Octal Number System:

Base of the octal number system is 8 as it uses the digits 0 to 7. It is also a positional value system, in which each octal digit has its own value or weight expressed as a power of 8.

Positional values in octal system.

$(462.378)_{10} = (\quad . \quad)_8$

4	6	2	.	3	7	8
8^2	8^1	8^0	(octal point)	8^{-1}	8^{-2}	8^{-3}
weights				weights		

octal representation of decimal number

Decimal Number	0	1	3	4	5	6	7	8	9	10	11	12	13	14	15
Octal Number	0	2	3	4	5	6	7	10	11	12	13	14	15	16	20

Octal to Decimal and Decimal to octal conversion:

(i) Octal to Decimal:-

To convert the integral part each octal digit left of octal point is multiplied by its weight $(8^0, 8^1, 8^2)$ towards the left of octal point and then added while for the fractional part, each octal digit is multiplied by $8^{-1}, 8^{-2}$ towards the right of octal point.

$(308\cdot128)_8 = (\qquad)_8$

$3\times8^2 + 0\times8^1 + 8\times8^0 + 1\times8^{-1} + 2\times8^{-2} + 8\times8^{-3}$

$= 192 + 0 + 8 + 0\cdot125 + 0\cdot1325 + 0\cdot15625$

$= (200\cdot171875)_8$

(ii). $(43\cdot215)_8 = (\quad\cdot\quad)_{10}$

$8^1 \quad 8^0 \quad 8^{-1} \quad 8^{-2} \quad 8^{-3}$

$4\times8 + 3\times1 + \frac{2}{8} + \frac{1}{16} + \frac{5}{512}$

$= 32 + 3 + \cdot25 + 0\cdot015625 + 0\cdot0097656$

$= (35\cdot2753)_{10}$

$(43\cdot215)_8 = (35\cdot2753)_{10}$.

(iii) $(6327\cdot4051)_8 = (3287\cdot5100098)_{10}$.

(iv) $(305\cdot371)_8 = (197\cdot4799)_{10}$

Decimal to octal

Divide the integral part by 8 and multiply the fractional part by 8.

	Quotient	Remainder	
$\frac{247}{8} =$	30	7	— 1st (LSB).
$\frac{30}{8} =$	3	6	— 2nd.
$\frac{3}{8} =$	0	3	— Last (MSB).

$(247)_{10} = (367)_8$

Example (i) $(214 \cdot 98)_{10} = (\quad \cdot \quad)_8$

		Qu.	Remainder			carry
$\frac{214}{8}$	=	26	6 — LSB	$\cdot 98 \times 8$	= 7·84	7 — MSB
$\frac{26}{8}$	=	3	2	$0 \cdot 84 \times 8$	= 6·72	6
$\frac{3}{8}$	=	0	3 — MSB.	$0 \cdot 72 \times 8$	= 5·76	5
				$0 \cdot 76 \times 8$	= 6·08	6
				$0 \cdot 08 \times 8$	= 0·64	0

$(214 \cdot 98)_{10} = (326 \cdot 76560)_8$

(ii). $(3287 \cdot 6875)_{10} = (\quad \cdot \quad)_8$

		Q.	R.			C
$\frac{3287}{8}$	=	410	7 - LSB	$0 \cdot 6875 \times 8$	= 5·500	5 — MSB
$\frac{410}{8}$	=	51	2	$0 \cdot 500 \times 8$	= 4·000	4 — LSB
$\frac{51}{8}$	=	6	3			
$\frac{6}{8}$	=	0	6 — MSB			

$(3287 \cdot 6875)_{10} = (6327 \cdot 54)_8$

(iii). $(203 \cdot 97)_{10} = (\quad \cdot \quad)_8$

		Q.	R.			C
$\frac{203}{8}$	=	25	3 — LSB	$0 \cdot 97 \times 8$	= 7·76	7 (MSB)
$\frac{25}{8}$	=	3	1	$0 \cdot 76 \times 8$	= 6·08	6
$\frac{3}{8}$	=	0	3 - MSB.	$0 \cdot 08 \times 8$	= 0·64	0
				$0 \cdot 64 \times 8$	= 5·12	5
				$0 \cdot 12 \times 8$	= 0·96	0 — LSB

$(203 \cdot 97)_{10} = (313 \cdot 76050)_8$

Octal to Binary:

Base = 8 = $2^{(3)}$ octal numbers when converted into binary will have 3 bits. For conversion each octal digit is replaced by its 3 bit binary digit.

Examples:

(i) $(5321)_8 = (\quad)_2$.

101 011 010 001

$(5321)_8 = (101\,011\,010\,001)_2$.

(ii) $(56 \cdot 27)_8 = (\quad \cdot \quad)_2$.

