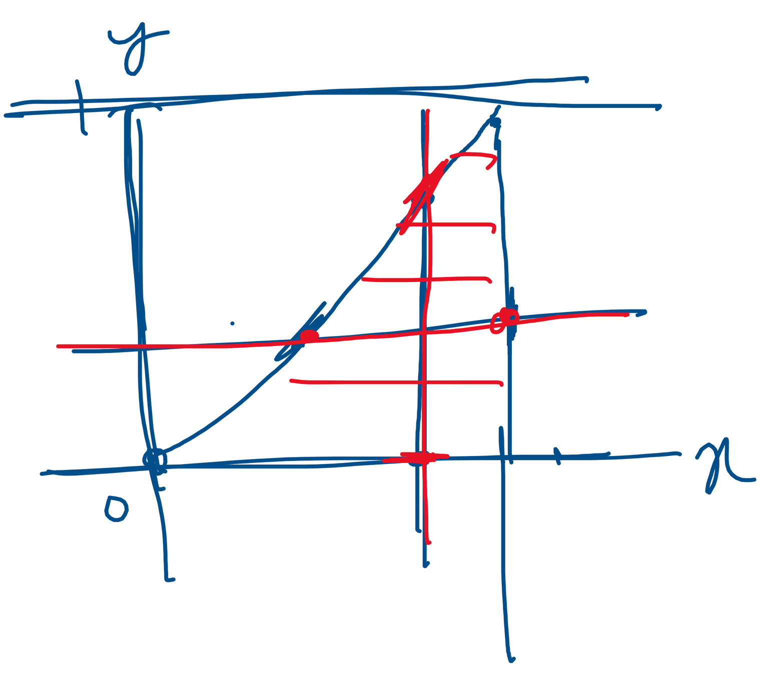
12.2

Friday, 9 October 2020 11:07 AM

Q6



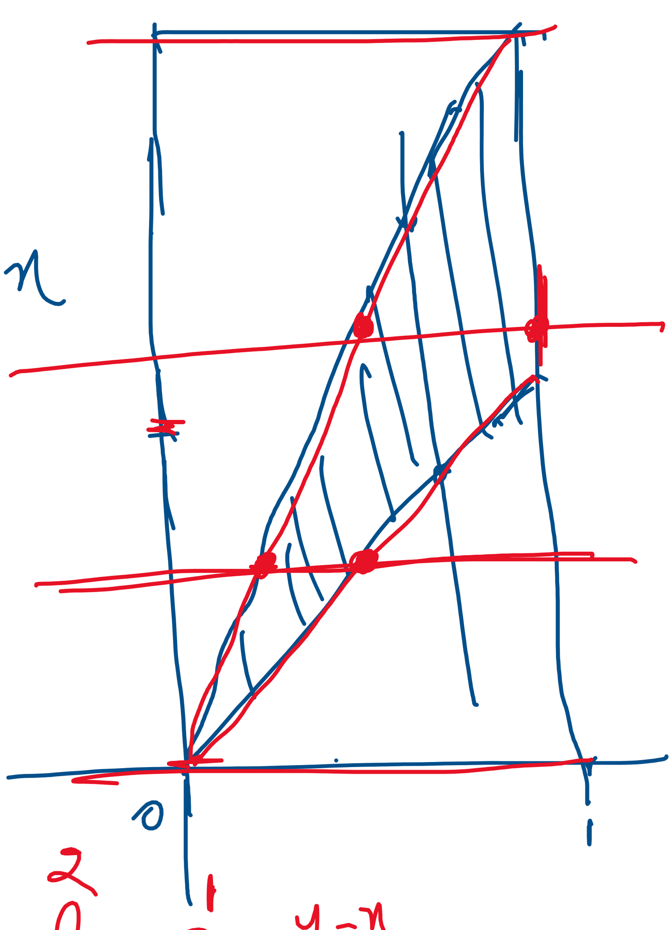
$x = \tan^{-1} y$

$y = \tan x$

$x = \frac{\pi}{4}$

$$\int_0^{\pi/4}\int_0^{\tan x} \sec x \, dy\, dx$$

(11) $$\int_0^1\int_x^{2x} e^{y-x}\, dy\, dx$$

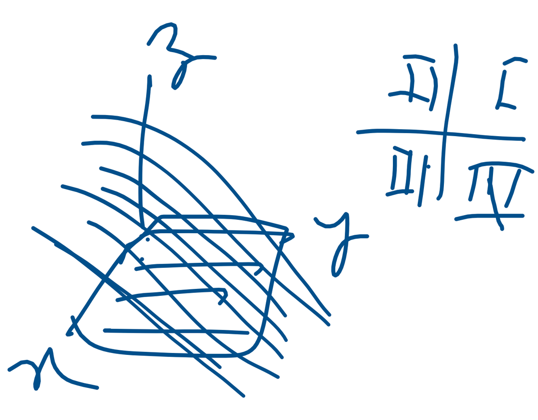


Reverse

$$\int_0^2 \int e^{y-x}\, dx\, dy$$

$$\int_0^1\int_{y/2}^{y} e^{y-x}\, dx\,dy + \int_1^2\int_{y/2}^{1} e^{y-x}\, dx\, dy$$

