

example - 11

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$$Q = 2000 - 20P \Rightarrow P = \frac{-Q + 2000}{20}$$

$$C(Q) = 0.05Q^2 + 10000$$

$$MC = 0.1Q$$

$$\begin{aligned} TR &= P \cdot Q \\ &= \left(\frac{2000 - Q}{20} \right) \cdot Q \end{aligned}$$

$$TR = \frac{-Q^2 + 2000Q}{20}$$

$$MR = \frac{-2Q + 2000}{20}$$

$$\boxed{MR = -\frac{Q}{10} + 100}$$

$$\text{Profit} = TR - TC$$

$$= P \cdot Q - C(Q)$$

$$(P - AC)Q$$

$$= 75 - (0.05(500))$$

$$= 15,000$$

markup

$$= P - MC$$

$$= 75 - 0.1 \times 500$$

$$\boxed{\text{markup} = 25}$$

So at Equilibrium

$$MR = MC$$

$$\frac{Q}{10} - 100 = 0.1Q$$

$$-\frac{Q}{10} - 100 = \frac{Q}{10}$$

$$\frac{2Q}{10} = 100$$

$$\boxed{Q^* = 500}$$

and

$$P = \frac{-500 + 2000}{20}$$

$$P = \frac{1500}{20}$$

$$\boxed{P^* = 75}$$

Profits

$$(P - AC)Q$$

$$PQ - (AC)Q$$

$$= PQ - \left(\frac{TC}{Q} \right) Q$$

$$= \boxed{TR - TC}$$

[P and AC - Profit
P and MC - Power (Markup) strength analysis]

$P = MC$ = zero strength

hence Perfect Comp. mkt

linear demand function (Case)

$$P = a - bQ$$

$$C(Q) = cQ$$

$$\begin{aligned} TR &= P \cdot Q \\ &= (a - bQ)Q \end{aligned}$$

$$TR = aQ - bQ^2$$

$$\boxed{MR = a - 2bQ}$$

$$MC = c$$

$$\text{at } \boxed{MR = MC} - \text{eqn}$$

$$a - 2bQ = c$$

$$-2bQ = c - a$$

$$\boxed{Q^* = \frac{a - c}{2b}}$$

$$P = a - b \left[\frac{a - c}{2b} \right]$$

$$P = \frac{2a - a + c}{2}$$

$$\boxed{P^* = \frac{a + c}{2}}$$

$$\text{and } AC = \frac{CQ}{Q}$$

$$\boxed{AC = c}$$

for a linear demand function.

* Constant MC implies that we are having linear cost function ($AC = MC$)

So cost function CQ

$$\text{not } C(Q) = \bar{C} + CQ$$

$$AC = \frac{\bar{C}}{Q} + c \quad \text{--- so not equal to } \underline{C(MC)}$$

So cost function is $CQ \rightarrow$ fixed cost is zero.

* $MC = AVC \neq AC$ --- fixed cost exist with cost function is linear

Constant elasticity demand function

$$Q = a P^e$$

$$c(Q) = cQ$$

(1) elasticity?

$$\eta = \left(\frac{dQ}{dP} \right) \frac{P}{Q}$$

$$\eta = e a P^{e-1} \times \frac{P}{a P^e}$$

$$\boxed{|\eta| = e}$$

(2) P^*, Q^* ?

$$MC = c$$

$$Q = a P^e$$

$$\frac{Q}{a} = P^e$$

$$e \sqrt[e]{\frac{Q}{a}} = P \Rightarrow \left(\frac{Q}{a} \right)^{1/e}$$

$$TR = P \cdot Q$$

$$= e \sqrt[e]{\frac{Q}{a}} \cdot Q \left(\frac{Q}{a} \right)^{1/e} \cdot Q$$

$$TR = (Q^{1/e+1}) \frac{1}{a^{1/e}}$$

$$MR = \frac{\frac{1}{e} + 1 (Q^{1/e})}{a^{1/e}}$$

$$\frac{\frac{1}{e} + 1 [Q^{1/e}]}{a^{1/e}} = c$$

$$Q^{1/e} = \frac{c \cdot a^{1/e}}{(\frac{1}{e} + 1)}$$

$$Q^{1/e} = \frac{e(c \cdot a^{1/e})}{(1+e)}$$

$$\boxed{Q^* = \left(\frac{e(c \cdot a^{1/e})}{1+e} \right)^e}$$

$$P = \left(\frac{\left(\frac{e(c \cdot a^{1/e})}{1+e} \right)^e}{a} \right)^{1/e}$$

