ELECTROSTATICS

* Electrostatics deals with the physics of stationary electric charges i.e charges at rest called electrostatics.

* Duality of charge: One fundamental property of electric charge is its existence in two varieties i.e. +ve & -ve.
All members of one class repel each other, while that of diff. classes attract each other.

* What we call -ve charge could just as well have been called +ve & vice-versa. The choice was historical accident. The charge of ordinary e- has nothing intrinsically -ve about it. Benjamin Franklin, a pioneer of electrostatics gave the name of "+ve charge" to the charge on an glass rod rubbed with slik & "-ve charge" to that of an ebonite rod rubbed with fur.

* Quantization of charge: # The atomic theory of matter has shown that fluids (water & air) are not cts; but are made up of atoms.
Milikan's oil drop expt shows that there is exact equality in the charges carried by the charged drops. (ne). n: integer; e: electron charge

#Any physically existing charge in the universe is an integral multiple of e. So that charge exist in discrete packets rather than in cts amount. This is known as charge quantization.

There are two kinds of charge quanta: One is that on e- & other is that on proton. (Both have same magnitude).

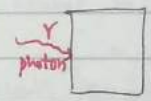
* **Conservation of charge**: when a glass rod is rubbed with silk, a +ve charge appears on the rod and a -ve charge charge of equal magnitude appears on the silk. So that rubbing doesn't create charge but merely transfers charge from one object to other, disturbing slightly the electrical neutrality of each.

OR

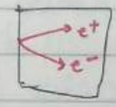
Total charge in an isolated system remains constt. By isolated we mean that no matter is allowed to enter or to leave the system. The charges can be neither created nor destroyed. An equal qty of each sign (+ve & -ve) is simultaneously produced & disappeared. This is known as Conservation of Charge.

Ex]:- Creation of a pair of e^- & positron when high energy photons enter a thin walled box in vaccum.

A pair creation event in which a high energy photon ends its existence with the creation of -ve e^- (electron) & +ve e^- (positron).



Thus two electrically charged particles are newly created but the net change in the total charge is zero.



n the "reverse process", when a e^- & e^+ combins gives photon. This process is known as "annihilation",

$\Rightarrow$ Confirms the conservation of charge.

Ex2:- Radioactive Decay:

$$_{92}U^{238} \longrightarrow {_{90}Th^{234}} + {_2He^4}$$

$$_{88}Ra^{226} \longrightarrow {_{86}Rn^{222}} + {_2He^4}.$$

In these processes, the total amount of charge present before & after disintegration is same.

Ex3:- Nuclear Rxns:-

Proton & α-particle rxn given below:-

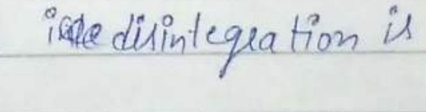
$$_{80}Hg^{200} + {_1H^1} \longrightarrow {_{79}Au^{197}} + {_2He^4}.$$

Again amount of charge before & after ~~the~~ disintegration is same.

COULOMB'S LAW OF FORCE:

Coulomb reported the results of a series of measurements on the dependence of the electrical force b/w the charged bodies at rest upon the distance b/w them & the magnitude of charges.

$$f \propto \frac{q_1 q_2}{r^2}.$$



Coulomb gave a law, which can be stated as :-

1) The force ~~b~~ which two Charged bodies 1 & 2, whose dimensions are small compared to their separation (i.e. pt. charges), exert on one another has a dir^n of the line joining the charges & is inversely prop to the square of their sepr^n.

i.e. $F \propto \frac{1}{r^2}\hat{r}$.

$\hat{r} = \frac{\vec{r}}{r}$ is the unit vector in the dir^n from 1 to 2. if the force F is acting on charge 2 due to 1.

2) F is prop. to the product of the charges on these charged bodies : $F \propto q_1 q_2$.

The force act as repulsive force when q_1 & q_2 are of same sign
" " " attractive " " " " " opp. sign.