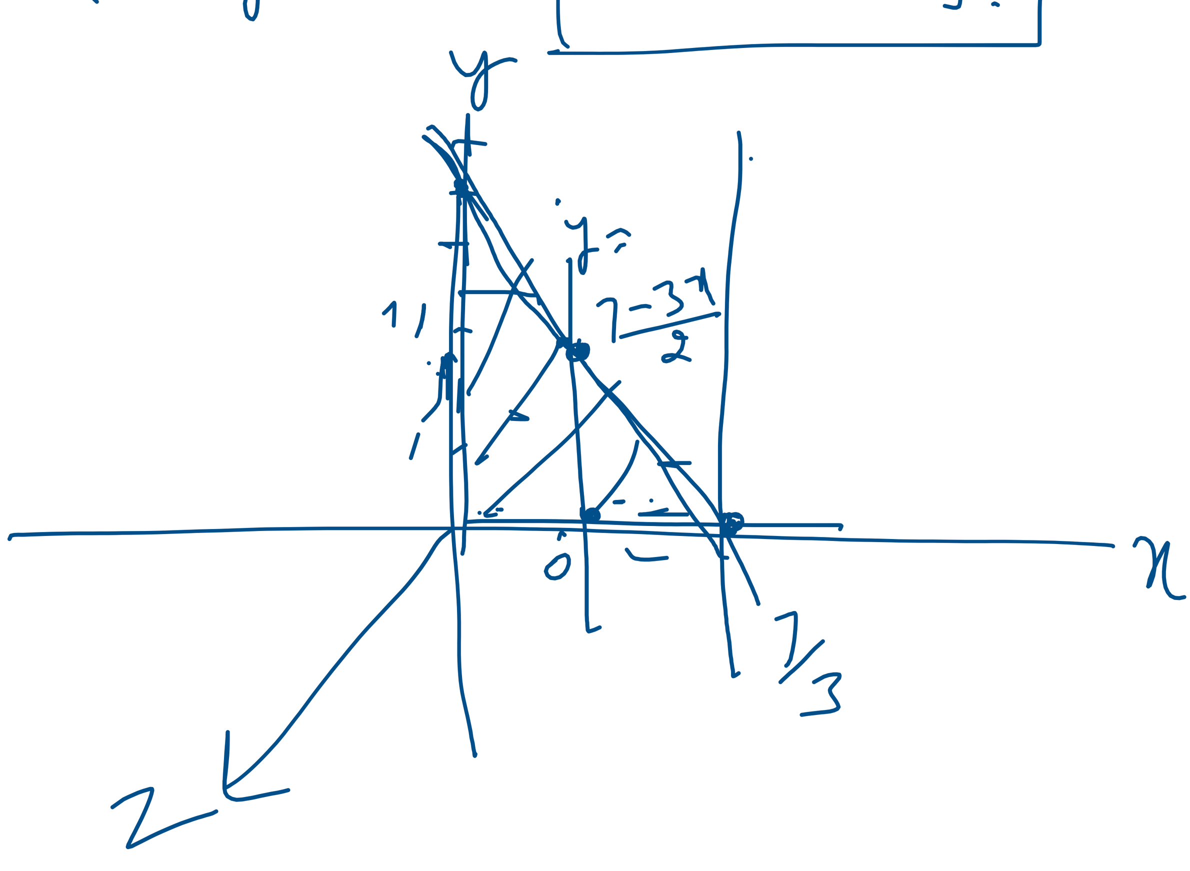
(14) $x>0, y>0, z>0$



$\boxed{z = 7-3x-2y}$ ✓

$= f(x,y)$

$7-3x-2y=0 \Rightarrow \boxed{3x+2y=7.}$ ✓



$\boxed{z = 7-3x-2y}$ $x\geq 0, y\geq 0, z\geq 0$

$x=0, y=0, z=0$

$f = 7-3x-2y$ ✓

$\boxed{3x+2y=7, \; x=0, y=0}$ ✓

$$\int_0^{7/3}\int_0^{\frac{7-3x}{2}} (7-3x-2y)\, dy\, dx$$

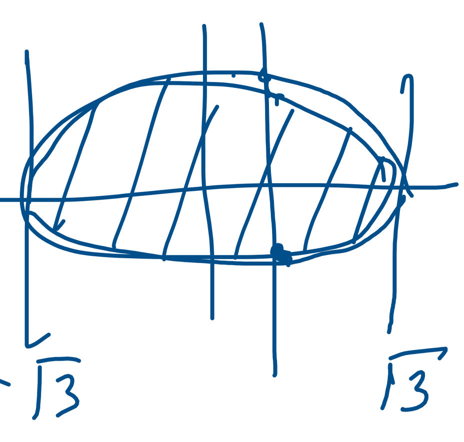
(15) $z = 6-2x^2-3y^2$ (1), $z=0$

