

## Scalar: Tensor of Rank 0

It remains invariant under coordinate transformation.

for example-size, mass, temperature, distance, speed et\_cetera are the scalar quantities, that is, they have only magnitude and no direction.

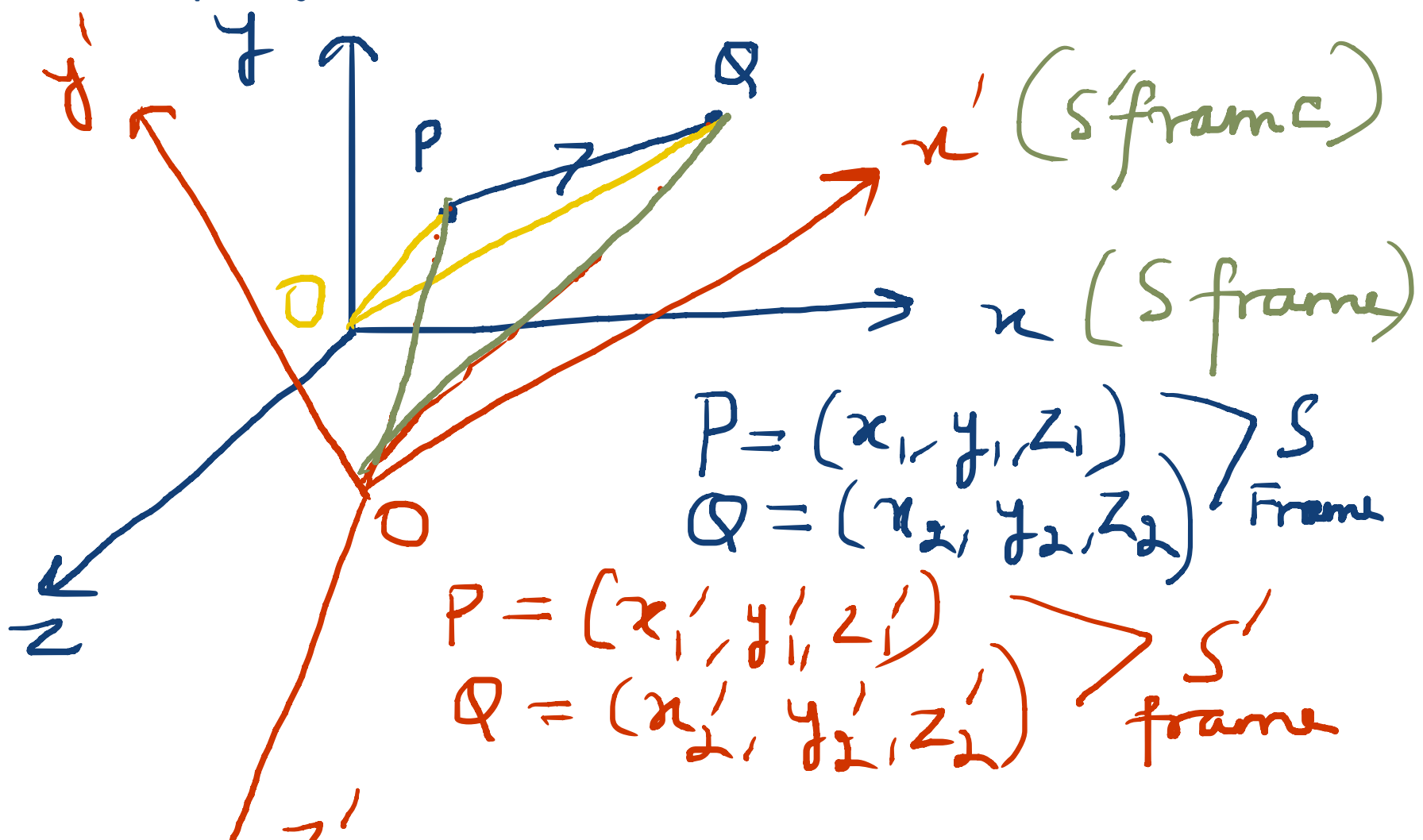
Hence they are independent of the coordinate transformation.

