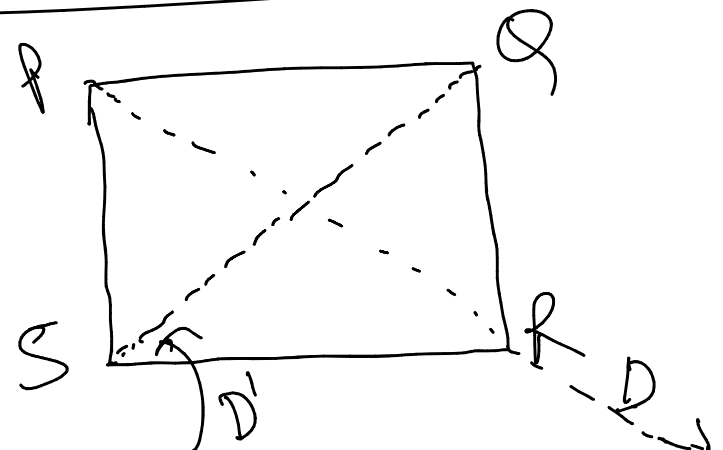
15 September 2021 09:08

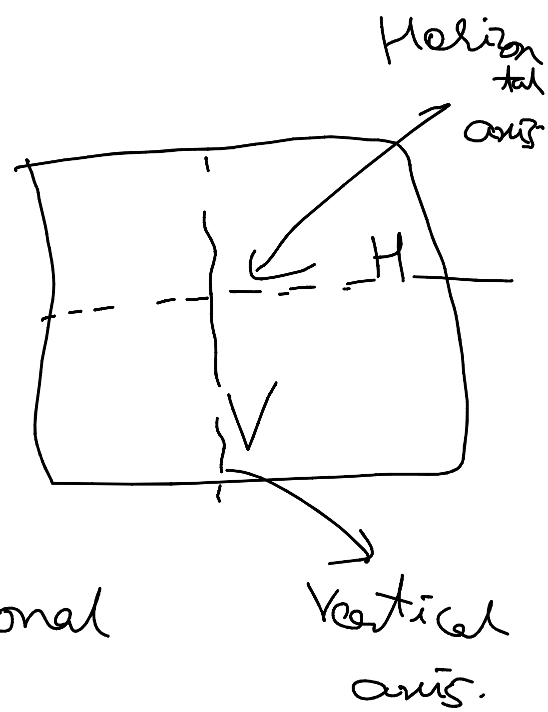
Composition of functions:

$g(x) = x^2 + 1, \quad f(x) = \frac{1}{x}$

$$g \circ f(x) = g\{f(x)\} = g\left\{\frac{1}{x}\right\} = \left(\frac{1}{x}\right)^2 + 1 = \frac{1}{x^2} + 1.$$

Symmetry of a Square:



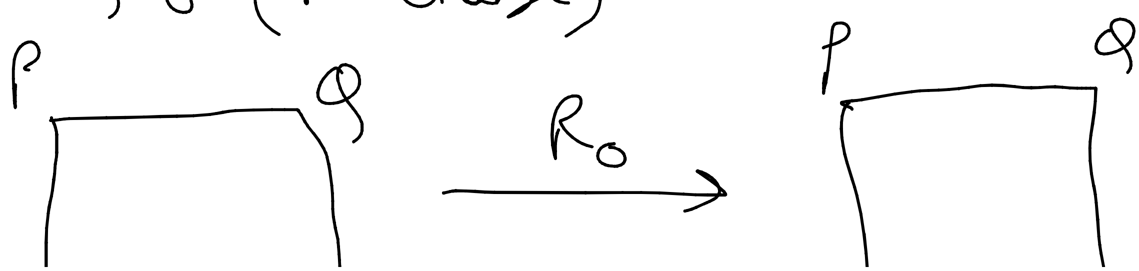


Rotations:

$0°, 90°, 180°, 270°$

$R_0 \Leftrightarrow$

Rotation of $0°$ (no change)



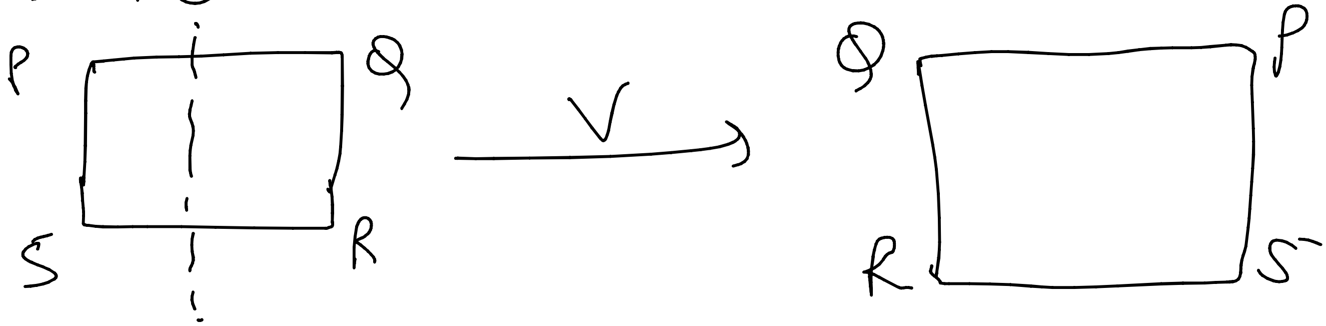
R_{90} = Rotation of 90° (Anti clockwise)

