Problem 1. Solve the following cases using Lagrange's equation of motion:

a) Linear harmonic oscillator (solve by Hamiltonian also)

b) Compound & double pendulum (Determine the time period of oscillation)

c) A mass *2m* is suspended from a fixed support by a spring (S_1), of spring constant, *2k*. From this mass, another mass, *m*, is suspended by another spring (S_2) constant, *k*, (Fig. 1). Find equation of motion of the coupled system.

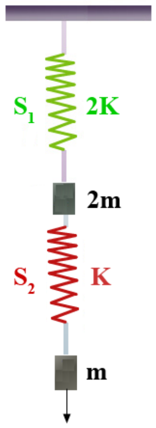


Figure 1

Problem 2. Consider a bead of mass *m* frictionlessly gliding on a wire which rotates with constant angular velocity ω. Fig. 2 shows the bead move in the earth gravitational field.

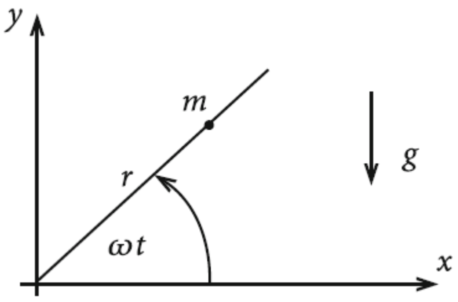


Figure 2 Bead on a rotating wire in the earth's gravitational field

a) Which constraint forces are present?

b) Formulate the Lagrangian function for the bead.

c) Determine the Lagrange equation of motion and find its general solution.

d) Use the initial conditions

$$r(t=0) = r_0 \qquad \& \qquad \dot{r}(t=0) = 0$$

How large ω be at the least to force the bead to move outwards for $t \to \infty$?

e) How would we have to treat the problem in Newton's mechanics?

Problem 3. A box is gliding without friction along the *x* axis with constant velocity v_0. On the bottom of the box there oscillates, also in *x* direction and frictionlessly, a mass *m* being fixed by a spring (spring constant: *k*) at the back-wall of the box (Fig. 3).

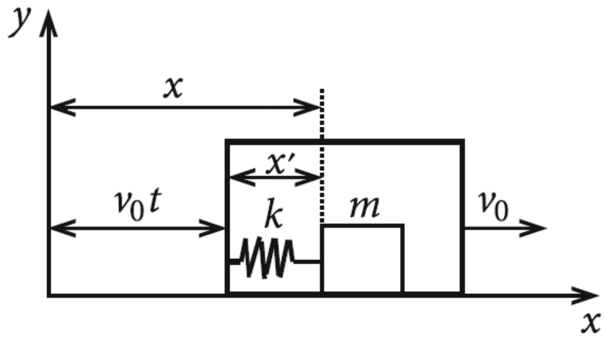


Figure 3 Oscillating mass in a with constant velocity frictionlessly gliding box

a) Find the Hamilton function in the rest system of coordinates $\sum$. Is H a conserved quantity? Is *H* identical to the total energy *E*? Derive Hamilton's equations of motion.

b) Investigate the same problem in the co-moving system of coordinates $\sum'$.

Problem 4. A planet of mass *m* & an angular momentum *L* moves in a potential, $V(r) = -k / r$, where *k* is a constant. If it is slightly perturb radially, find the angular frequency of radial oscillation.

Problem 5. Incompressible liquid is flowing steadily through a circular pipe. Determine the rate of flow of liquid through pipe with uniform gradient of pressure difference is maintained across the length *l*.

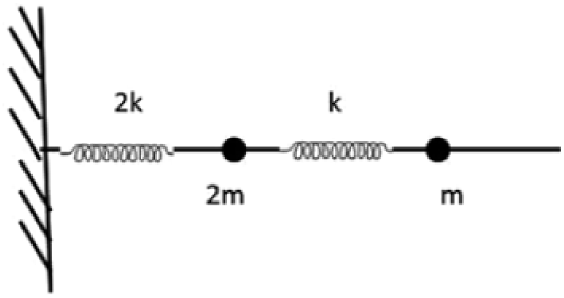
Problem 6. A particle of mass 1 kg is undergoing small oscillation about the equilibrium point in the potential $V(x) = \frac{1}{2x^{12}} - \frac{1}{x^6}$ for x > 0 meters. Determine the time period of the oscillation.

Problem 7. Two beads of mass 2*m* and *m* can move without friction along a horizontal wire. They are connected to a fixed wall with two springs of spring constants 2*k* and *k* as shown in Fig. 1:

(a) Find the Lagrangian for this system and derive from it the equations of motion for the beads.

(b) Find the eigenfrequencies of small amplitude oscillations.

(c) For each normal mode, sketch the system when it is at the maximum displacement.



Problem 8. A tape of 0.015 cm thick and 1.00 cm wide is to be drawn through a gap with a clearance of 0.01cm on each side. A lubricant of dynamic viscosity 0.021 Ns/m^2 completely fills the gap for a length of 80 cm along the tape. If the tape can withstand a maximum tensile force of 7.5 N calculate the maximum speed with which it can be drawn through the gap.