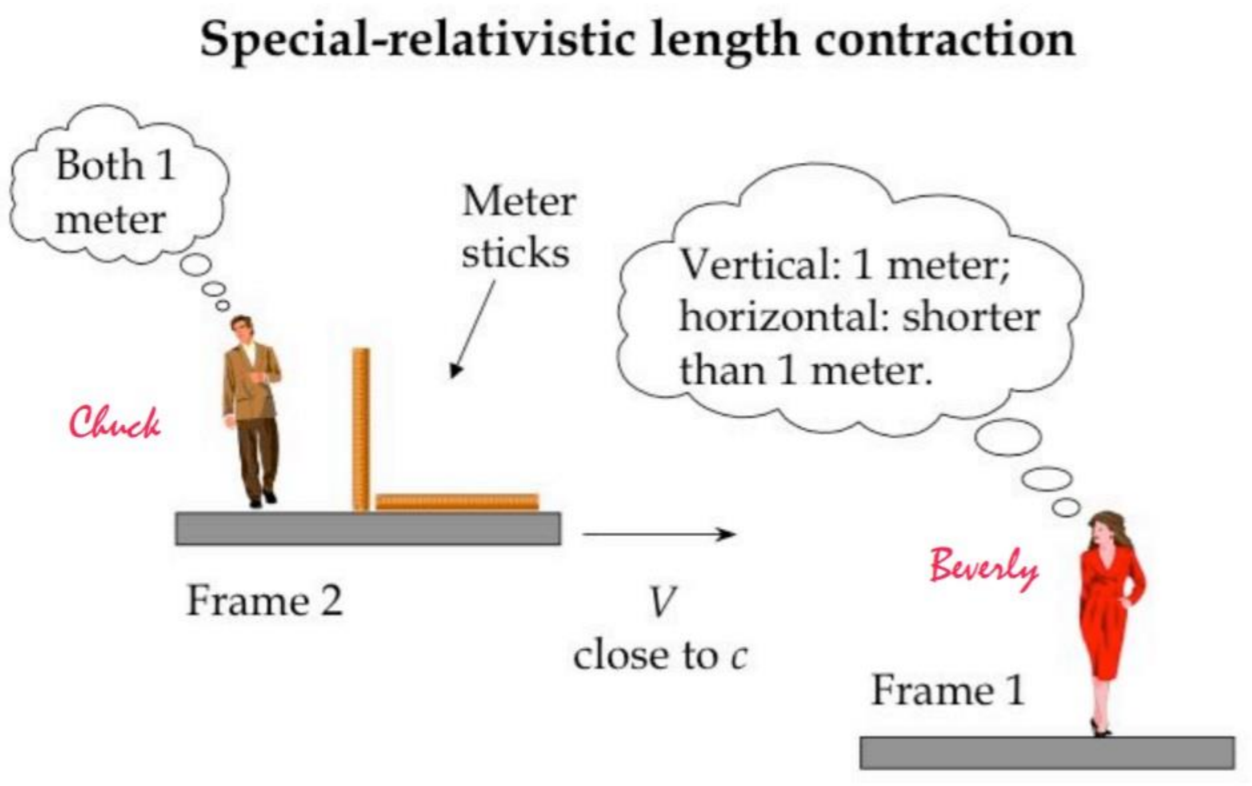
Lorentz contraction

Objects seen in moving reference frames appear shorter along the direction of motion than the same object seen at rest.

