Commutative Ring:

A ring R is called commutative if multiplication operation is commutative in R

i.e. $a \cdot b = b \cdot a \quad \forall\, a, b \in R$

Examples: (i) $(\mathbb{Z}, +, \cdot)$ is a commutative ring

(ii) $(R, +, \cdot)$ is a non-commutative ring if $R = GL(2, \mathbb{R})$

Ring with unity:

Identity element under multiplication is called unity in a ring.

A ring R is called ring with unity if it contains multiplicative identity.

Example: (i) $(\mathbb{Z}, +, \cdot)$ is a ring with unity

(ii) $(\mathbb{Q}, +, \cdot)$ is a ring with unity

Unit:

An element $a \in R$ is called unit in a ring R if inverse of a under multiplication exists in R.

i.e. a is unit if a^{-1} (under multiplication) exists.

~~Examples~~ (i) In a ring $(\mathbb{Z}, +, \cdot)$.

1 and -1 are units.

(ii) In a ring $(\mathbb{Q}, +, \cdot)$, the set of units is $\mathbb{Q} \setminus \{0\}$.

~~Factors:~~

a is a factor of b (i.e. $a | b$))

In a commutative ring R, a nonzero element $a \in R$ is called factor of $b \in R$, if there exists an element $c \in R$ such that $b = ac$. (notation $a | b$)

If a does not divide b (or a is not the factor of b) then we denote it by $a \nmid b$.

~~Example:~~ (i) In $(\mathbb{Z}, +, \cdot)$.

2 is the factor of 6

because $6 = 2 \cdot 3$

(ii) In a ring $(\mathbb{Z}_{12}, +_{12}, \cdot_{12})$

$\mathbb{Z}_{12} = \{0, 1, 2, 3, \cdots 10, 11\}$

7, 11, 1, 2, 5 and 10 are the factors of $1\underline{0}$

11, 1, 2, 7, 5, 10, are the factors of $\underline{2}$

$2 = \underline{14}, \underline{26}, \underline{38}, \underline{50}, 62, 74, 86, 98,$

11, 1, 2, 7, 5, 10,

10 (modulo 12) = 10, 22, 34, 46, 58, 70,
82, 94, 106, 118, 130, 142

62, 74,
86, 98,
110, 122, 134