$(101\ 110 \cdot 010\ 111)_2$

Ans: $(56 \cdot 27)_8 = (101110 \cdot 010111)_2$.

Binary to octal

Conversion of a binary number to an octal number is the reverse of the octal to binary conversion.

~~Ro~~ $(\quad \cdot \quad)_2$

Start making the group of 3 bits from ~~left to right~~ Right to left of the octal point for ~~integ~~ integer part. For fractional part make the group of 3 bits from left to right of the octal point.

If there are not three bits available for completing the group, add one or two zero to make the complete group. This will not affect the value of binary number.

(i) $(1001110 \cdot 10100110)_2 = (\quad \cdot \quad)_8$

Integer part.

Make a group of 3 bits from left to right of radix point

1 $\overline{001}$ $\overline{110}$

Two groups are complete. so make the group add two zero's on the left of 1.

$\overline{001}$ $\overline{001}$ $\overline{110}$ $= (116)_8$

Two Zero added.

1 1 6

For fractional part move right to left of radix point.

$0.\overline{101}\ \overline{001}\ 10$, Two groups of 3 bits are complete. to make the 3^{rd} group add Zero on the right hand side of 10 so the new group formation is as under

$0.\overline{101}$ $\overline{001}$ $\overline{100}$ extra zero added

↓ ↓ ↓

0.5 1 4

$(1001110 \cdot 1100110)_2 = (101 \cdot 514)_8$

(ii) 101101110 · 110010100 11

Make the group of three bits

$\overline{001}$ $\overline{011}$ $\overline{011}$ $\overline{110}$ · $\overline{110}$ $\overline{010}$ $\overline{100}$ $\overline{110}$

Add Zero added Zero

1 3 3 6 · 6 2 4 6

Ans: $(1336 \cdot 6246)_8$.

Binary-to-hexadecimal conversion

To convert the binary to hexadecimal number we take a group of 4-bits from left hand side of decimal point (integer part) and group of 4-bits from right hand side of decimal point (fractional part), in the binary number and write its hexadecimal equivalent. If group of 4 bits is not complete, add zero's to complete it.

(i) $(1001010)_2 = 100\ \overline{1010}$

Incomplete group, so add zero

$= \overline{0100}\ \overline{1010}$

$\quad\ \ 4\quad A$

$(1001010)_2 = (4A)_H$.

(ii) $(11010 \cdot 01101)_2 = (\quad \cdot \quad)_H$.

$1\ \overline{1010} \cdot \overline{0110}\ 1$

Incomplete group, add three zero.

Incomplete group, add three zero.

$\overline{0001}\ \overline{1010} \cdot \overline{0110}\ \overline{1000}$

Ans $\rightarrow (1A \cdot 68)_H$

(iii). $1010\underline{01}\ \underline{1010}\ \underline{1111} \cdot \underline{0001}\ \underline{1110}\ \underline{1011}\ 01$

$0010\quad 9\quad A\quad F \cdot 1\ E\ B\ 0100$

Incomplete group.

$(29AF \cdot 1EB4)_H$.

Hexadecimal-to-decimal conversion:

(i) Convert the hexadecimal number to binary and then convert from binary to decimal.

Example

$(1C)_{16} = \quad 1 \quad C$

$\quad 0001 \quad 1100 \quad = 2^2 + 2^3 + 2^4$

$2^7\, 2^6\, 2^5\, 2^4 \quad 2^3\, 2^2\, 2^1\, 2^0$

$= 4 + 8 + 16 = (28)_{10}$

(ii). To convert a hexadecimal number to its equivalent is to multiply the decimal value of each hexadecimal digit by its weight and then take the sum of these products. The weights of a hexadecimal number are increasing powers of 16 from right to ~~left~~ left. [for integer part]. For fractional part the weights of a hexadecimal number are decreasing powers of 16 from left to right

(i) Example: $(E5)_{16} = (\quad)_{10}$

$(E \times 16^1 + 5 \times 16^0) = E \times 16 + 5 \times 1$

$= 14 \times 16 + 5 + 1 = 224 + 5 = (229)_{10}$

(ii) $(3A \cdot 2F)_{16} = (\quad)_{10}$

$= 3 \times 16^1 + A \times 16^0 + 2 \times 16^{-1} + F \times 16^{-2}$

$= 3 \times 16 + 10 \times 1 + \frac{2}{16} + \frac{F}{(16)^2} = 48 + 10 + \frac{2}{16} + \frac{15}{16}$

$= 48 + 10 + .0125 + .05859$

$= (58.18359)_{10}$

Decimal-to-Hexadecimal conversion.