F can be written F_{21} (force on 2 " of 1)

$$\Rightarrow \boxed{F_{12} = -F_{21}}$$

F_{12} → force on 1 " of 2.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \quad \text{—— (A)}$$

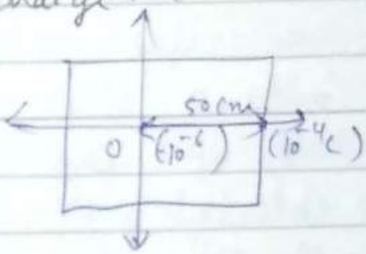
$\frac{1}{4\pi\epsilon_0}$ → constt of proportionality.

Permitivity of free space ← $\epsilon_0 = 8.85418 \times 10^{-12}\ C^2/N.m^2$

$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ N\,m^2/C^2$.

$1C = 3 \times 10^9$ e.s.u of charge
$1N = 10^5$ dynes.

Q:- A -ve pt. charge of $10^{-6}C$ is situated in air at the origin of a rectangular co-ordinate system. A second -ve pt. charge of $10^{-4}C$ is situated on the +ve axis at a distance of 50 cm from the origin. What is the force on the 2nd charge. (which dirn).



Sol:-

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$= \frac{9 \times 10^9 \times (-10^{-6}) \times (-10^{-4})}{(0.5)^2}$$

$$= (3.6)\hat{i} \ N$$

Thus the force of 3.6 N in the +ve x-dirn on the 2nd charge

PRINCIPLE OF SUPERPOSITION OF CHARGES

If more than two charges are present, the force exerted on any one say q_1 by all the others q_2, q_3, q_4 ---- can be given by adding vectorially all the forces obtained by using eqn (A)

$$f_1 = f_{12} + f_{13} + F_{14} + ----$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + ----\right]$$

Unit vectors $\hat{r}_{12}$ & $\hat{r}_{13}$ have the dirns of the lines from q_2 to q_1 & q_3 to q_1 respectively.

$$\Rightarrow \boxed{F_1 = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_1 q_i}{r_{1i}^2} \hat{r}_{1i}}$$

This is just the superposition principle for the forces.

* Consider a situ^n when the charge is spread over a region instead of being concentrated at particular pts.
To calculate F_1, let us consider the charge as made up of small charge elements & then the force due to each element dq is added vectorially.

As the dist^n is cts, $\therefore \Sigma \longleftrightarrow \int$.

$$\Rightarrow F_1 = \frac{1}{4\pi\epsilon_0} \int \frac{q_1 dq}{r^2} \hat{r} = \frac{q_1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}.$$

where $\hat{r}$ is the variable unit vector that pts from each charge element dq toward the location of charge q.

* If the charge is distributed over a vol. $d\tau$, eq^n can be written a

$$F_1 = \frac{1}{4\pi\epsilon_0} \int q_1 \frac{\rho \, d\tau}{r^2} \hat{r}.$$

where ρ is the charge density.

Note: a) Coulomb's law applies to pt. charges. Pt. charge is one whose spatial dim. are very small compared with any other length used in the problem.
b) It also applies to elementary particles (e^- & p).

Comparison of Coulomb's Law with Newton's Gravitational Law

COULOMB's Law	Newton's Gravitational Law
(1) It is based on experimental observations.	(1) It is based on speculations concerning the fall of an apple towards earth.
(2) Force is directly prop. to the product of charges.	(2) Force is directly prop. to the product of masses.
(3) Force varies ~~directly~~ inversely as the square of the distance b/w two charges.	(3) Force varies inversely as the square of the distance b/w the masses.
(4) Force b/w the charges $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$	(4) Force b/w two masses: $F = G \frac{m_1 m_2}{r^2}$
(5) Force is repulsive when charges are of similar nature & attractive when are unlike.	(5) Force b/w two masses is always attractive.
(6) Force b/w any two charges is indep. of the presence of other charges.	(6) Force b/w any two masses is indep. of the presence of other masses
(7) ϵ_0 is constt. for a free space	(7) G is a universal constt.
(8) ϵ_0 depends upon units of force, charge & distance	(8) G also depends upon units of force, mass & length.