$$P = \frac{Q(c \cdot a^{1/e})}{\frac{1+e}{a^{1/e}}}$$

$$P = \frac{e \cdot c(a^{1/e})}{(1+e)a^{1/e}}$$

$$\boxed{P^* = \frac{e \cdot c}{(1+e)}}$$

for monopoly firm we know, ⑦
 $MR = MC$ or $MR = AR \left(1 + \frac{1}{e}\right)$

$$MR = P \left(1 + \frac{1}{e}\right)$$

at profit maximisation condition

$$\pi = P \left(1 + \frac{1}{e}\right)$$

$$\pi = P \left(\frac{e+1}{e}\right)$$

$$\pi \cdot e = P(e+1)$$

$$P = \frac{e \cdot \pi}{(e+1)}$$

$$\frac{dP}{dC} = \left(\frac{e}{1+e}\right)$$

etc)

for $e < -1$ is the case for profit maximisation
(elastic segment)
because in inelasticity segment MR goes

negative $\left\{ \begin{array}{l} e > -1 \\ e = -1 \\ \text{AR} \\ \text{MR} \end{array} \right\}$

① $e = -2$

$$P = \frac{-2\pi}{(1-2)}$$

$$P = 2\pi$$

② $e = -1.5$

$$P = \frac{-1.5 \cdot \pi}{(1-1.5)}$$

$$P = \frac{+1.5 \cdot \pi}{+0.5}$$

$$P = 3\pi$$

as elasticity increases towards -1 , the divergence b/w P and MC is increasing with elasticity rising

