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09:16

Q. (40) A non cyclic group of order 4.

cyclic group of order 4.

$(1\,2\,3\,4) \rightarrow$ order 4.

$H_1 = \langle (1\,2\,3\,4) \rangle = \{(1\,2\,3\,4), (1\,3)(2\,4), (1\,4\,3\,2), \varepsilon\}$

Non cyclic subgroup of order 4.

$H_2 = \{\varepsilon, (12), (34), (12)(34)\} \rightarrow$ finite subgroup test.

$(12)\,(12)(34) = (34)$

$(34)\,(12)\,(34) = (12)\,(34)^2 = (12)$

$(12)\,(34)\;\;(12)(34) = (12)\,(12)\,(34)(34) = \varepsilon$

Q. 41. Suppose that $\beta$ is a 10-cycle. For which integers i between 2 and 10 is $\beta^i$ also a 10-cycle?

Soln. $\beta$ is a 10-cycle,

$|\beta| = 10$

$\beta^i$ is a 10-cycle iff $|\beta^i| = 10$

$$|\beta| = |\beta^i| = 10 \quad \text{iff } \gcd(i,10) = 1$$

$$\Rightarrow i = 1, 3, 7, 9$$

$\therefore$ values of $i$ between 2 and 10 are 3, 7 and 9.

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Q (48) $S_7$ $x^2 = (1234)$

Show that in $S_7$, the equation ~~x2 5 (1234)~~ has no solutions but the equation ~~x3 5 (1234)~~ has at least two.

$x^3 = (1234)$ has at least two sln's

Sol^n's:

~~In $S_7$~~ $(1234) = (1\ 4)(1\ 3)(1\ 2)$

$\Rightarrow (1234)$ is an odd-permutation.

But in $S_7$ for every permutation $\alpha$, $\alpha^2$ is an even permutation.

$\because$ odd + odd = even
even + even = even

$\therefore x^2 = (1234)$ has no sol^n.

sol^n of. $x^3 = (1234)$. $\Rightarrow |(1234)| = 4$.

$$|x^3| = 4 \Rightarrow (x^3)^4 = \varepsilon \Rightarrow x^{12} = \varepsilon$$

$\Rightarrow |x|$ can be 1 or 2 or 4 or 6 or 3 or 12.

$x^2 = (1234) \to 2$-shift

$x^3 = (1234) \to 3$-shift $\Rightarrow x = (1\,4\,3\,2)$

$(1\,4\,3\,2)^3 = \,?$

$(1\,4\,3\,2)^2 = (1\,3)(2\,4)$

$(1\,4\,3\,2)^3 = (1\,3)(2\,4)(1\,4\,3\,2) = (1\,2\,3\,4)$

$\therefore x = (1\,4\,3\,2)$ is a solⁿ of $x^3 = (1\,2\,3\,4)$

In $S_7$, $x = (1\,4\,3\,2)(5\,6\,7)$ is also a solⁿ of $x^3 = (1\,2\,3\,4)$

and $x = (1\,4\,3\,2)(5\,7\,8)$ is also a solⁿ of $x^3 = (1\,2\,3\,4)$ $\left[\because (5\,6\,7)^3 = \varepsilon\right]$

there are atleast three solⁿs. of eqⁿ $x^3 = (1\,2\,3\,4)$.

~~Chapter-6. After 37~~

38, 39, 40, 41, 42, 43, 45, 51 to 55, 57 to 61,

<u>Chapter-10</u>

Q. 1 to 10, 12, 14 to 16, 18 to 31, 33, 35, 36, 37, 39, 40 to 44, 46, 49, 50 to 56, 58 to 60, 62 to 65.