Permutation:

06 October 2021 09:09

Odd Permutation:

A permutation that can be expressed as a product of an odd number of 2-cycles is called an odd permutation

Example: $(1\ 2\ 3\ 4) = (1\ 4)(1\ 3)(1\ 2)$

three 2-cycle

Odd permutation.

Q. Is the set of odd permutations in S_n, forms a subgroup of S_n?

NO

Sol// × Closure (odd permutation)(odd permutation) = (even permutation)

× Identity ε is an even permutation.

T.

Theorem:

For $n > 1$, A_n has order $\frac{n!}{2}$

$A_n \rightarrow$ Alternating group of degree n

set of all even permutation.

$|S_n| = n!$, $A_n = \frac{n!}{2}$

If B_n = set of all odd permutations in S_n.

$n! = \frac{n!}{2}$.

Q. Find all possible orders in symmetric group S_7?

Sol^n//

$|S_7| = 7! = (7)(6)(5)(4)(3)(2)(1)$
$= 5040$.

$$\boxed{\begin{array}{ll} (123)(47)(6) & (12)(457)(6) \\ \text{order} = LCM\{3,2,1\} & \text{order} = LCM(2,3,1) \end{array}}$$

We shall consider possible disjoint cycle structure of elements of S_7.

Now, arranging all possible disjoint cycle structures of elements of S_7 according to longest cycle lengths from left to right, we have.

$\boxed{A = \{1,2,3,4,5,6,7\}}$

$(7) \longrightarrow$ order $= 7$

$(6)(1) \longrightarrow$ order $= LCM\{6,1\} = 6$.

$(5)(2) \longrightarrow$ order $= LCM\{5,2\} = 10$

$(1\,2\,3\,4\,5)(6\,7)$, $(2\,3\,4\,5\,6)(1\,7)$, $(3\,4\,7\,6\,1)(2\,5)$, ...

$(5)(1)(1) \longrightarrow$ order $= LCM\{5,1,1\} = 5$.

$(4)(3) \longrightarrow$ order $= LCM\{4,3\} = 12$

$(4)(2)(1) \longrightarrow$ order $= LCM\{4,2,1\} = 4$

$(4)(1)(1)(1) \longrightarrow$ order $= LCM\{4,1,1,1\} = 4$

$(3)(3)(1) \longrightarrow$ order $= LCM\{3,3,1\} = 3$

$(3)(2)(2) \longrightarrow$ order $= LCM\{3,2,2\} = 6$

$(3)(2)(1)(1) \longrightarrow$ order $= LCM\{3,2,1,1\} = 6$

$(3)(1)(1)(1)(1) \longrightarrow$ order $= LCM\{3,1,1,1,1\} = 3$.

$(3)(1)(1)(1)(1) \to$ order $= \operatorname{lcm}\{3,1,1,1,1\} = 3.$

$(2)(2)(2)(1) \to$ order $= \operatorname{lcm}\{2,2,2,1\} = 2$

$(2)(2)(1)(1)(1) \to$ order $= 2$

$(2)(1)(1)(1)(1)(1) \to$ order $= 2$

$(1)(1)(1)(1)(1)(1)(1) \to$ order $= \operatorname{lcm}\{1,1,1,1,1,1,1\} = 1.$

Hence, possible ~~order~~ orders of elements of S_7 are $1, 2, 3, 4, 5, 6, 7, 10$ and 12.

Q. What is the maximum order of elements in S_7

Ans. 12.

Possible disjoint cycle structure $(4)(3) \to \operatorname{lcm}(4,3) = 12$

Q. Find all possible orders of elements in A_7?

Soln

$(7) \to 6$ 2-cycles.

$\boxed{(5)(2)} \to 5$ 2-cycle odd permutation.