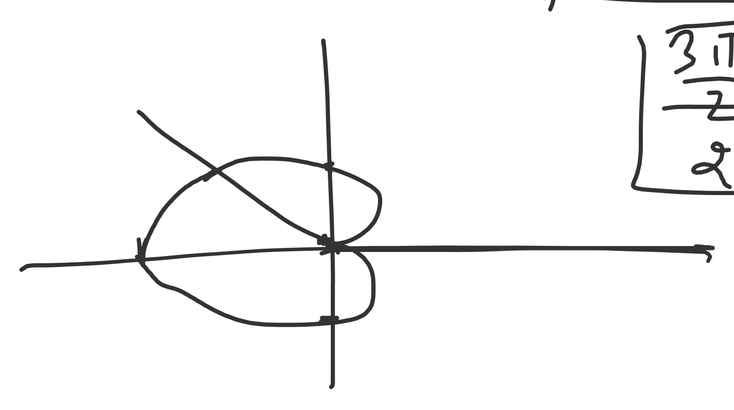
12.3 exercise and 12.4

Thursday, 15 October 2020 11:11 AM

Q5 $r = 2(1-\cos\theta)$

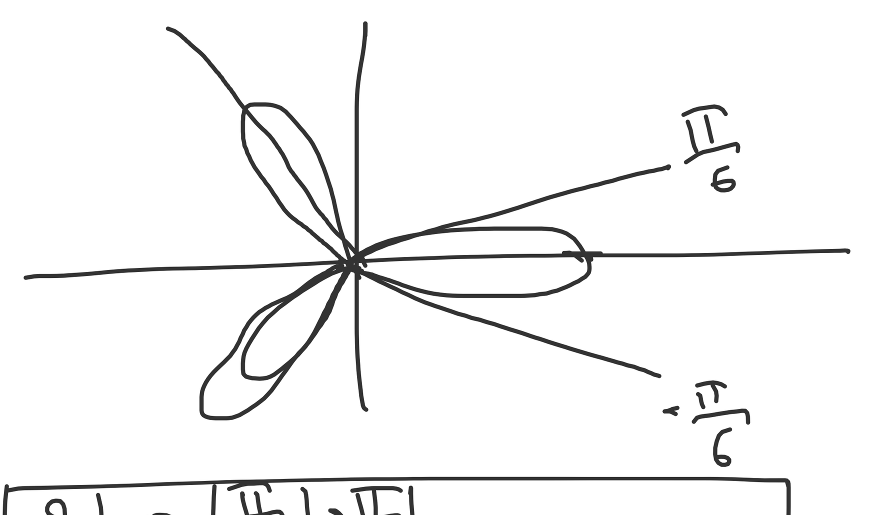
$$\int_0^{2\pi}\int_0^{2(1-\cos\theta)} r\,dr\,d\theta$$



θ	0	$\pi/2$	π
r	0	2	4

$\frac{3\pi}{2}$	2π
2	0

7 $r = 4\cos 3\theta$

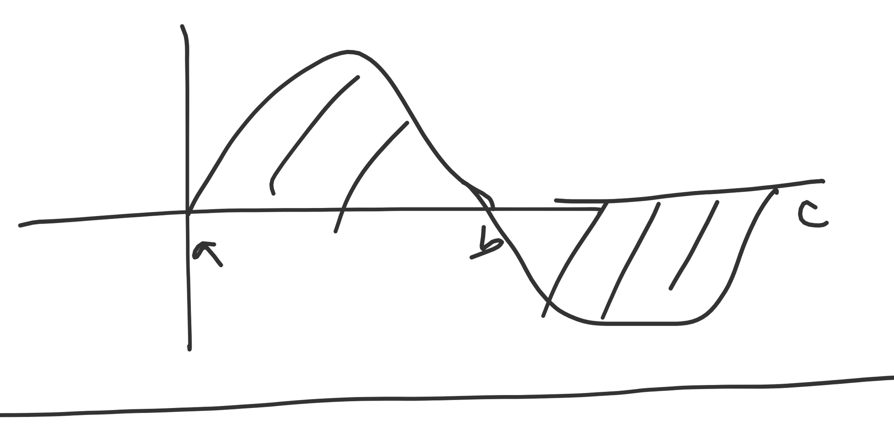


θ	0	$\frac{\pi}{6}$	$-\frac{\pi}{6}$	
3θ	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	
r	4	0	0	

$$A = 3\int_{-\pi/6}^{\pi/6}\int_0^{4\cos 3\theta} r\,dr\,d\theta = 6\int_0^{\pi/6}\int_0^{4\cos 3\theta} r\,dr\,d\theta$$