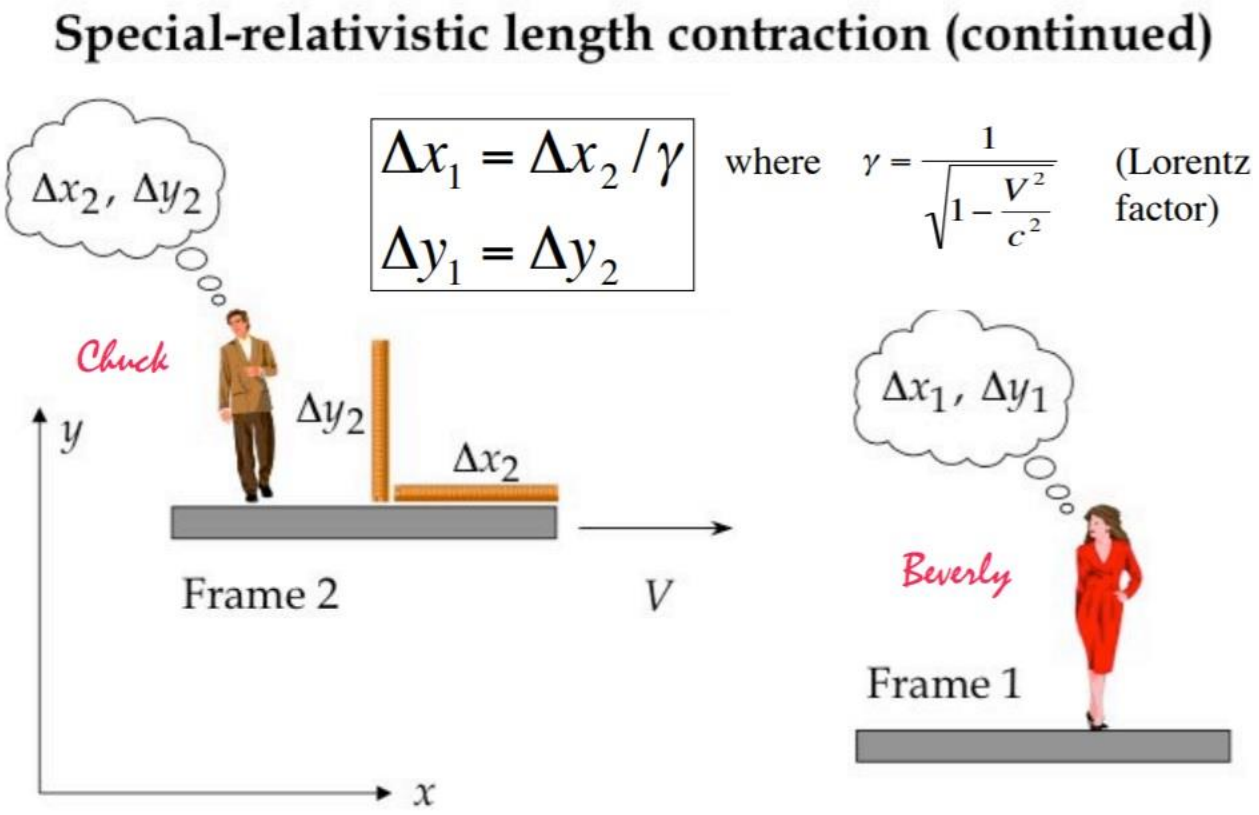


[Image from http://www.mncs.k12.mn.us/physics/relativity]

Length contraction continue..



Length contraction continue..



Understanding length contraction with example

Some numerical examples of length contraction

Meter stick
$\Delta x_2 = 1$ meter

V

Observer measures the length of meter sticks moving as shown.

Her results:

$V = 0$	$\Delta x_1 = 1$ meter
10 km/s	1 meter
1000 km/s	0.999994 meter
100000 km/s	0.943 meter
200000 km/s	0.745 meter
290000 km/s	0.253 meter