A scalar quantity is a tensor of rank zero, this means, that a scalar quantity requires zero or no basis to describe itself.

It is represented as  $A$ .

# Vector: Tensor of Rank 1

It is a quantity which carries both magnitude as well as direction



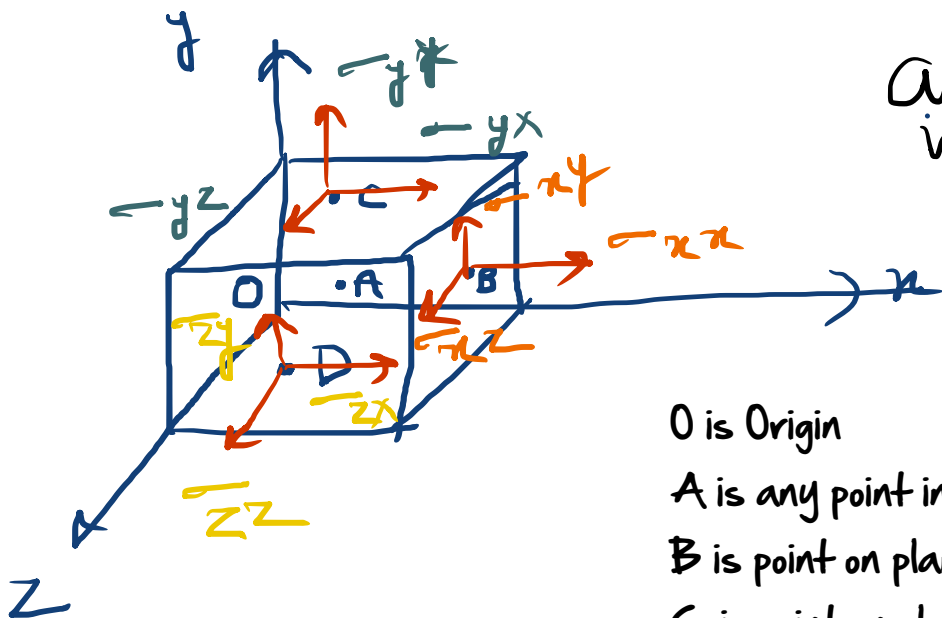
When we go from one coordinate system  $S$  to another coordinate system  $S'$ , then the components of  $P$  and  $Q$  in itself changes but the vector as such i.e. the nature of the vector does not change, its magnitude does not change, it is pointing from  $P$  to  $Q$  only in another frame of reference. This implies that if an object does not vary under coordinate transformation by following a set of transformation rules then that object is called as a tensor. The only thing that changes is the components of tensor.

A vector is a tensor of rank one that is it requires one basis vector ( $x$  or  $y$  or  $z$ ) to describe its each component. For example  $A_y, A_z$

In 3D ( $d=3$ ), number of components of such a vector / tensor of rank 1 ( $n=1$ ) is 3 (i.e  $d^n$ )

Note that pseudo vector is not a tensor because it does not satisfy those transformation rules.

Stress Tensor: Tensor of Rank 2



$\vec{x}, \vec{y}$  → direction of force  
 area vector (plane)

O is Origin

A is any point in the interior facing all the stresses

B is point on plane with area vector  $x$  undergoing 3 stresses

C is point on plane with area vector  $y$  undergoing 3 stresses

D is point on plane with area vector  $z$  undergoing 3 stresses

Total Required Components to describe all forces at A: 9

3 elongations or stretching

6 shearing

Set of Basis Vectors required to describe each component  $n=2$  ( $n$  is the rank of the tensor)

For eg  $xy, xz, yy, \dots$  etc

Dimension of system  $d=3$

So, the components of a rank 2 tensor in 3D is  $3^2 = d^n = 9$

Representation of a tensor of Rank 2:  $A_{xy}, A_{yz}, A_{ij}$   
 etc