Page No. _____
Date _____

Question :- A particle moves along x axis according to law $x = a\cos\omega t$. Find the distance that particle covers during time interval $t = 0$ to t.

Solution: From law of motion $x = a\cos\omega t$ it is obvious that line covers the distance equal to a starting the extreme position equal to $T/4$

So $t = n\cdot\frac{T}{4} + t_0$ where $t_0 < \frac{T}{4}$ $n = 0, 1, 2, 3 \ldots$ for even value of n $t_0 = 0$ extreme position

Case I - When n is odd no

$x = \pm a\cos\omega t$ if t is called $\frac{nT}{4}$ the distance covered in this interval

$S_1 = a\sin\omega t_0 = a\sin\omega\left(t - \frac{nT}{4}\right)$

So the distance

$S = na + S_1$

$= na + a\sin(\omega t - n\pi/2)$

Case II when n is even.

$x = a\cos\omega t$

S_2 is to called

$a - S_2 = a\cos\omega t_0$

$S_2 = a\left[1 - \cos(\omega t - n\pi/2)\right]$

$$S = na + S_2$$
$$= na + a\left[1 - \cos\left(\omega t - \frac{n\pi}{2}\right)\right]$$
$$= a\left[n + 1 - \cos\left(\omega t - \frac{n\pi}{2}\right)\right]$$
$$S = a\left[n + 1 - \cos\left(\omega t - n\pi/2\right)\right] \quad n \text{ is even}$$
$$a\left[n + \sin\left(\omega t - \frac{n\pi}{2}\right)\right] \quad n \text{ is odd}$$

Q2. Find the trajectory equation of point if its motion according to
$x = a\sin\omega t, \; y = a\sin 2\omega t$

Solution

$$x = a\sin\omega t$$
$$\sin\omega t = \frac{x}{a}$$
$$y = 2a\sin\omega t\cos\omega t$$
$$\cos\omega t = \sqrt{1 - \sin^2\omega t}$$
$$= \sqrt{1 - \frac{x^2}{a^2}}$$
$$y = a\sin 2\omega t$$
$$= 2a\sin\omega t\cos\omega t$$
$$y = 2a \times \frac{x}{a} \times \sqrt{1 - x^2/a^2}$$

$$y = 2x\sqrt{1 - \frac{x^2}{a^2}}$$

$$y^2 = 4x^2\left(1 - \frac{x^2}{a^2}\right)$$

Question 3. A particle of mass m is located in unidirectional potential field where the potential energy $U(x) = U_0(1 - \cos ax)$, U_0 and a are constant. Find the period of small oscillations that particle perform about the equilibrium position.

Solution: $U(x) = U_0(1 - \cos ax)$

So from formula:

$$F = -\frac{dU}{dx}$$

$$F_x = -U_0 a \sin ax$$

$$= -U_0 a\, ax$$

When angle is small then $\sin ax \simeq ax$

$$F_x = -U_0 a^2 x$$

Page No. _____
Date _____

$F_x = -m\omega_0^2 x$ for small oscillation

$$\omega_0^2 = \frac{U_0 a^2}{m}$$

$$\omega_0 = a\sqrt{\frac{U_0}{m}}$$

$$\text{Time period} = \frac{2\pi}{\omega_0} = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$$