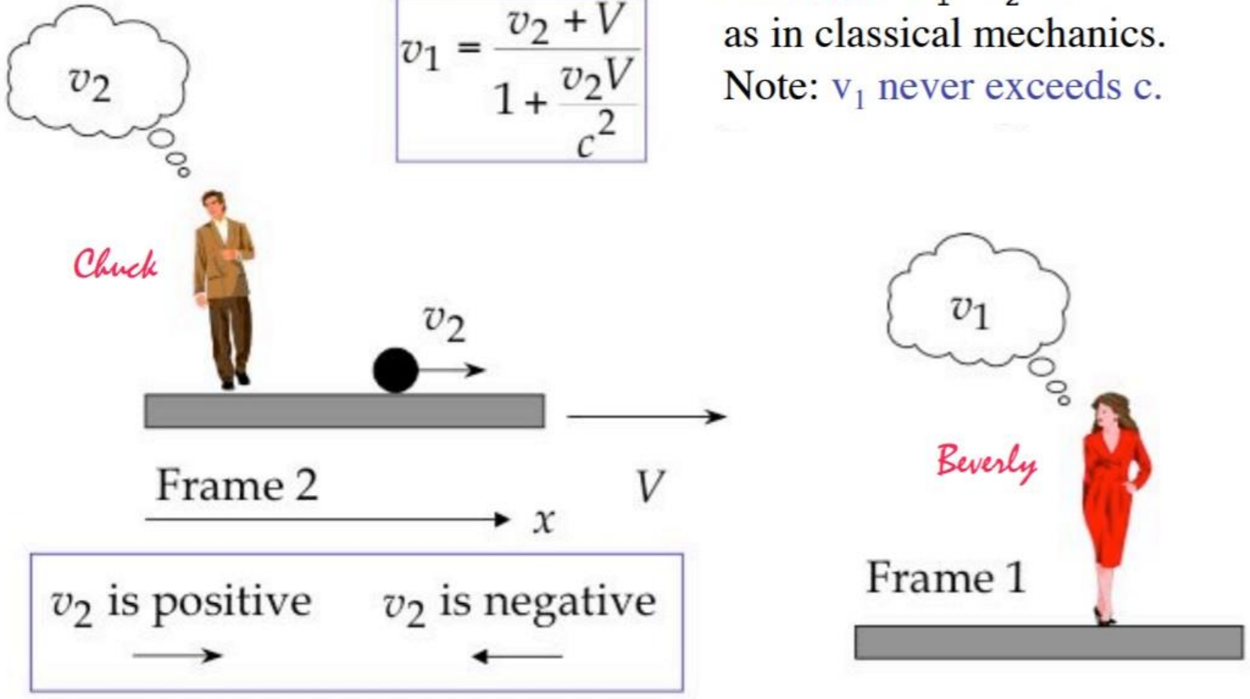
Δx_1 would always be 1 meter if Galileo's relativity applied.

Addition of Velocity

Special-relativistic velocity addition (in the direction of motion)

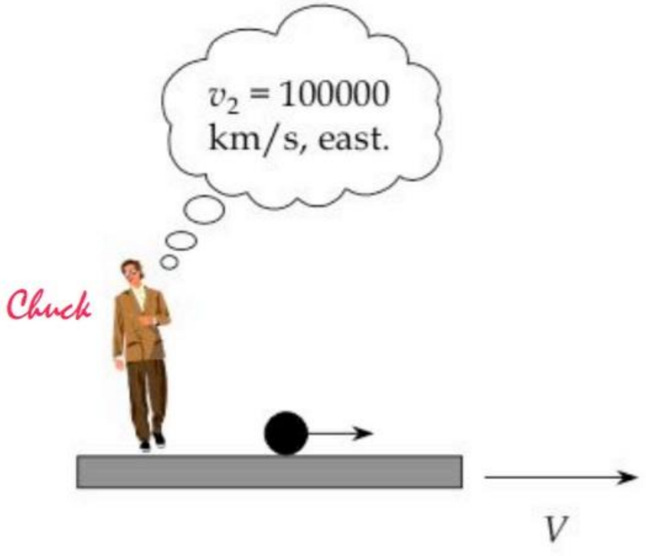
$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

and NOT $v_1 = v_2 + V$
as in classical mechanics.
Note: v_1 never exceeds c.



Note: velocities can be positive (towards east in this example) or negative (towards west)

Special-relativistic velocity addition: an example



Observer measures the speed of a ball rolled at 100000 km/s in a reference frame moving at speed V, using her surveying equipment and her own clock for comparison.

Understanding Addition of Velocity with examples..