R_{180} = Rotation of 180° (Anticlockwise)

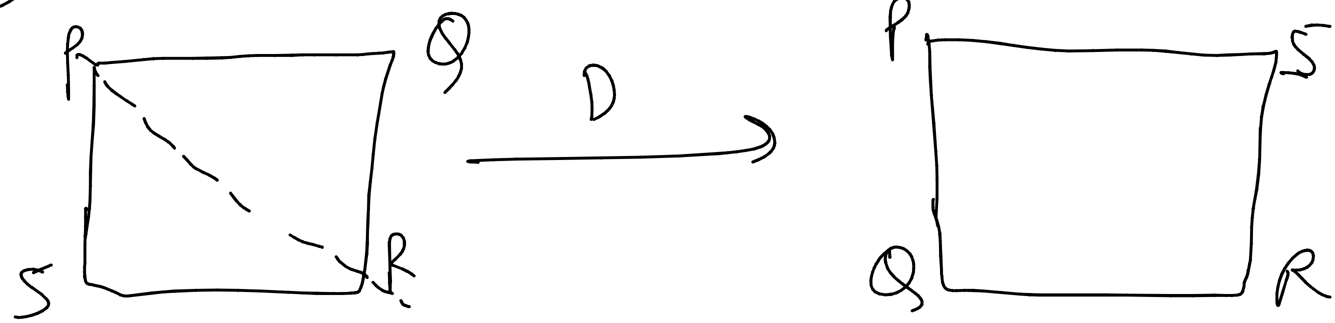
R_{270} = Rotation of 270° (Anticlockwise)

H = Rotation of 180° about Horizontal axis

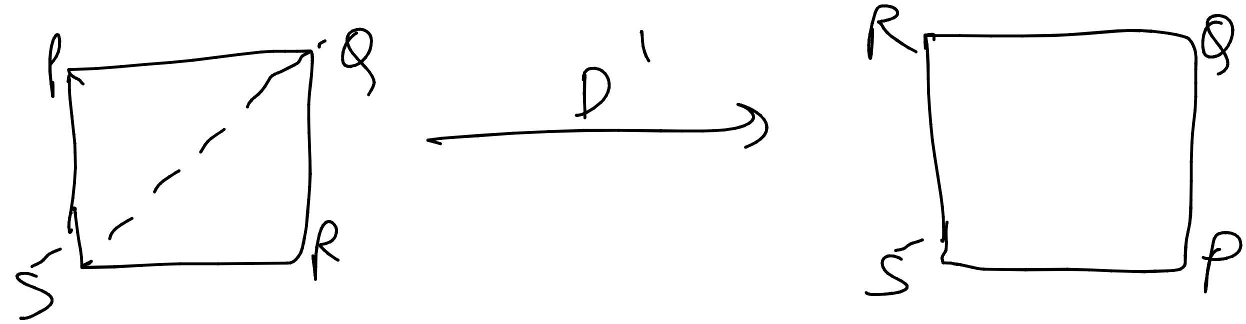
V = Rotation of 180° about vertical axis



D = Rotation of 180° about main diagonal



D' = Rotation of 180° about other diagonal



These eight symmetries of a square

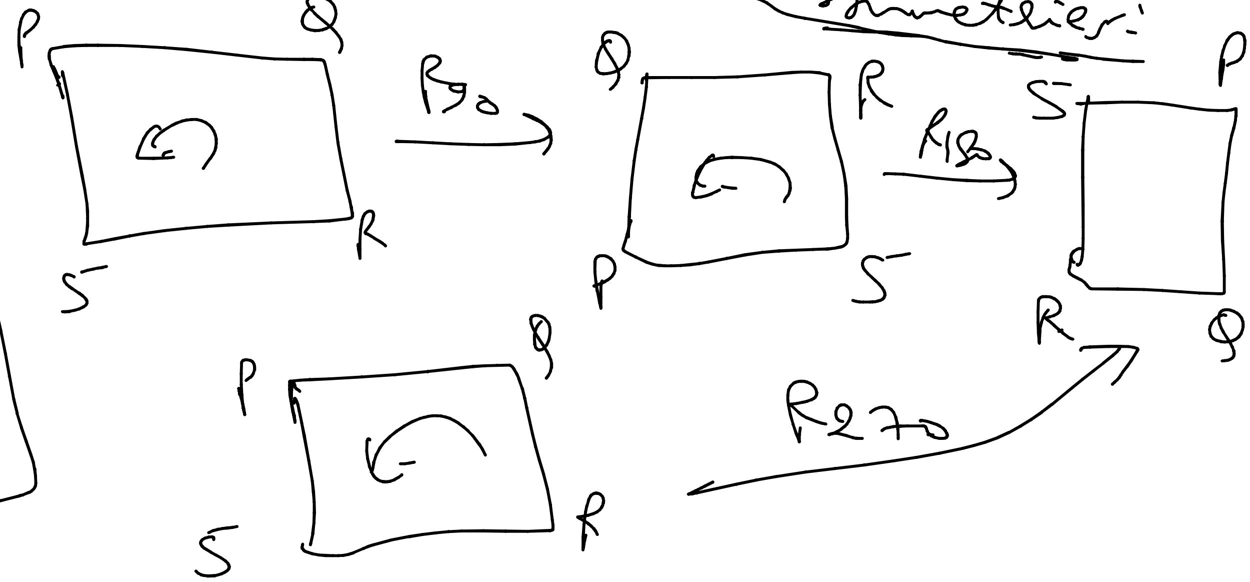
$R_0, R_{90}, R_{180}, R_{270}, H, V, D$ and D' together with the operation 'Composition' form a mathematical System called group.

