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Q. How many elements of order 10 in $\mathbb{Z}_{100}$?

Sol^n: $\mathbb{Z}_{100} = \{0, 1, 2, 3, \ldots, 98, 99\}$

If G is a cyclic group of order n and d is a divisor of n, then no. of elements of order d are $\phi(d)$.

$\because$ $\mathbb{Z}_{100}$ is a cyclic group of order 100.

and $10 \mid 100$ (10 divides 100)

$\therefore$ No. of elements of order 10 in $\mathbb{Z}_{100}$ are

$$\phi(10) = 10\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 4.$$

Theorem: In a finite group, the no. of elements of order d is a multiple of $\phi(d)$.

Proof: Let G be a finite group of order n.

If there is no element of order d in G, then we are done, because $\phi(d)$ divides 0.

Now suppose that $a \in G$ such that $|a| = d$. then the subgroup $\langle a \rangle$ has exactly $\phi(d)$ elements of order d.

If these $\phi(d)$ elements are only elements of order d in group G, then we are done.

Suppose there exist an element $b \in G$ such that $b \notin \langle a \rangle$ and $|b| = d$.

then the subgroup $\langle b \rangle$ has exactly $\phi(d)$ elements of order d. In this case, there are $2\phi(d)$ elements of order d in $\langle a \rangle$ and $\langle b \rangle$.

If these are the only elements of order d in group G, then we are done.

because $\langle a \rangle$ & $\langle b \rangle$ have no elements in common as $b \notin \langle a \rangle$.

Continuing in this manner, we can see that no. of elements of order d in group G are multiple of $\phi(d)$.

Chapter 5 Permutation Groups:

Permutation:

Let A be a set. A function $\alpha : A \to A$ is called a permutation of set A if

(i) α is one-one

(ii) α is onto.

Example: Suppose $A = \{1, 2, 3\}$ is a set

A permutation α of set A is defined as:

$\alpha(1) = 1$, $\alpha(2) = 3$, $\alpha(3) = 2$.