Moment of a force about a line

Two Things to Keep in mind

(1) Line of force (passing through a given point)

(2) Line about which moment is to be found.

First step : Find $\hat{n}$ = Unit vector along line

Second step : Take any point on line and using its coordinate find a position vector by subtracting

given point
↕
or any point

on the line of force.

⇓ This is $\vec{r}$ vector now

Third step : After this find $\vec{r} \times \vec{F}$

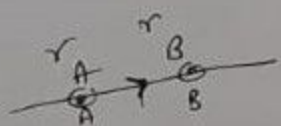
fourth step : Find $(\vec{r} \times \vec{F}) \cdot \hat{n} = M_{point(Any)}$

↓

This is moment of force about the given line along $\hat{n}$

Fifth Step : Answer is $M_{point(Any)}$ along $\hat{n}$

Note:



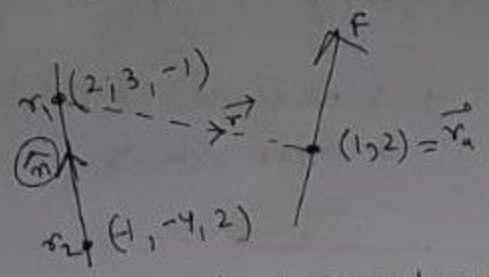
$\overrightarrow{AB} = \vec{B} - \vec{A}$

Eg) Compute moment of a force $F = 50\hat{i} + 30\hat{j}$ Newton, which goes through position $r_a = \hat{i} + 2\hat{j}$ m about a line going through points $\vec{r_1} = (2\hat{i} + 3\hat{j} - \hat{k})$ m and $\vec{r_2} = (-\hat{i} - 4\hat{j} + 2\hat{k})$ m respectively.

Ans:

① $\hat{n}$

Line



Position vector of line / vector representing line ($\vec{r_1}$ and $\vec{r_2}$)

$= \vec{r_1} - \vec{r_2}$ or $\vec{r_2} - \vec{r_1}$ (u can choose any)

$= [2-(-1)]\hat{i} + [3-(-4)]\hat{j} + [-1-(2)]\hat{k}$

$= 3\hat{i} + 7\hat{j} - 3\hat{k}$

$$\hat{n} = \frac{3\hat{i} + 7\hat{j} - 3\hat{k}}{\sqrt{3^2 + 7^2 + (-3)^2}} = \frac{3\hat{i} + 7\hat{j} - 3\hat{k}}{\sqrt{67}}$$

② Any point on line is suppose $\vec{r_1}$ (or you can choose $\vec{r_2}$ also)

$$\vec{r} = \vec{r_a} - \vec{r_1} = (\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j} - \hat{k}) = (-\hat{i} - \hat{j} + \hat{k})$$

③ $$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 50 & 30 & 0 \end{vmatrix} = \hat{i}(0 - 30) - \hat{j}(0 - 50) + \hat{k}(-30 + 50)$$

$$= -30\hat{i} + 50\hat{j} + 20\hat{k}$$

④ $$(\vec{r} \times \vec{F}) \cdot \hat{n} = (-30\hat{i} + 50\hat{j} + 20\hat{k}) \cdot \left(\frac{3\hat{i} + 7\hat{j} - 3\hat{k}}{\sqrt{67}}\right) = \frac{1}{\sqrt{67}}(-90 + 350 - 60)$$

$$= \frac{200}{\sqrt{67}}$$

⑤ Moment of force about given line is $\frac{200}{\sqrt{67}}$ along $\hat{n}$

Units of moment of force is Newton. Meter