Limitations of the Coulomb's Law

The Coulomb's law fails to explain the stability of the nucleus, Since, in a nucleus, there are several protons, all having the positive charge. According to coulomb's law, they should repel each other. But actually, they don't push themselves apart, ∵ nucleus has a stable identity. Hence here Coulomb's law fails. However, in nuclei there are some non-electrical and non-gravitational forces, in addn to electrical forces called nuclear forces. Nuclear forces are stronger than electrical forces. Hence, they are able to hold the protons together, inspite of the electrical repulsion.

Coulomb's Law and Nuclear fission :

→ However, the nuclear forces are of short range. Their magnitude falls off much more rapidly than $1/r^2$ and this has an important consequence.

→ If a nucleus has too many protons in it gets too big and it will not stay together.

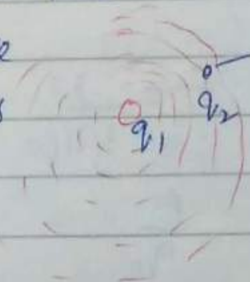
→ Nuclear forces act mainly b/w each proton or neutron and its nearest neighbour, while electrical forces act over larger distances, giving a repulsion b/w each proton and all of the others in the nucleus.

→ Greater is the no. of protons in the nucleus, stronger is the electrical repulsion and so it is easy to break such nucleus.

ELECTRIC FIELD

Gravitational forces are explained by assuming that every pt. in space near the earth is associated with a field, known as Gravitational field.
Similarly, the space surrounding a charged body is associated with a field known as Electric field.
The space where charge influence can be felt, is called site of electric field.

If a charge q_1 is placed at any pt, it sets up an E.F. in the space around itself. Field is indicated by the shaded region. If a charge q_2 is placed in the field region of q_1, former will experience the force F. In this way, the field plays an intermediary role in the forces b/w the charges.



* An E.F. is a physical qty which has diff. values at diff. pts in space. The force b/w two charges is considered as follows :-
1) Charge q_1 setup an E.F. in the space itself.
2) The field of q_1 acts on q_2 and q_2 experiences a force $\vec{F}$.

Thus, the force $\vec{F}$ on q_2 is due to action of the field of q_1 on q_2.
However we can imagine q_2 sets up a field and that this field acts on q_1, producing a force $-\vec{F}$ on q_1.

Electric Field Strength

A test charge q_0 is placed at any pt. in the region of any charge where we want to calculate the E.F. strength. If this charge experiences a force F, then the E.F. intensity or strength E at that pt. is defined as :-

$$E = \frac{F}{q_0} \quad (N/C).$$

The dir^n of E is the dir^n of F

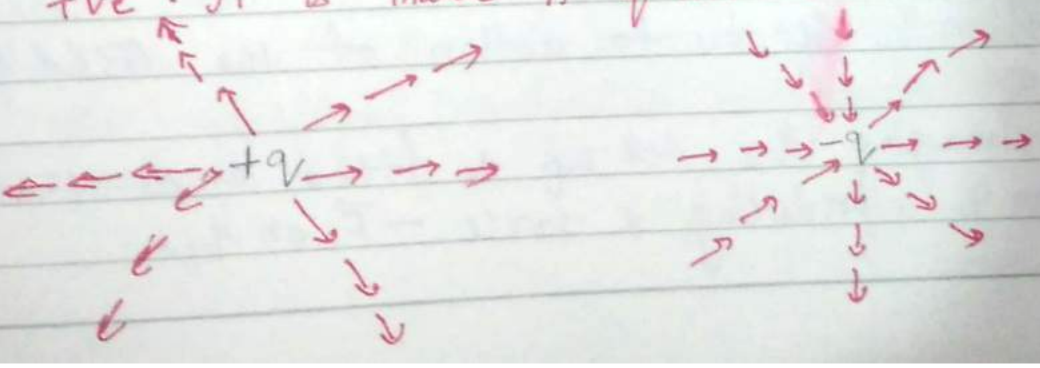
* $E = \lim_{q_0 \to 0} \frac{F}{q_0}$ (test charge as small as possible).

$$F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r}.$$

$$\frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} = \frac{-q}{4\pi\epsilon_0} \nabla\left(\frac{1}{r}\right).}$$

Its dir^n is radial from q pointing outward if q was +ve. It is inward if q was -ve.