$$\int_a^b \underline{f(x)}\,dx$$

$f \geq 0$



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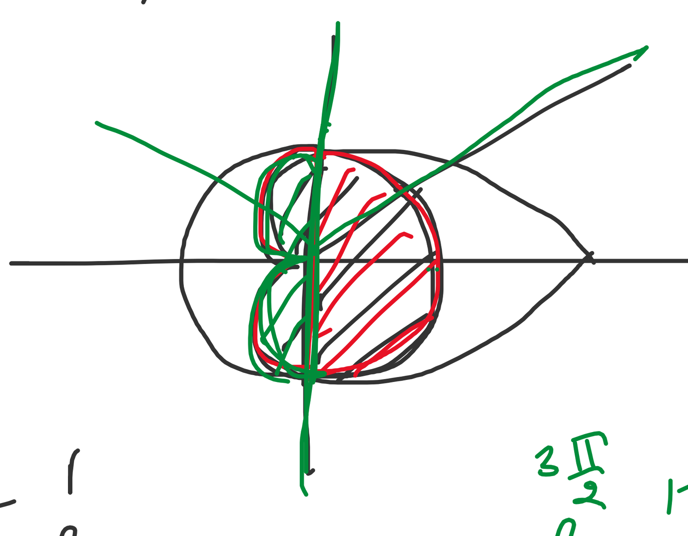
11 $r = 1$, $r = 1+\cos\theta$



Red part = is part of circle
green area = part of cardiode

$$\int_{-\pi/2}^{\pi/2}\int_0^1 r\,dr\,d\theta + \int_{\pi/2}^{3\pi/2}\int_0^{1+\cos\theta} r\,dr\,d\theta$$

$$= 2\int_0^{\pi/2}\int_0^1 r\,dr\,d\theta + 2\int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta$$

19 $$\int_0^2\int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dx\,dy$$

12.4 Triple Integral

$$\iiint_D f(x,y,z)\,dx\,dy\,dz$$

$\boxed{\text{over rectangular box}}$

$D = B : a \leq x \leq b,\ c \leq y \leq d,\ r \leq z \leq s$

$$\iiint_D f(x,y,z)\,dV = \int_r^s\int_c^d\int_a^b f(x,y,z)\,dx\,dy\,dz$$

Int. w.r.t x, y, z can be in any order

$$= \int_a^b\int_r^s\int_c^d f(x,y,z)\,dy\,dz\,dx$$

Ex 1 $\iiint_B z^2 y e^x\,dV$ $B: 0 \leq x \leq 1,\ 1 \leq y \leq 2,\ -1 \leq z \leq 1$

$$\int_{-1}^1\int_1^2\int_0^1 z^2 y e^x\,dx\,dy\,dz$$

$$= \int_{-1}^1\int_1^2 z^2 y\, e^x\Big|_0^1\,dy\,dz$$

$$= (e-1)\int_{-1}^1 z^2\frac{y^2}{2}\Big|_1^2\,dz$$

$$= (e-1)\int_{-1}^1\left(\frac{4-1}{2}\right)z^2\,dz$$

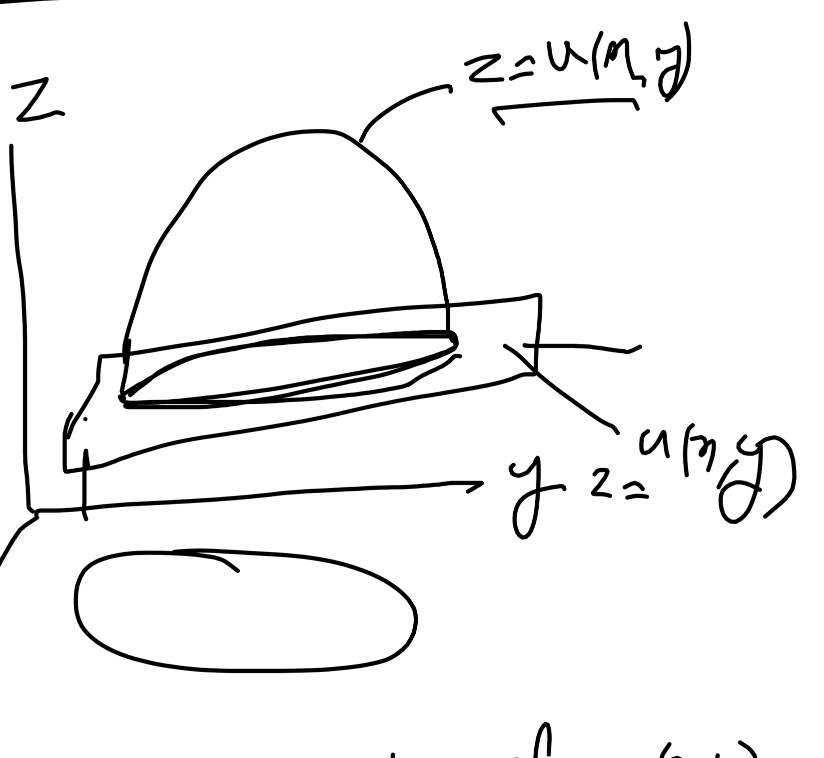
$$= \frac{3}{2}(e-1)\,\frac{z^3}{3}\Big|_{-1}^1$$

