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11:09

Classification of States:

State j is said to be accessible from state i if $P_{ij}^{n} > 0$ for some $n \geq 0$.

$$P_{ij}^{n} = P\{X_n = j \mid X_0 = i\}$$

If $P_{ij}^{n} = 0 \quad \forall\ n \geq 0$

then j is not accessible from i.

$\because P\{\text{ever enter } j \mid \text{start in } i\}$

$$= P\left\{\bigcup_{n=0}^{\infty} \{X_n = j \mid X_0 = i\}\right\}$$

$$\leq \sum_{n=0}^{\infty} P(X_n = j \mid X_0 = i)$$

$$= \sum_{n=0}^{\infty} P_{ij}^{n} = \sum^{\infty} 0 = 0$$

$n=0$

$\therefore P\{\text{ever enter } j \mid \text{Start in } i\} \leq 0$

$\Rightarrow P\{\text{ever enter } j \mid \text{Start in } i\} = 0$

$\Rightarrow$ j is not accessible from i if $P_{ij}^{n} = 0$

$\forall n \geq 0$.

Similarly state i is accessible from state j if

$P_{ji}^{n} > 0$ for some $n \geq 0$.

Two states i and j that are accessible

to each other are said to communicate,

and we write $i \longleftrightarrow j$.

~~Note~~ every state communicates with itself

$\therefore P_{ii}^{0} = P\{X_0 = i \mid X_0 = i\} = 1.$

The relation of communication satisfies the following three properties:

(i) state i communicates with itself.

(ii) If state i communicates with state j, then state j communicates with state i.

(iii) If state i communicates with j and j communicates with k, then i communicates with k.

Therefore, relation of communication is an equivalence relation.

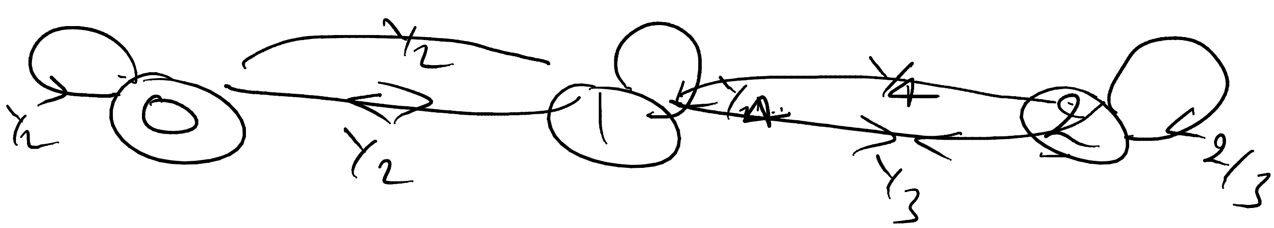
Two states that communicate are said

to be in the <u>same class</u>.

The Markov chain is said to be irreducible if there is only one class. that is, all states communicate with each other.

<u>Example ①</u>

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \end{bmatrix} \end{array}$$



State Transition Diagram

This Markov chain is irreducible.

$\because P_{00} = \frac{1}{2} > 0, \quad P_{01} = \frac{1}{2} > 0,$

$P \quad = \quad P^{2} \quad \sum P \; P$