$$* \boxed{e = -1}$$

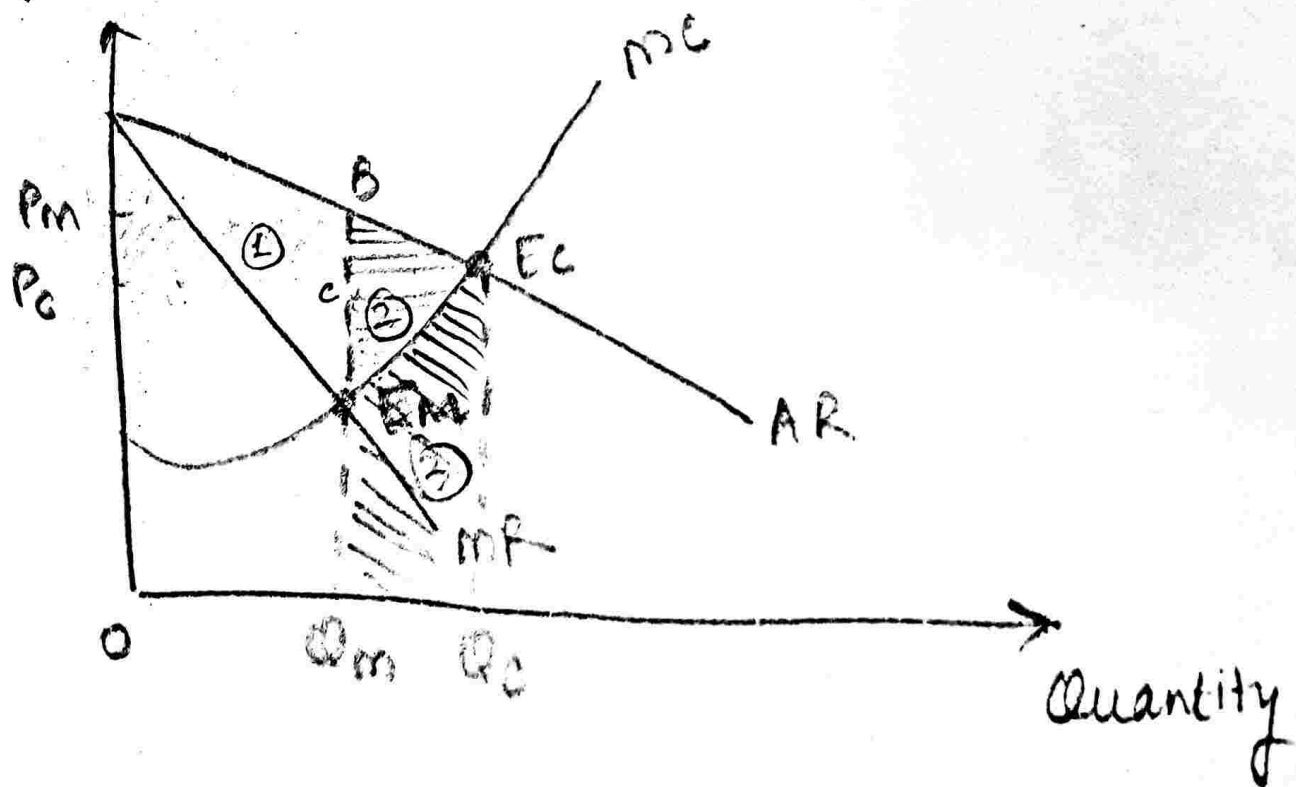
$$P = \frac{-1C}{0.}$$

$$\boxed{P = \infty}$$

Monopoly and resource allocation

under monopoly $MR = MC < P$, in this scenario market price does not convey accurate information abt cost of prodⁿ. Hence consumer's decision will no longer reflect true opportunity cost of prodⁿ and resources will be misallocated.

Price



region 1

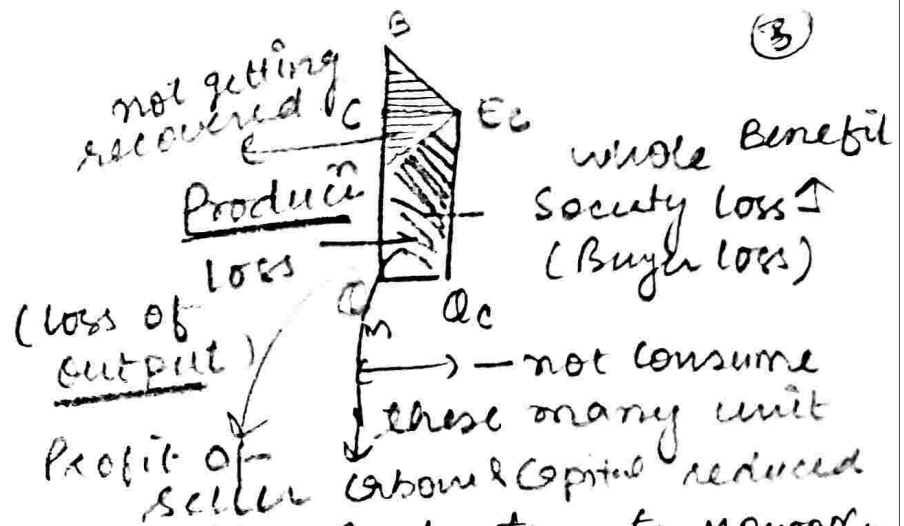
$P_m B C P_c$

region 2

$B E_c E_m$

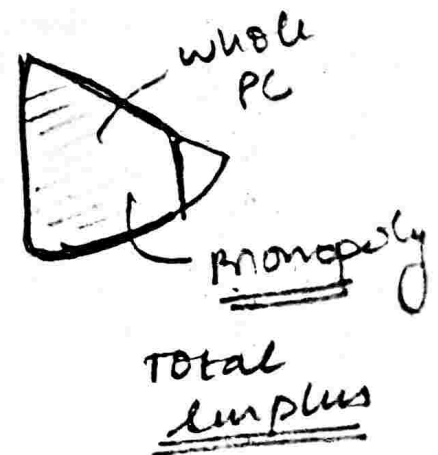
region 3

$E_m E_c Q_c Q_m$



if we shift from competitive industry to monopoly then there is change in welfare