$f(x,y) = 6-2x^2-3y^2$

Put $z=0$ in (1)

$6-2x^2-3y^2=0$

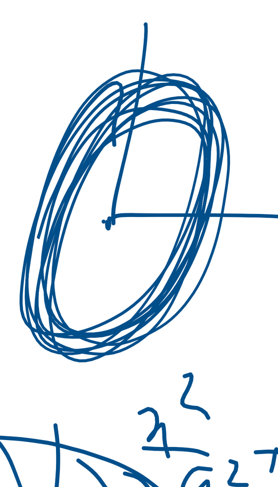
$\boxed{2x^2+3y^2=6}$



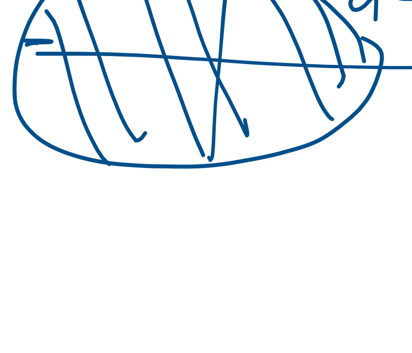
$$\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{\frac{6-2x^2}{3}}}^{\sqrt{\frac{6-2x^2}{3}}} (6-2x^2-3y^2)\, dy\, dx$$

$$= 2\times 2\int_{0}^{\sqrt{3}}\int_0^{\sqrt{\frac{6-2x^2}{3}}} (6-2x^2-3y^2)\, dy\, dx$$

