Question 1:- A ball of radius R carries a positive charge whose volume density depend only on a separation r from the ball's centre as

$\rho = \rho_0 \left(1 - \frac{r}{R}\right)$, where ρ_0 is constant

Assuming the permittivities of the ball and environment to be equal to the unity. Find

a) Magnitude of Electric field strength as a function of distance r both inside and outside the ball.

b) The maximum intensity E_{max} and corresponding distance r_m

Solution: Given $\rho = \rho_0 \left(1 - \frac{r}{R}\right)$

$\rho_0 = \text{const}$

$\varepsilon = 1$

We know that

$$\int E \cdot ds = \frac{q(r)}{\varepsilon_0}$$

$$q(r) = \int_V \rho \, dV$$

$$dV = \frac{4}{3}\pi (r + dr)^3 - \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

$$q(r) = \int_0^r (4\pi r^2 dr)\left(\rho_0 \left(1 - \frac{r}{R}\right)\right)$$

$$= \frac{1}{3} r^3 \left(\frac{4R - 3r}{R}\right) \pi \rho_0$$

DATE : ___/___/___

PAGE ______

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}\frac{1}{3} r^3 \left(\frac{4R-3r}{R}\right)\pi \rho_0$$

$$E = \frac{1}{3}\left(1-\frac{3r}{4R}\right)\frac{\rho_0 r}{\varepsilon_0}$$

Electric field outside ball

$r > R$

in plane of r we put R

$$q(R) = \frac{1}{3} R^3 \left(\frac{4R-3R}{R}\right)$$

$$= \frac{1}{3} R^3 \pi \rho_0$$

$$\int E \cdot ds = \frac{1}{3}\pi R^3 \rho_0$$

$$E \times 4\pi r^2 = \frac{1}{3}\pi R^3 \rho_0$$

$$E(r) = \frac{1}{12}\frac{R^3 \rho_0}{r^2 \varepsilon_0}$$

for calculating maximum intensity

$$E = \frac{1}{3}\left(1-\frac{3r}{4R}\right)\frac{\rho_0 r}{\varepsilon_0}$$

we can diff w.r.to r

$$\frac{dE}{dr} = \frac{d}{dr}\frac{1}{3}\left(1-\frac{3r}{4R}\right)\frac{\rho_0 r}{\varepsilon_0}$$

$$= -\frac{1}{4R}\frac{\rho_0 r}{\varepsilon_0} + \left(\frac{1}{3}-\frac{1}{4}\frac{r}{R}\right)\frac{\rho_0}{\varepsilon_0}$$

$$r = \frac{2}{3}R$$

Substituting this value in

$$E = \frac{1}{3}\left(1 - \frac{3r}{4R}\right)\frac{\rho_0 r}{\varepsilon_0} \qquad r = \frac{2}{3}R$$

$$= \frac{1}{3}\left(1 - \frac{3 \times \frac{2}{3}R}{4R}\right)\rho_0 \times \left(\frac{2}{3}R\right)$$

$$E_{max} = \frac{1}{9}\frac{\rho_0 R}{\varepsilon_0}$$

Question 2:- Inside the ball, charged uniformly with density ρ, there is a spherical cavity. The centre of cavity is shifted relative to the centre of ball by distance a. Find the field strength E inside cavity.

Ans We know that

$$\int E\,dS = \frac{q}{\varepsilon_0} = \phi$$

$$ES = \frac{\rho dV}{\varepsilon_0}$$

$$E\,4\pi r^2 = \rho\,4\pi r^2 dr$$

$$E_1 = \frac{\rho}{\varepsilon_0} \times \frac{4\pi r^3}{3 \times 4\pi r^2}$$

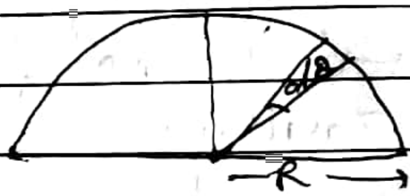
$$= \frac{\rho r}{3\varepsilon_0}$$

The value shifted by a then the value of electric field becomes:

$$\vec{E}' = \frac{\rho a}{3\varepsilon_0}$$

Question No. 3: Find the electric field potential and strength of hemisphere of radius R with surface density σ.

Solution :- In case of hemisphere



$$d\phi = \frac{1}{4\pi\varepsilon_0} \frac{da}{R}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sigma ds}{R}$$

$$\phi = \int \frac{\sigma \, ds}{4\pi\varepsilon_0 R}$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{\sigma}{4\pi\varepsilon_0 R} R^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{\sigma R}{\varepsilon_0}$$

$$dE_z = dE \cos\theta$$

$$E = E_z = \int_S \frac{\sigma \cos\theta}{4\pi\varepsilon_0 R^2} dS$$

$$E = \frac{\sigma}{4\varepsilon_0}$$