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Thm Distributive Inequalities

If L is a lattice then the foll. distributive inequalities ~~are~~ hold in L

(i) $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$

(ii) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$.

Pf i) $x \wedge y \leq x$

& $x \wedge y \leq y \leq y \vee z$.

$\therefore$ $x \wedge y$ is a l.b of x & $y \vee z$

$\Rightarrow$ $x \wedge y \leq x \wedge (y \vee z)$ — (1)

$x \wedge z \leq x$ & $x \wedge z \leq z \leq y \vee z$

$\Rightarrow$ $x \wedge z$ is a l.b of x & $y \vee z$.

$\Rightarrow$ $x \wedge z \leq x \wedge (y \vee z)$ — (2)

By (1) & (2)

$x \wedge (y \vee z)$ is an u.b of $x \wedge y$ & $x \wedge z$

$\Rightarrow$ $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$

(ii) Prove

Remark Every Singleton in a lattice is a Sublattice of L.

Pf Let L be a lattice

& $S = \{a\} \subseteq L$

$(S, \leq)$ is clearly a poset w.r.t induced order ($\leq$).

Also $a \in S \subseteq L$

& L is a lattice

$\therefore$ $a \wedge a = a$ & $a \vee a = a$

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$\therefore$ $a \wedge a = a \in S$
& $a \vee a = a \in S$
$\therefore$ S is closed w.r.t the meet & join of L
$\therefore$ S is a sublattice of L.

Defn <u>Interval in a lattice</u>

Let L be a lattice & $x, y \in L$
s.t $x \leq y$
Then $[x, y] = \{ a \in L / x \leq a \leq y \}$

<u>Note</u> $x \in [x, y]$
& $y \in [x, y]$

<u>To prove</u> $[x, y]$ is a sublattice of L.

Clearly $[x, y] \subseteq L$
$\therefore$ $([x, y], \leq)$ is a Poset.
Let $a, b \in [x, y]$
$x \leq a$, $x \leq b$
$\Rightarrow$ x is a l b of a & b
$\Rightarrow$ $\boxed{x \leq a \wedge b}$

Also $a \wedge b \leq a \leq y$
$\Rightarrow a \wedge b \leq y$
$\therefore$ $x \leq a \wedge b \leq y$
$\Rightarrow a \wedge b \in [x, y]$
Similarly $a \vee b \in [x, y]$

$\therefore$ $[x, y]$ is a sublattice of L.

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Defn

Let L & M be lattices.

A fn $f: L \to M$ is called

1) Meet homomorphism if
$$f(a \wedge b) = f(a) \wedge f(b) \quad \forall a, b \in L.$$
2) Join homomorphism if
$$f(a \vee b) = f(a) \vee f(b) \quad \forall a, b \in L$$
3) A lattice homomorphism if f is both a meet & a join homomorphism.
4) Order homomorphism if whenever $a \leq b$ in L, $f(a) \leq f(b)$ in M

Defn

A lattice homo. $f: L \to M$ is called a Monomorphism if f is 1-1.

Defn A lattice homo. $f: L \to M$ is called an epimorphism if f is onto

Defn A lattice homo. $f: L \to M$ is called an isomorphism if f is bijective.

Remark If $f: L \to M$ is a lattice homo., then $f(L)$ is called the homomorphic image of L.

Prove $f(L)$ is a sublattice of M