This group is called dihedral group of order 8 and is denoted by D_4.

$\therefore D_4 = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$.

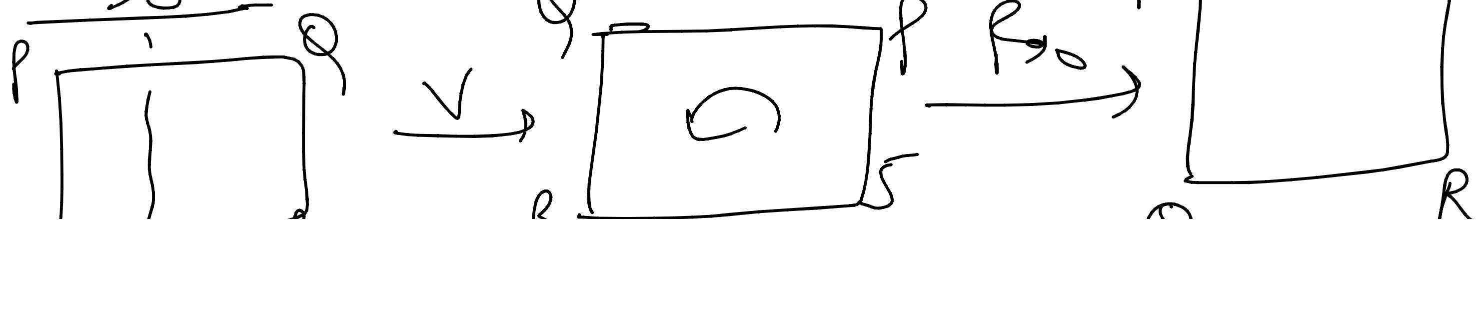
Now, we will see some Compositions of these symmetries:

$R_{180} R_{90}$

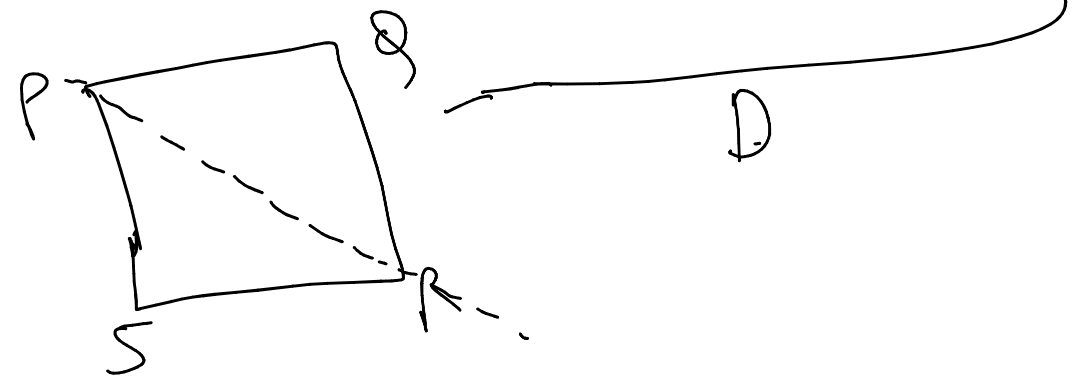


$\boxed{R_{180} R_{90} = R_{270}}$

$R_{90} V$



$\boxed{R_{90} V = D}$



Similarly other compositions may be obtained.

$$\boxed{g \circ f = g\{f(x)\}}$$

Now, we construct a Cayley table for group D_4.

$\circ$	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	R_0	D'	D	H	V
R_{180}	R_{180}	R_{270}	R_0	R_{90}	V	H	D'	D
R_{270}	R_{270}	R_0	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	R_0	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	R_0	R_{270}	R_{90}
D	D	V	D'	H	R_{270}	R_{90}	R_0	R_{180}
D'	D'	H	D	V	[illegible]	[illegible]	[illegible]	[illegible]