- 1) Consumer surplus reduces by region 1
- 2) Region 3 reflects value of inputs reallocated to production of other goods
- 3) Region 2 represents Dead weight loss, it has two components
 - (1) $\Delta B C E_c$ (loss in consumer surplus)
 - (2) $\Delta C E_c E_m$ (loss in producer surplus)



in example (1) cost is 50.

if a Buyer has is equal to 60, to Buy a good.

P^* is 75 and

willingness to pay which then he will not able

here, willingness to pay is greater than MC hence transaction must take place.

But due to monopoly such transactions are prevented hence monopoly Eq^m is not Pareto equilibrium optimum. (B/c $WTP > MC$)

Example

welfare loss and elasticity

$Q = P^e$ (Constant elasticity demand fn) P_m

$$MC = AC = C$$

1) Competitive Price expression

2) Monopoly " "

Equilibrium under perfect competition

$$P = MC$$

$$(Q)^{1/e} = C$$

$$\boxed{Q_c = C^e}$$

Equilibrium under monopoly market

$$MR = MC$$

$$TR = P \cdot Q$$

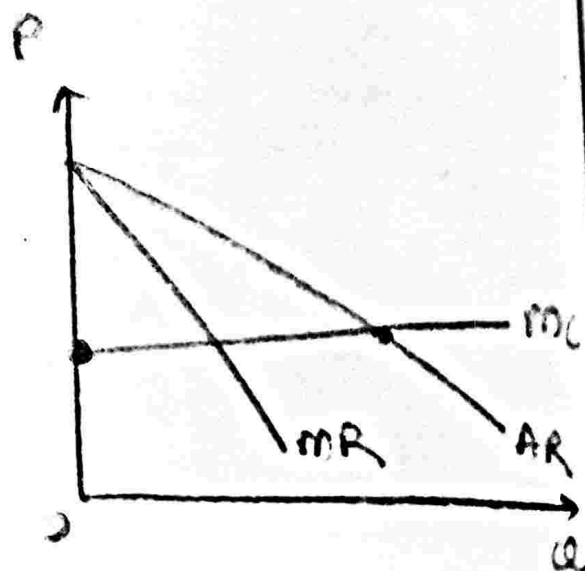
$$= (Q)^{1/e} \cdot Q$$

$$TR = (Q)^{\frac{1+e}{e}}$$

$$MR = \frac{1+e}{e} Q^{\frac{1+e-e}{e}}$$

$$MR = \frac{1+e}{e} Q^{1/e}$$

$$MR = MC$$



for competitive market
 $\Rightarrow$ Producer Surplus is zero
 (Profit of Producer is zero)

$$P = MC$$

in Perfect comp in short-terms - Profits - in long-run zero profit or
 LR - zero profit $\rightarrow$ LR = zero profit

$$\frac{1+e}{e} Q^{1/e} = c$$

$$Q^{1/e} = \frac{c \cdot e}{1+e}$$

$$\boxed{Q_m = \left(\frac{e \cdot c}{1+e} \right)^e}$$

$$P = Q^{1/e}$$

$$P_m = \left[\left(\frac{e \cdot c}{1+e} \right)^e \right]^{1/e}$$

$$\boxed{P_m = \frac{e \cdot c}{1+e}}$$

$$P_c = (c^e)^{1/e}$$

$$\boxed{P_c = c}$$

Consumer surplus expression

Consumer Surplus at P_0 (general)