Special-relativistic velocity addition: an example (continued)

Her results for the speed of the ball, for several values of the speed V of frame 2 relative to frame 1:

$V = 0$	$v_1 =$ 100000 km/sec	= 100000 km/sec
10 km/sec	100008.9 km/sec	= 100010 km/sec
1000 km/sec	100888 km/sec	= 101000 km/sec
100000 km/sec	179975 km/sec	= 200000 km/sec
200000 km/sec	245393 km/sec	= 300000 km/sec
290000 km/sec	294856 km/sec	= 390000 km/sec
		if Galileo's relativity applied

Relativistic mass

Mass is relative

An object seen in a moving reference frame appears to be more massive than the same object seen at rest.

Mass increase due to very high speeds

Mass at 0% of the speed of light = 10kg

Mass at 99.5 % of the speed of light = 100kg

The relation between the two masses is given by

$$\boxed{m = \gamma m_0}$$ where m_0 is the mass measured at rest.

Two consequences:

- When trying to bring an object to a velocity v $\longrightarrow$ c, its mass m appears infinite and therefore you would need an infinite amount of energy $\longrightarrow$ nothing can move faster than light!
- **$E=Mc^2$** Mass and energy are equivalent

Relativistic Momentum

$$\vec{p} = m\frac{d\vec{r}}{dt}\gamma$$

$$p = \gamma m\vec{u}$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

Relativistic dynamics can be derived by ***assuming*** that mass is increasing with velocity. The Lorentz factor gets larger when velocities get larger and so does mass ***apparently*** as we can see from the relativistic momentum equation. Einstein derived relativistic dynamics that way. His derivations are sure correct, but the foundations are somewhat shaking as there is no really good definition for mass.

Relativistic Energy

- Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
  - Therefore, we modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u}) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

Kinetic energy is work done on a particle by a force

Relativistic Kinetic Energy

Equation (2.58) does not seem to resemble the classical result for kinetic energy, $K = \frac{1}{2}mu^2$. However, if it is correct, we expect it to reduce to the classical result for low speeds. Let's see if it does. For speeds $u \ll c$, we expand γ in a binomial series as follows:

$$K = mc^2\left(1-\frac{u^2}{c^2}\right)^{-1/2} - mc^2$$

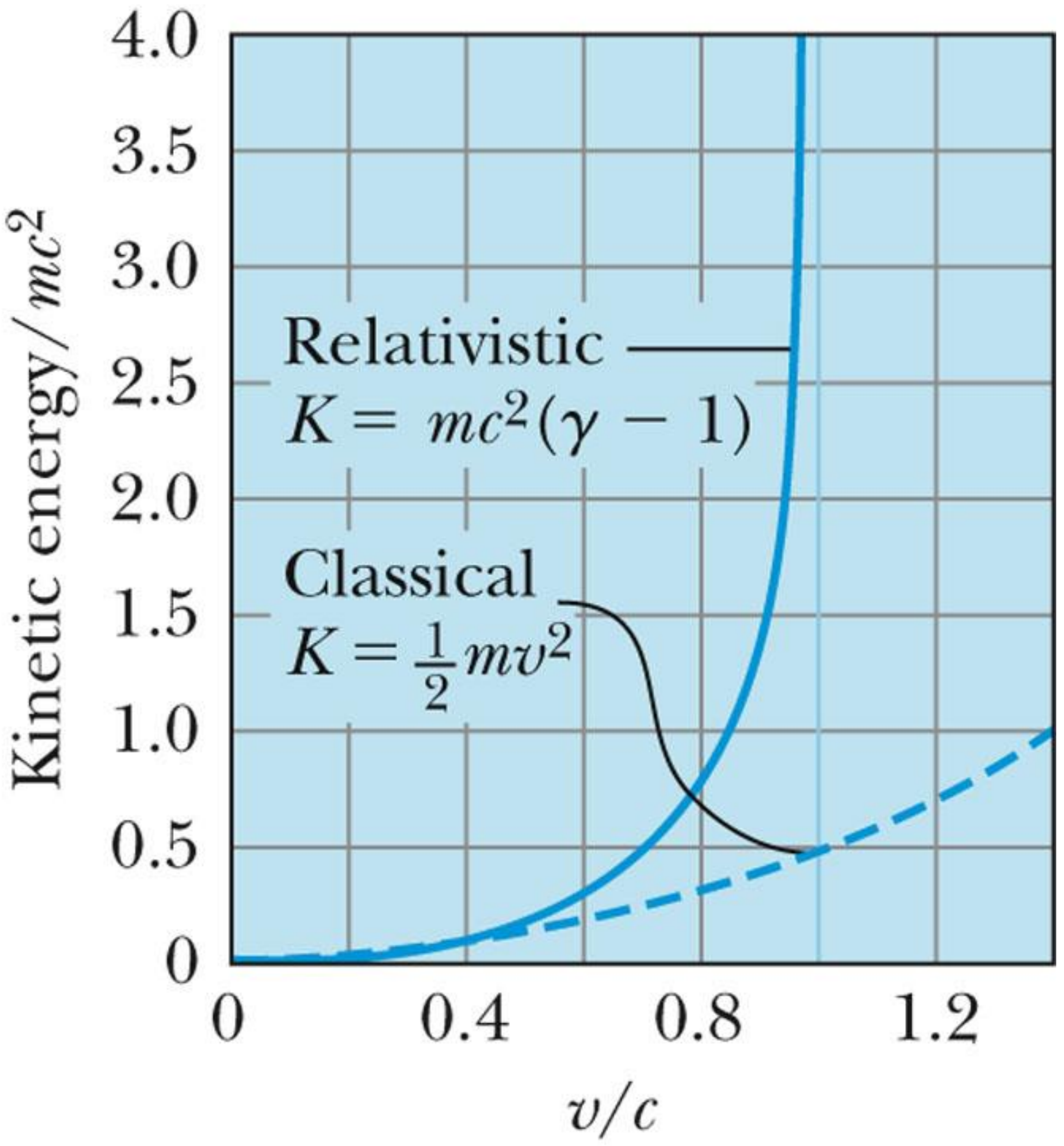
$$= mc^2\left(1+\frac{1}{2}\frac{u^2}{c^2}+\dots\right) - mc^2$$

where we have neglected all terms of power $(u/c)^4$ and greater, because $u \ll c$. This gives the following approximation for the relativistic kinetic energy at low speeds:

$$K = mc^2 + \frac{1}{2}mu^2 - mc^2 = \frac{1}{2}mu^2$$

which is the expected classical result. We show both the relativistic and classical kinetic energies in the next Figure. They diverge considerably above a velocity of $0.1c$. Best to use relativistic dynamics as soon as the speed of something is larger than 1 % of the speed of light.

Relativistic and Classical Kinetic Energies



Total Energy and Rest Energy

We rewrite in the form

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

This leaves the sum of the kinetic energy and rest energy to be interpreted as the total energy of the particle. The total energy is denoted by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Momentum and Energy

$$p = \gamma mu = \frac{mu}{\sqrt{1 - u^2 / c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2$$

$$= \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} \right) = \gamma^2 m^2 c^4 \beta^2$$

We replace β^2 and find

$$p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2} \right)$$

Accelerator equation

$$= \gamma^2 m^2 c^4 - m^2 c^4$$

Momentum and Energy continue..

The first term on the right-hand side is just E^2, and the second term is E_0^2. The last equation becomes

$$p^2c^2 = E^2 - E_0^2$$

We rearrange this last equation to find the result we are seeking, a relation between energy and momentum.

$$E^2 = p^2c^2 + E_0^2$$

or

$$E^2 = p^2c^2 + m^2c^4$$

is a useful result to relate the total energy of a particle with its momentum. The quantities ($E^2 - p^2c^2$) and m are invariant quantities. Note that when a particle's velocity is zero and it has no momentum, "accelerator Equation" correctly gives E_0 as the particle's total energy.

There can be mass less particles that still have momentum. These can collide with massive particles. For such a collision one needs to invoke special relativity!

Thank you