(16) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$



$z = \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Put $z=0$

domain $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$2\int_{-a}^{a}\int_{-\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}}^{\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}} \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}\, dy\, dx$$

= Required volume

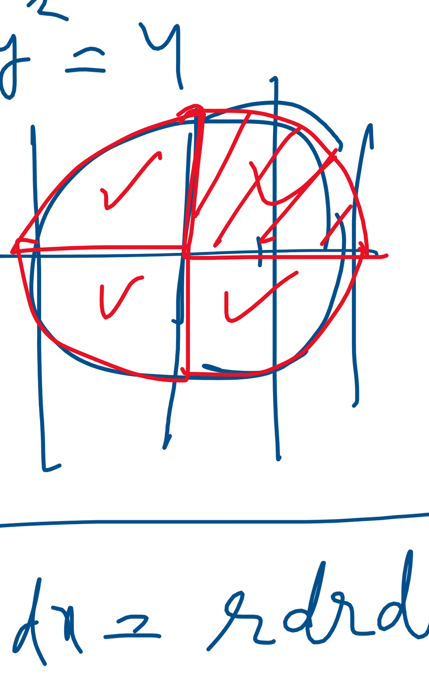
12.3 double Integration in Polar Coordinates

(x,y) (r,θ)

$x = r\cos\theta$, $y = r\sin\theta$

$r^2 = x^2+y^2$, $\theta = \tan^{-1}\frac{y}{x}$

$\iint (x^2+y^2+1)\, dA$, $x^2+y^2=4$

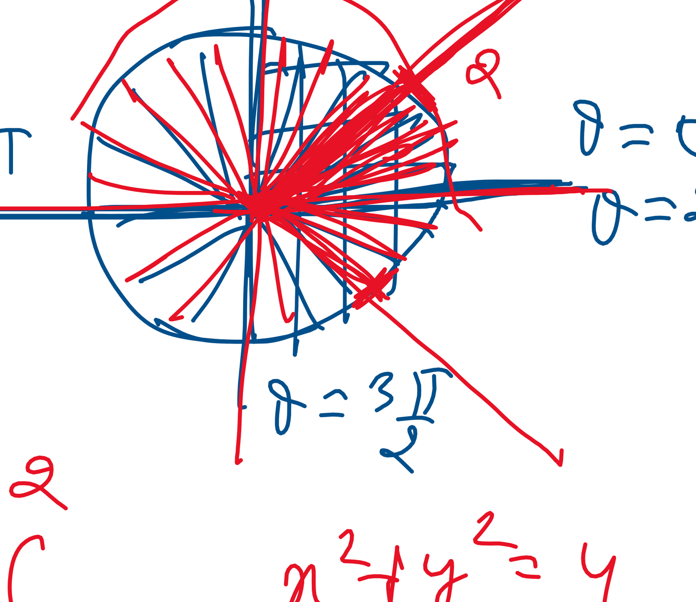


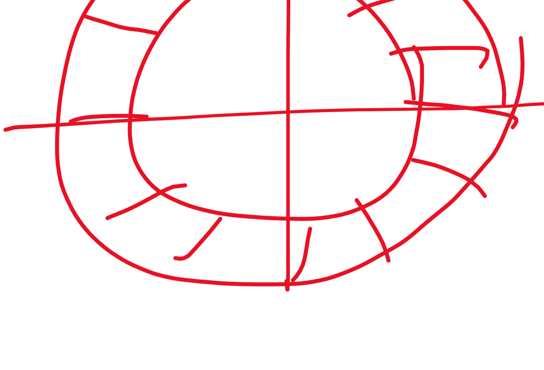
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2+y^2+1)\, dy\, dx$$

$\boxed{dy\,dx = r\,dr\,d\theta}$ ✓

$$\iint (r^2+1)\, r\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{2} (r^2+1)\, r\, dr\, d\theta$$





$$\int_1^2$$

$x^2+y^2=4$

$r^2=4$

$\boxed{r=2}$