FIELD LINES :

The line of force in an E.F is a curve s.t. the tangent at any pt. on it gives the dir^n of the resultant E.F. strength at that pt. This is also the path on which a test charge will tend to move, if free to move.

PROPERTIES OF ELECTRIC LINES OF FORCES :

① ~~Electric line of force originates or terminates~~

* When the vector E is known for all pts in an E.F, the field is completely specified. The dir^n & magnitude of field at any pt. indicate how a small charge q_0 would begin to move if put there. By following this dir^n from pt. to pt. curves are obtained. These are termed as LINES OF FORCE.

* So, lines of force in an E.F. is a curve s.t. the tangent at any pt. on it gives the dir^n of resultant E.F. strength at that pt. This is also the path on which a test charge will tend to move, if free to move.

* These lines are imaginary, their existence can be seen by sprinkling saw dust or gypsum salt in the electrostatic field. These particles acquire charges & place themselves along the lines of force.

Properties:

(1) No line of force originates or terminates in the space surrounding a charge. Every line of force is cts & smooth curve originating from a +ve charge and ending on a -ve charge.

2) The tangent to a line of force at any pt. gives the dirⁿ of E at that pt.

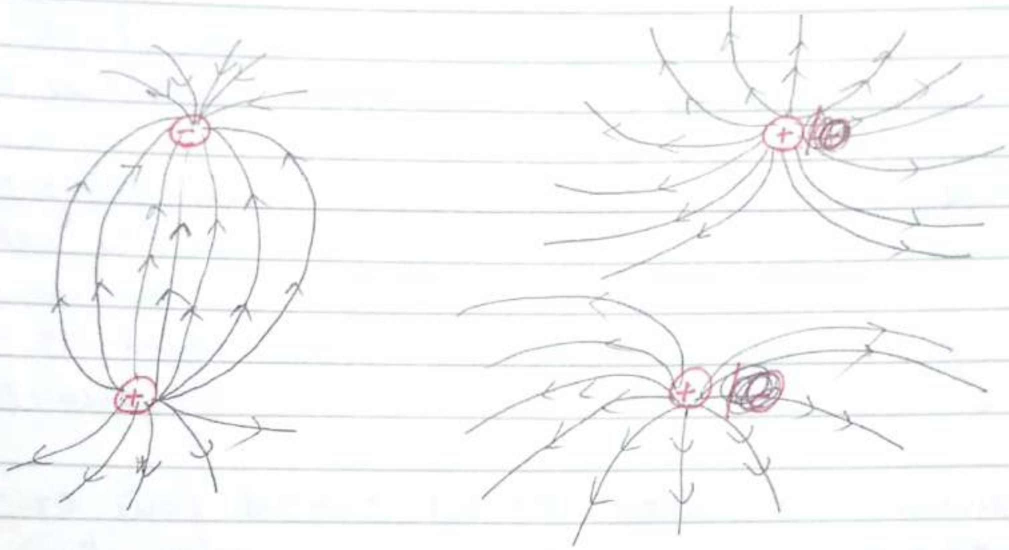
3) They don't pass but leave or end on a charged conductor normally when the charges on the conductor are in equillibm.

Suppose the lines of force are not $\perp$ to the conductor surface. In this sitⁿ the component of E.f. $\parallel$ " " surface would cause the e^- to move & would $\therefore$ gives rise to current. Since there is no current on the surface, hence lines of force are always $\perp$ to the conductor surface.

) Lines of force never intersect. If it happens then we say that at the pt. of intersection the electric intensity is zero; otherwise it would have to be tangetial to diff. curves at the same instance which is impossible.

