Ring: Notation : $(R, +, \cdot)$

A ring R is a set with two binary operations, (addition and multiplication) such that R satisfies the following properties.

1. $a+b = b+a \quad \forall\, a, b \in R$
2. $(a+b)+c = a+(b+c) \quad \forall\, a, b, c \in R$
3. There is an ~~ad~~ additive identity (denoted by 0) in R such that
   $a+0 = a \quad \forall\, a \in R$
4. There is additive inverse $(-a)$ of every element $a \in R$ such that
   $a + (-a) = 0$
5. $a\cdot(b\cdot c) = (a\cdot b)\cdot c$
6. Distributive laws:
   $a\cdot(b+c) = a\cdot b + a\cdot c$ and $(b+c)\cdot a = b\cdot a + c\cdot a$

additive identity is called "zero element" of R.

If multiplicative identity exists, it is called unity of ring R.

Alternative

$(R, +, \cdot)$ is a ring if

① $(R, +)$ is an abelian group

② $(R, \cdot)$ is associative

③ Distributive laws follows in R

$a \cdot (b + c) = a \cdot b + a \cdot c$

and $(b + c) \cdot a = b \cdot a + c \cdot a$

~~Factor~~:

~~a ...~~ **Commutative ring:**

If $(R, +, \cdot)$ is a ring and.

If $a \cdot b = b \cdot a \quad \forall \; a, b \in R$,

then R is called Commutative ring.

~~Fact~~ If commutative ring R have units, then it is called Commutative ring with units.

Example:

① $(\mathbb{Z}, +, \cdot)$ is a ring

② $(\mathbb{Q}, +, \cdot)$ is a ring

③ $(\mathbb{R}, +, \cdot)$ is a ring

④ $(\mathbb{Z}_n, +_n, \cdot_n)$ is a ring.

⑤ $M_2(\mathbb{R})$ is a ring $\quad M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \begin{array}{l} a, b, c, d \in \mathbb{R} \\ ad - bc \neq 0 \end{array} \right\}$