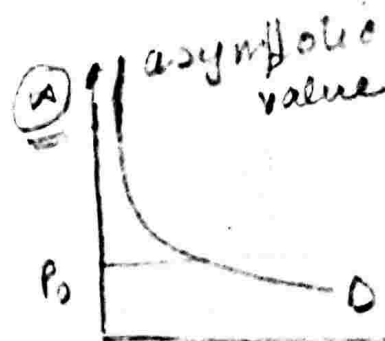
$$CS = \int_{P_0}^{\infty} Q(P) \cdot dP$$

$$= \int_{P_0}^{\infty} P^e \cdot dP$$

$$\boxed{CS = \left[\frac{P^{e+1}}{e+1} \right]_{P_0}^{\infty}}$$

$$CS = \left[\frac{\infty^{e+1}}{e+1} - \frac{P_0^{e+1}}{e+1} \right] \quad \text{--- as price increases } Q \text{ approaches to zero}$$

$$\boxed{CS = - \left[\frac{P_0^{e+1}}{e+1} \right]}$$



Consumer surplus in perfect comp. market

$$P_0 = P_c = c$$

$$CS = - \left[\frac{P_c^{e+1}}{e+1} \right]$$

$$CS = - \left[\frac{c^{e+1}}{e+1} \right]$$

Consumer Surplus in monopoly

$$P_0 = P_m = \frac{e \cdot c}{1+e}$$

$$CS = - \left[\frac{\left(\frac{e \cdot c}{1+e} \right)^{e+1}}{e+1} \right]$$

* depending upon elasticity the CS is depending, because monopoly produces in region where $e < -1$ (elastic demand) then the (-) sign is omitted.

Ratio of CS_M to CS_c

$$\frac{CS_M}{CS_{PC}} = \frac{\left(\frac{ec}{1+e} \right)^{e+1} / (1+e)}{\frac{c^{e+1}}{(e+1)}}$$

$$= \frac{\left(\frac{ec}{1+e} \right)^{e+1} \times (e+1)}{(1+e) \cdot c^{e+1}}$$

$$\frac{CS_M}{CS_{PC}} = \left[\frac{ec}{1+e} \right]^{e+1} = \left(\frac{e}{1+e} \right)^{e+1}$$

expected sign of the $\frac{CS_m}{CS_c}$.

(less than 1)

because $CS_m < CS_c$

consider $e = -2$

$$\frac{CS_m}{CS_c} = \left(\frac{-2}{1-2} \right)^{-2+1}$$

$$= \left(\frac{-2}{-1} \right)^{-1}$$

$$\frac{CS_m}{CS_c} = \frac{1}{2} = 0.5$$

$e = -1.5$

$$\left(\frac{-1.5}{1-1.5} \right)^{-1.5+1}$$

$$= \left(\frac{-1.5}{-0.5} \right)^{-0.5}$$

$$\frac{CS_m}{CS_c} = \left(\frac{1}{3} \right)^{0.5} = 0.57$$

as elasticity approaches -1 the ratio increases.

Monopoly Profit

$$\text{monopoly profit} = TR - TC$$

$$= P_m Q_m - c Q_m$$

$$= \left(\frac{ec}{1+e} \right) Q_m - c Q_m$$

$$= \left(\frac{ec}{1+e} - c \right) Q_m$$

$$= \left(\frac{ec - c - ec}{1+e} \right) Q_m$$

$$= \left(\frac{-c}{1+e} \right) Q_m$$

$$= \left(\frac{-c}{1+e} \right) P_m^e \quad \text{— according to demand function}$$

$$= \left(\frac{-c}{1+e} \right) \left(\frac{ec}{1+e} \right)^e = - \left(\frac{c}{1+e} \right)^{e+1} \cdot e^e$$