) The no. per unit area crossing a surface at right angle to the field dirⁿ at every pt. is prop. to the electric intensity. Hence the lines of force are closely spaced where the intensity is large & are widely separated where intensity is large.

(6) They repel each other in the case of two like charges.



SIGNIFICANCE:

* To determine the no. of lines per unit charge, let us draw a spherical surface of radius r around a charge q at its centre.

E.F. at the surface, $E = \frac{q}{4\pi\epsilon_0 r^2}$.

As the density of lines of force is E, the total no. of lines of force originating at q & crossing the spherical surface $= 4\pi r^2 \cdot E = \frac{q}{\epsilon_0}$.

Hence, the no. of lines originating from a unit charge $= \frac{1}{\epsilon_0}$

* Electric lines of force are very useful as they give qualitative picture of E.F. distribn.

ELECTRIC FLUX :

* The relation b/w $\vec{E}$ & its source can be expressed in a simple way in terms of electric flux which is of very great use.

* In case of fluid, we call the net amount of fluid going out through the surface the "flux of velocity" through the surface.

* The flow through an element of surface is just equal to the comp. of the velocity ⊥ to the surface, times the area of surface.

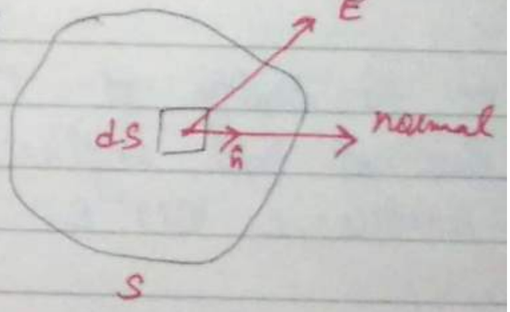
For an arbitrary closed surface, the net outward flow or flux is the avg. outdrawn normal component of the vector, times the area of surface.

$$\text{flux}, \phi = (\text{avg. normal comp}) \times (\text{Surface area}).$$

ELECTRIC FLUX OVER A CLOSED SURFACE :

Consider an infinitesimal surface element ds of surface S.

Electric flux due to this element = $d\phi_E$.



Acc. to above eqn :-

$$d\phi_E = E \cos\alpha \cdot ds$$

α is the ∠ b/w $\vec{E}$ & $\hat{n}$ drawn along a normal to the surface.

In vector form, $d\phi_E = \vec{E} \cdot \hat{n}\, ds$ $= \vec{E} \cdot \vec{ds}$ [$\hat{n}\, ds = \vec{ds}$ = vector area]

$$\phi_E = \oint d\phi_E = \oint_s \vec{E} \cdot \vec{ds}$$

* Gauss Law states that the flux of the E.F. $\vec{E}$ through any closed surface, that is the integral of $\vec{E} \cdot \vec{ds}$ over the surface $= \frac{1}{\epsilon_0} \times$ (total charge enclosed by the surface).

If charge enclosed by surface $= 0$

$$\phi_E = 0.$$

OR

The flux of the E.F. E through any closed surface is equal to the total net charge inside the enclosed vol. divided by ϵ_0.

Gauss Law in Differential Form :-

When a charge is distributed over a volume s.t. ρ is the density of charge. Then the charge enclosed by surface enclosing the volume is given by,

$$q = \int_v \rho\, dv.$$

$$\Rightarrow \oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} \cdot \int_v \rho\, dv.$$

Acc. to divergence theorem, (for converting the surface integral to vol. integral) :- $\oint_s \vec{E} \cdot \vec{ds} = \int_v \operatorname{div}\vec{E} \cdot dv.$

$$\Rightarrow \int_V \text{div}\,\vec{E}\cdot dv = \frac{1}{\epsilon_0}\int_V \rho\, dv$$

$\because$ it is true for any arbitrary vol, integrands must be equal,

$$\therefore \quad \text{div}\,\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}} \rightarrow \text{This is required differential form of Gauss Law.}$$

$\Rightarrow$ Divergence of E.F. is prop. to the charge at that pt.