Repeated division of a decimal number by 16 will produce the equivalent hexadecimal number, formed by remainders of the division. The first remainder produced is the lest significant digit ~~bit~~ (LSD. Last remainder produced is the most significant digit. (MSD).

When a quotient has a fractional part, the fractional part is multiplied by the divisor to get the remainder/carry. First carry is then MSB and last carry is LSB.

Example:- $(675.625)_{10} = (\quad . \quad)_{16}$

	Q.	R.
$\frac{675}{16} =$	42	3 – LSB
$\frac{42}{16} =$	2	10 = A
$\frac{2}{16} =$	0	2 – MSB

$0.625 \times 16 = 10.000$ C: $10 = A$.

stop when fractional part is zero.

Ans:- $(2A3.A)_{16}$

(ii). $(342.56)_{10} = (\quad . \quad)_{16}$

	Q	R
$\frac{342}{16} =$	21	6 – LSB
$\frac{21}{16} =$	1	5
$\frac{1}{16} =$	0	1 – MSB

		C
0.56×16	$= 8.96$	8 – MSB
0.96×16	$= 15.36$	15 = F
0.36×16	$= 5.76$	5
0.76×16	$= 2.50$	2 – LSB

Ans → $(156.8F52)_{16}$.

Octal to Hexadecimal conversion.

For this conversion first octal is converted into binary, then make the group of 4 bits.

Example:- $(247.36)_8$ =

Each digit is converted into binary equivalent.

$\to$ 010 100 111 . 011 110

(ii) Make a group of four bits.

0 $\overline{1010}$ $\overline{0111}$. $\overline{0111}$ 10

(iii) Add zero to complete the group

$\overline{0000}$ $\overline{1010}$ $\overline{0111}$. $\overline{0111}$ $\overline{1000}$ Added zero.

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

0 A 7 7 8.

Ans $\to$ $(0A7.78)_H$.

Hexadecimal to octal conversion:

$(A72E.BF85)_{16}$

(i) convert the hexadecimal ~~convers~~ digit into its binary equivalent.

A	7	2	E	.	B	F	8	5
1010	0111	0010	1110	.	1011	1111	1000	0101

(ii) Make a group of three bits.

1 $\overline{010}$ $\overline{011}$ $\overline{100}$ $\overline{101}$ $\overline{110}$. $\overline{101}$ $\overline{111}$ $\overline{111}$ $\overline{000}$ $\overline{010}$ 1

(iii) Add zero to make a group of three ones.

$\overline{001}$ $\overline{010}$ $\overline{100}$ $\overline{101}$ $\overline{110}$. $\overline{101}$ $\overline{111}$ $\overline{111}$ $\overline{000}$ $\overline{010}$ $\overline{100}$ Added zero

(iv) Write octal equivalent.

$(12456.577024)_8$.

Hexadecimal System

It has 16 symbols, so the base is 16.

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.]
Symbols used in Hexadecimal System.

Is is also a positional value system just like above mentioned number systems.

Positional values of Hexadecimal System.

5	B	3	•	3	A	9.
16^2	16^1	16^0	Hexadecimal point	16^{-1}	16^{-2}	16^{-3}.

Hexadecimal-to-binary conversion:

16 base to base 2 conversion

$16 = 2^n$ n = no of bits in binary system

$n = 4$.   so equivalent of hexadecimal ~~eg number~~ digit in binary must have four bits.

example   5A   5 = 0101
A = 1010.

To convert a hexadecimal number to binary number, convert each hexadecimal digit to its 4-bit binary equivalent.

$(E5C3)_{16} =$ E → 1110   5 → 0101   C → 1100   3 → 0011

$$(E5C3)_{16} = (1110\ 0101\ 1100\ 0011)_2.$$

Octal to Hexadecimal conversion.

For this conversion first octal is converted into binary, then make the group of 4 bits.

Example:- $(247.36)_8$ =

010 100 111 . 011 110

Each digit is converted into binary equivalent.

(ii) Make a group of four bits.

0 1010 0111 . 0111 10

(iii) Add zero to complete the group

0000	1010	0111	.	0111	1000
↓	↓	↓		↓	↓
0	A	7		7	8.

Added zero.

Ans → $(0A7.78)_H$.

Hexadecimal to octal conversion:

$(A72E . BF85)_{16}$

(i) convert the hexadecimal ~~conversi~~ digit into its binary equivalent.

A	7	2	E	.	B	F	8	5
1010	0111	0010	1110	.	1011	1111	1000	0101

(ii) Make a group of three bits.

1 010 011 100 101 110 . 101 111 111 000 010 1

(iii) Add zero to make a group of three ones.

001 010 100 101 110 . 101 111 111 000 010 100

Added zero

(iv) Write octal equivalent.

$(12456.577024)_8$.