Group:

Binary Operation

Operations are '+', '-', 'x', '/'.

Let G be a set. A binary operation on G is a function that assigns each ordered pair of elements of G to an element of G.

That is, An operation on G is called binary operation if G is closed under ~~operat~~ that operation.

For example

① Addition ('+') is a binary operation on $\mathbb{N}$ (set of natural numbers).

② Multiplication ('.') is a binary operation on $\mathbb{Z}$ (Set of Integers).

Semi Group:

Let G be a non-empty set together with a binary operation $*$. and the operation $*$ is associative in G. Then G is called a Semigroup under operation $*$. It is usually denoted by $(G, *)$.

For eg:

① $(\mathbb{R}, +)$ is a semigroup

② $(\mathbb{R}, -)$ is not a semigroup [since '$-$' is not associative in $\mathbb{R}$]

Monoid:

Let G be a non empty set together with a binary operation $*$. We say G is a Monoid under this operation if the following properties are satisfied.

① **Associativity:**

The operation is associative. That is,

$$(a*b)*c = a*(b*c) \quad \forall\, a, b, c \in G.$$

② Identity:

There is an element e (called the identity) in G, such that

$$a * e = e * a = a \quad \forall a \in G.$$

For example:

① $(\mathbb{Q}, \cdot)$ is a monoid

② $(\mathbb{N}, +)$ is not a monoid

[$\because$ Identity '0' does not exist in $\mathbb{N}$].

Group:

Let G be a nonempty set together with a binary operation $*$. We say that G is a group under this operation if the following properties are satisfied.

① Associativity:

$$(a * b) * c = a * (b * c) \quad \forall a, b, c \in G$$

② Identity:

$\exists e \in G$ such that

$$a * e = e * a = a \quad \forall a \in G.$$

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③ ~~Ident~~ Inverse:

For each element $a \in G$, there is an element ~~in~~ b in G such that

$$a * b = e = b * a$$

This b is called inverse of a and usually denoted by $b = a^{-1}$.

For example:

① $(\mathbb{Z}, +)$ is a group

② $(\mathbb{Z}, \cdot)$ is not a group

③ $(\mathbb{Q}, \cdot)$ is not a group
(Inverse of zero does not exist)

④ $(\mathbb{Q}^*, \cdot)$ is a group $\left[\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}\right]$.

⑤ $(\mathbb{R}, +)$ is a group

⑥ $(\mathbb{R}^*, \cdot)$ is a group $\left[\mathbb{R}^* = \mathbb{R} \setminus \{0\}\right]$.