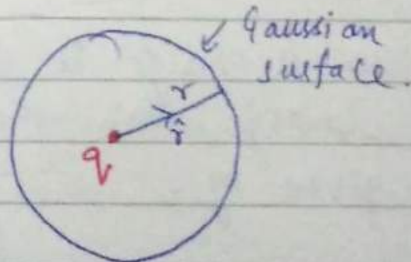
As in vaccum, $\vec{D} = \epsilon_0 \vec{E}$

$$\nabla\cdot\vec{D} = \epsilon_0\, \nabla\cdot\vec{E} = \rho$$

GAUSS LAW & COULOMB'S LAW

Let us apply Gass's law to an isolated pt. charge q as shown in fig. Construct a Gaussian surface taking r as radius, $\vec{E}$ is radial, on surface.

$$\vec{E}\cdot\vec{ds} = E\,ds$$



$$\epsilon_0 \oint \vec{E}\cdot\vec{ds} = \epsilon_0 \oint E\,ds$$

E is constt for all pts on the surface.

$$\epsilon_0 E \oint ds = q$$

$$\epsilon_0 E\, 4\pi r^2 = q$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

Let us put a 2nd +ve charge q_0 at the pt. where $\vec{E}$ is calculated.

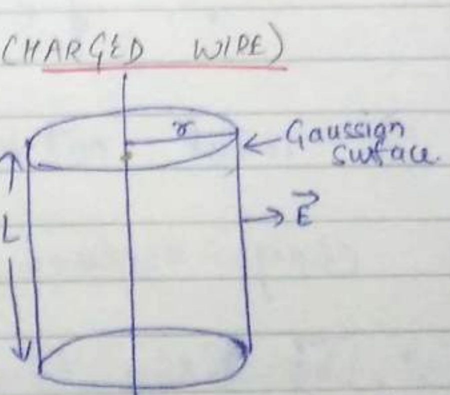
Force ~~acts~~ that acts on q_0 :- $\boxed{\vec{F} = \vec{E}\, q_0 = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r}}$ → Coulomb's Law

=> Gauss Law is equivalent to Coulomb's Law.

APPLICATIONS OF GAUSS LAW:

Gauss law doesn't provide expression for E but provide only its surface integral. To calculate E we must choose surface ~~integral~~ in such a way that the surface integral may be replaced by a product of which E is a factor.

FIELD DUE TO LINE CHARGE (UNIFORMLY CHARGED WIRE)



* This fig shows the sec^n of a uniformly charged wire (or rod). If the thickness is negligible, then it is characterised by the linear charge density λ (i.e. charge per unit length (c/m), which is constt for all pts in the line.

* From symmetry, the E.F. will be directed radially outward from the wire (i.e. radial and is indep. of both positions along the wire & angular pos^n around the wire.

* Let us calculate E at a distance r from the line.

In order to apply Gauss law, we construct a Gaussian surface in the form of a circular cylinder of radius r and length l with the wire as its axis.

E is const over the cylindrical surface ∵ it is at equi distance from the line (or axis).

Flux of E through cylindrical surface :-

$$\oint \vec{E} \cdot \vec{ds} = \oint E\,ds = E \oint ds = E \cdot 2\pi r l.$$

∵ E.F is ∥ to the circular ends of the cylinder ; the contribⁿ of these ends to the flux is zero.
(∵ E is ⊥ to the faces).

$$\therefore \oint \vec{E} \cdot \vec{ds} = 0$$

⇒ Total outward flux, $\oint \vec{E} \cdot \vec{ds} = E \cdot 2\pi r l$

Charge enclosed by the Gaussian surface $= \lambda l$.

∴ by Gauss law, $\oint \vec{E} \cdot \vec{ds} = E\,2\pi r l = \frac{\lambda l}{\epsilon_0}$.

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

⇒ $E \propto \frac{1}{r}$. The dirⁿ of E is radially outward for a line of +ve charge.

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