$$= \frac{3}{2}(e-1)\left(\frac{1+1}{3}\right)$$

$$= \boxed{e-1}$$

Triple Integration over Z-simple region

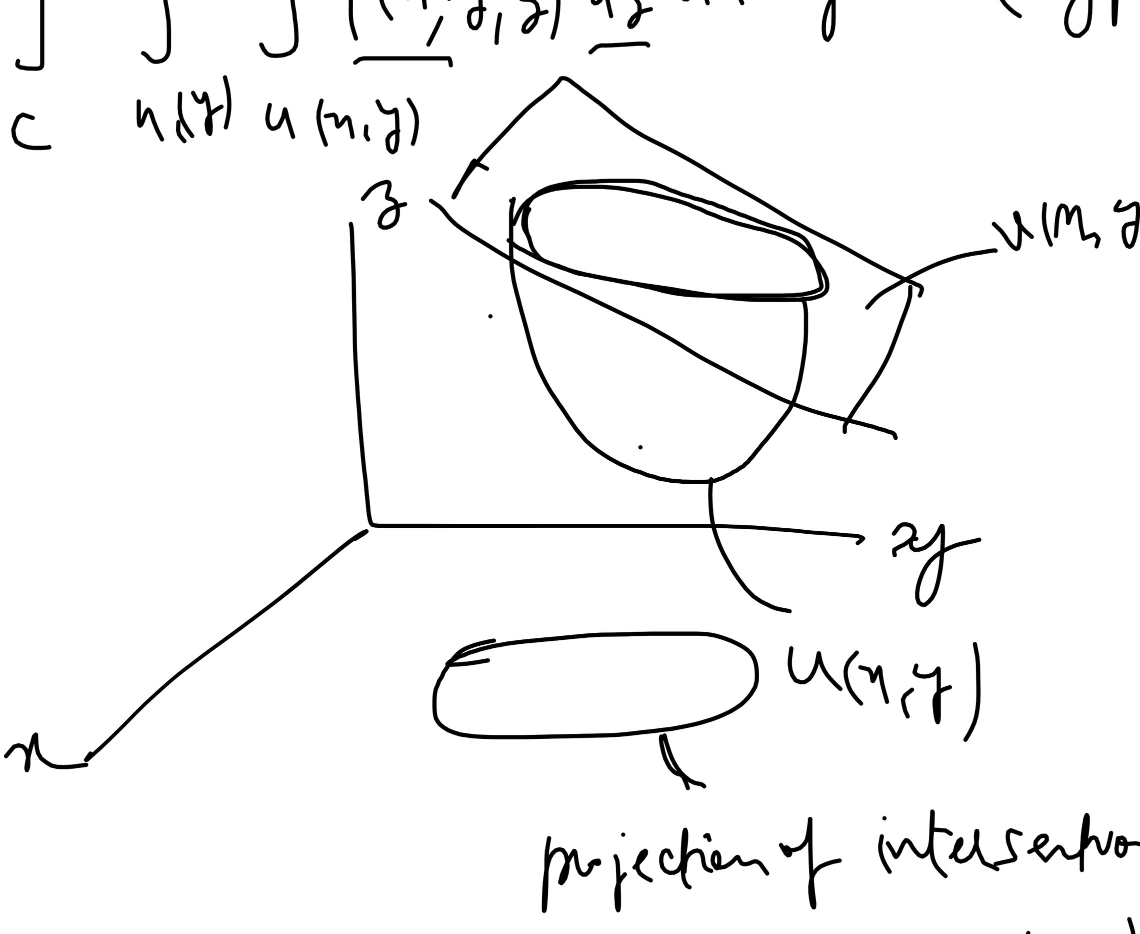
bdd below by surface
$z = u(x,y)$
bdd above by a surface
$z = v(x,y)$



and projection on xy plane of intersection of $u(x,y)$ and $v(x,y)$ is a region, type 1 or type 2.

$$\iiint f(x,y,z)\,dV = \int_a^b\int_{g_1(x)}^{g_2(x)}\int_{u(x,y)}^{v(x,y)} f(x,y,z)\,dz\,dy\,dx \quad \text{(type 1)}$$

$$= \int_c^d\int_{h_1(y)}^{h_2(y)}\int_{u(x,y)}^{v(x,y)} f(x,y,z)\,dz\,dx\,dy \quad \text{(type 2)}$$



projection of intersection of
$z = u(x,y)$ & $z = v(x,y)$