

Group Theory - IWeek 1:Topic: Recap of Algebra.Integers:

$$\mathbb{Z} \text{ or } \mathbb{I} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Properties of Integers:Well Ordering Principle:

Every nonempty set of positive integers contains a smallest number.

Division Algorithm:

Let a and b be any integers with $b > 0$. Then there exist unique integers q and r with the property that

$$a = bq + r, \text{ where } 0 \leq r < b.$$

Revise GCD, relatively prime integers.

GCD is a linear combination:

For any nonzero integers a and b , there exist integers s and t such that $\gcd(a, b) = as + bt$.

Moreover, $\gcd(a, b)$ is the smallest positive integer of the form $as + bt$.

Euclid's Lemma:

$$p \mid ab \Rightarrow p \mid a \text{ or } p \mid b$$

If p is a prime that divides ab , then p divides a or p divides b .

Fundamental Theorem of Arithmetic:

Every integer greater than 1 is a prime or a product of primes. This product is unique, except for order in which the factors appear.

for any number $n > 1$,

$$n = p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$$

where p_i 's are primes and x_i 's are natural numbers.

Revisé LCM

Revisé modular Arithmetic

$$a \equiv b \pmod{n} \Rightarrow n \mid (a-b)$$

Revise Mathematical Induction

~~revise feb.~~

Set :

The set is a collection of well defined objects.

Relation :

$$R \subseteq A \times B$$

Relation is a subset of Cartesian Product of two sets.

Equivalence relation :

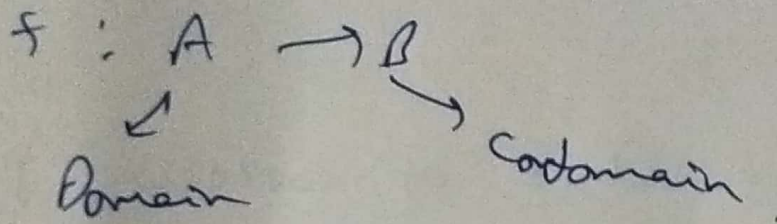
An equivalence relation on a set A is a set R of ordered pairs of elements of A such that

- ① $(a, a) \in R \quad \forall a \in A$
- ② $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall a, b \in A$
- ③ $(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$
 $\forall a, b, c \in A.$

revise equivalence class.

Function (mapping):

A function (or mapping) f from a set A to a set B is a rule that assigns to each element a of A exactly one element b of B .



$$\text{Im}(f) = \{f(x) \mid x \in A\}$$

revise one-one, onto functions

revise composition of functions.