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~~28 Aug~~

Date

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Q8 $\mathbb{C}$ = Set of Complex nos.

Define $(x_1, y_1) \le (x_2, y_2)$
iff $x_1 \le x_2$ & $y_1 \le y_2$ in $\mathbb{R}$ — (1)

Verify $\le$ is reflexive
antisymmetric
& transitive

$\therefore (\mathbb{C}, \le)$ is a Poset.

Verify '$\le$' relation defined by (1) is not symmetric

Lexicographic Order

We define another relation on $\mathbb{C}$

$(x_1, y_1) \le (x_2, y_2)$
iff either ~~$x_1 \le x_2$~~ $x_1 < x_2$
or $x_1 = x_2$ & $y_1 \le y_2$. — (2)

Verify $\le$ (defined by (2)) is a Partial order relation on $\mathbb{C}$.

$\therefore (\mathbb{C}, \le)$ is a Poset.

Q10 $C[a,b]$ = set of all continuous real valued fns on $[a,b]$

Define $f \le g$ in $C[a,b]$ iff $f(x) \le g(x)$ $\forall x \in [a,b]$

Verify $\le$ is a Partial order relation on $C[a,b]$.

$\therefore (C[a,b], \le)$ is a Poset.

Define $f \wedge g : [a,b] \longrightarrow \mathbb{R}$
as $(f \wedge g)(x) = \min \{f(x), g(x)\}$ $\forall x \in [a,b]$

& $f \vee g : [a,b] \longrightarrow \mathbb{R}$
as $(f \vee g)(x) = \max \{f(x), g(x)\}$ $\forall x \in [a,b]$

Then $f \wedge g$, $f \vee g \in C[a,b]$
($\because$ min & max are cts. fns)

Verify : Commutativity, Associativity & Absorption laws of $\wedge$ & $\vee$ in $C[a,b]$

$\therefore (C[a,b], \leq, \wedge, \vee)$ is a lattice.

Define $D[a,b]$ = set of all differentiable real valued fns on $[a,b]$
$\subseteq C[a,b]$

$(D[a,b], \leq)$ is a Poset with induced order.

But $D[a,b]$ is not a Sublattice of $C[a,b]$

ex $f = \sin : [0, 2\pi] \longrightarrow \mathbb{R}$, $f \in D[0, 2\pi]$
$g = \cos : [0, 2\pi] \longrightarrow \mathbb{R}$, $g \in D[0, 2\pi]$
$f \vee g : [0, 2\pi] \longrightarrow \mathbb{R}$
$(f \vee g)(x) = \max\{f(x), g(x)\}$.

$f \vee g$ is cts on $[0, 2\pi]$
But $f \vee g$ is not differentiable at $\frac{\pi}{4}$
$\therefore f \vee g \notin D[0, 2\pi]$
$\therefore D[0, 2\pi]$ is not closed w.r.t $\vee$ of $C[0, 2\pi]$
$\therefore D[0, 2\pi]$ is not a Sublattice of $C[0, 2\pi]$.

Q16 If $\exists$ 2 chains each containing 2 or more elts, then their cross product cannot be a chain.

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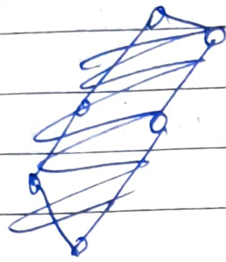
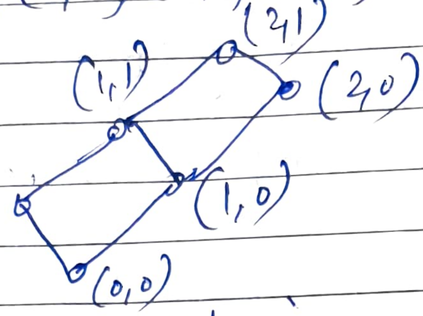
ex $C_1 = \{0, 1, 2\}$
$C_2 = \{0, 1\}$
$C_1 \times C_2 = \{(0,0), (0,1), (1,0), (1,1), (2,0), (2,1)\}$





It is not a chain.

Defn Distributive lattice -
A lattice L is called distributive if
(D1) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
& (D2) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$
$\forall x, y, z \in L$.

D1 & D2 are called distributive equalities.

Prove $D1 \Rightarrow D2$
Let $x, y, z \in L$
Consider RHS of (D2) $= (x \vee y) \wedge (x \vee z)$
$= [(x \vee y) \wedge x] \vee [(x \vee y) \wedge z]$ (By D1)
$= x \vee [(x \vee y) \wedge z]$ (By absorption)
$= x \vee [(x \wedge z) \vee (y \wedge z)]$ (By D1)
$= [x \vee (x \wedge z)] \vee (y \wedge z)$ (By Associativity)
$= x \vee (y \wedge z)$
$=$ LHS of D2
$\therefore D1 \Rightarrow D2$
Similarly $D2 \Rightarrow D1$
$\therefore D1 \Leftrightarrow D2$