Exercise ①:- Solve $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$

② Find general soln of

$$\frac{d^4y}{dx^4} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

③ Find general soln of

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 14\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + 25y = 0$$

④ Solve IVP.

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0, \quad y(0) = 1, \quad y'(0) = 6.$$

Exercises

Find the general solution of each of the differential equations in Exercises 1–24.

1. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$
2. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0.$
3. $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 5y = 0.$
4. $3\frac{d^2y}{dx^2} - 14\frac{dy}{dx} - 5y = 0.$
5. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3y = 0.$
6. $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0.$
7. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0.$
8. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0.$
9. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0.$
10. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 25y = 0.$
11. $\frac{d^2y}{dx^2} + 9y = 0.$
12. $4\frac{d^2y}{dx^2} + y = 0.$
13. $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = 0.$
14. $4\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = 0.$
15. $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} - 8y = 0.$
16. $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0.$
17. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0.$
18. $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0.$
19. $\frac{d^5y}{dx^5} - 2\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} = 0.$
20. $\frac{d^4y}{dx^4} - \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0.$
21. $\frac{d^4y}{dx^4} - 3\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 12y = 0.$
22. $\frac{d^4y}{dx^4} + 6\frac{d^3y}{dx^3} + 15\frac{d^2y}{dx^2} + 20\frac{dy}{dx} + 12y = 0.$
23. $\frac{d^4y}{dx^4} + y = 0.$
24. $\frac{d^5y}{dx^5} = 0.$

Solve the initial-value problems in Exercises 25–42:

25. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0, \quad y(0) = 3, \quad y'(0) = 5.$

26. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 0, \quad y(0) = -4, \quad y'(0) = 2.$

27. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0, \quad y(0) = 1, \quad y'(0) = 6.$

28. $3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0, \quad y(0) = 2, \quad y'(0) = -4.$

29. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0, \quad y(0) = 2, \quad y'(0) = -3.$

30. $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0, \quad y(0) = 4, \quad y'(0) = 9.$

31. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, \quad y(0) = 3, \quad y'(0) = 7.$

32. $9\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + y = 0, \quad y(0) = 3, \quad y'(0) = -1.$

33. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 29y = 0, \quad y(0) = 0, \quad y'(0) = 5.$

34. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 58y = 0, \quad y(0) = -1, \quad y'(0) = 5.$

35. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0, \quad y(0) = 3, \quad y'(0) = -1.$

36. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0, \quad y(0) = 2, \quad y'(0) = 6.$

37. $9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0, \quad y(0) = 6, \quad y'(0) = 0.$

38. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 37y = 0, \quad y(0) = 2, \quad y'(0) = -4.$

39. $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 2.$

40. $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0, \quad y(0) = 2, \quad y'(0) = 0, \quad y''(0) = 0.$

41. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0, \quad y(0) = 1, \quad y'(0) = -8, \quad y''(0) = -4.$

42. $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 5y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 6.$