Profit of monopoly
Competitive market
Consumer Surplus

$$= \frac{-\left(\frac{c}{1+e}\right)^{1+e} \times e^e}{-\left(\frac{c^{e+1}}{e+1}\right)}$$

$$= \frac{\frac{c^{e+1}}{(1+e)^{1+e}} \times e^e}{\left(\frac{-c^{e+1}}{e+1}\right)}$$

$$= \frac{e^e (e+1)}{(1+e)^{1+e}}$$

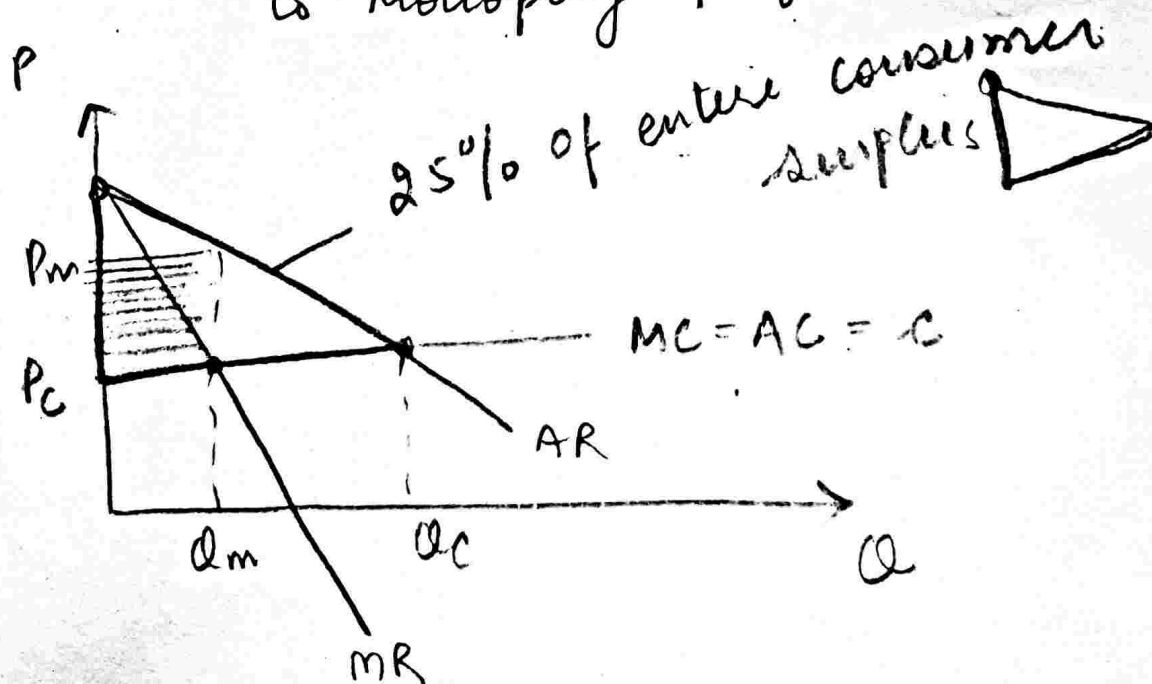
$$\frac{\pi_m}{CS_c} = \frac{e^e}{(1+e)^e} = \left(\frac{e}{1+e}\right)^e$$

so $\boxed{\frac{\pi_m}{CS_c} = \left(\frac{e}{1+e}\right)^e}$

$$e = -2$$

$$\boxed{\frac{\pi_m}{CS_c} = \frac{1}{4}}$$

ie $\frac{1}{4}^{th}$ of consumer surplus enjoyed under perfect comp. is transfer to monopoly profit.



$$(2) \quad e = -1.5$$

$$\begin{aligned} \frac{\pi_m}{CS_c} &= \left(\frac{-1.5}{1-1.5} \right)^{-1.5} \\ &= \left(\frac{1.5}{0.5} \right)^{-1.5} \\ &= \left(\frac{1}{3} \right)^{1.5} \\ &= (0.33)^{1.5} \end{aligned}$$

$$\boxed{\frac{\pi_m}{CS_s} = 0.1924}$$

$$\boxed{e \uparrow \Rightarrow \frac{\pi_m}{CS_c} \downarrow}$$