Duration: 3Hrs. **Marks:100** **Credits: 6**

Course Objective

The objective of this course is to familiarize the students with the basic mathematical tools with special emphasis on applications to business and economic situations.

Course Learning Outcomes

After completing the course, the student shall be able to:

CO1: comprehend the concept of systematic processing and interpreting the information in quantitative terms to arrive at an optimum solution to business problems.
CO2: develop proficiency in using different mathematical tools (matrices, calculus, linear programming, and mathematics of finance) in solving daily life problems.
CO3: acquire competence to use computer for mathematical computations, especially with Big data.
CO4: obtain critical thinking and problem-solving aptitude.
CO5: evaluate the role played by mathematics in the world of business and economy.

Course Contents

Unit I: Matrices and Determinants

1.1 Definition and types of matrix, Algebra of matrices, Inverse of a matrix- Business Applications.
1.2 Solution of system of linear equations (having unique solution and involving not more than three variables) using matrix inversion method and Cramer's Rule.
1.3 Leontief Input Output Model (Open Model Only).

Unit II: Basic Calculus

2.1 Mathematical functions and their types (linear, quadratic, polynomial, exponential, logarithmic and logistic function). Concepts of limit and continuity of a function.
2.2 Concept of Marginal Analysis. Concept of Elasticity, Applied Maxima and Minima problems including effect of Tax on Monopolist's Optimum price and quantity, Economic Order Quantity.

Unit III: Advanced Calculus

3.1 Partial Differentiation: Partial derivatives up to second order. Homogeneity of functions and Euler's theorem. Total differentials. Differentiation of implicit functions with the help of total differentials.
3.2 Maxima and Minima involving two variables – Applied optimization problems and Constraint optimization problems using La grangean multiplier involving two variables having not more than one constraint.
3.3 Integration: Standard forms & methods of integration- by substitution, by parts and by use of partial fractions. Definite integration. Finding areas in simple cases
3.4 Application of Integration to marginal analysis; Consumer's and Producer's Surplus. Rate of sales, The Learning Curve.

Unit IV: Mathematics of Finance

4.1 Rates of interest: nominal, effective and their inter-relationships in different compounding situations.
4.2 Compounding and discounting of a sum using different types of rates. Applications relating to Depreciation of assets and Equation of value.
4.3 Types of annuities: ordinary, due deferred, continuous, perpetual. Determination of future and present values using different types of rates of interest. Applications relating to Capital expenditure, Leasing, Valuation of simple loans and debentures, sinking fund.
(excluding general annuities).

Unit V: Linear Programming

5.1 Formulation of Linear programming problems (LPPs). Graphical solutions of LPPs. Cases of unique solutions, multiple optional solutions, unbounded solutions, infeasibility, and redundant constraints.
5.2 Solution of LPPs by simplex method - maximization and minimization cases. Shadow prices of the resources, Identification of unique and multiple optimal solutions, unbounded solution, infeasibility and degeneracy.
5.3 The dual problem: Formulation, relationship between Primal and Dual LPP, Primal and Dual solutions (excluding mixed constraints LPPs). Economic interpretation of the dual.

Practical Lab

In addition to the lectures, the students are expected to work on a software package for solving linear programming problems, problems related to mathematics of finance and analyze the results obtained there from. This will be evaluated through internal assessment.

References

Mathematics for Business Studies

Dr. J. K. Thukral

Matrices

$\Downarrow$

Rectangular arrangements of numbers.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix} \rightarrow \text{Rows}$$

$\downarrow$ Columns

$(3\times3) \rightarrow$ order of matrix

R × C

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & 6 & 7 \end{bmatrix}_{(2\times3)} \qquad C = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{(3\times1)}$$

Addition of Matrices :- When order of both the matrices are same.

Ex $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $A+B$ = Not Possible

Ex $A = \begin{bmatrix} 2 & 1 & 0 \\ 5 & 7 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -3 & 2 \\ 1 & 4 & 6 \end{bmatrix}$

$$A+B = \begin{bmatrix} 2+5 & 1-3 & 0+2 \\ 5+1 & 7+4 & -1+6 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & -2 & 2 \\ 6 & 11 & 5 \end{bmatrix}$$

Subtraction of Matrices. :- Same as in Addition

$$A-B = \begin{bmatrix} 2-5 & 1-(-3) & 0-2 \\ 5-1 & 7-4 & -1-6 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & 4 & -2 \\ 4 & 3 & -7 \end{bmatrix}$$

Multiplication of Matrices

Two matrices can be multiplied only if

No. of Columns of first matrix $=$ No. of Rows of second Matrix

Raw Matrix

$$A = \begin{bmatrix} 5 & 6 & 3 \\ 2 & 4 & 5 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 3 & 4 \\ 2 & 5 \\ 1 & 6 \end{bmatrix}_{3\times 2} \qquad AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{23} \end{bmatrix}_{2\times 2}$$

a_{11}

R C

$$AB = \begin{bmatrix} 5\times3+6\times2+3\times1 & 5\times4+6\times5+3\times6 \\ 2\times3+4\times2+5\times1 & 2\times4+4\times5+5\times6 \end{bmatrix}$$

$$= \begin{bmatrix} 15+12+3 & 20+30+18 \\ 6+8+5 & 8+20+30 \end{bmatrix} = \begin{bmatrix} 30 & 68 \\ 19 & 58 \end{bmatrix} \text{ As/}$$

Ex find AB if $A = \begin{bmatrix} 1 & -4 \\ 5 & 3 \\ 0 & 2 \end{bmatrix}_{3\times 2}$ $B = \begin{bmatrix} -2 & 4 & 1 & 6 \\ 2 & 7 & 3 & 8 \end{bmatrix}_{2\times 4}$

As $AB = \begin{bmatrix} -10 & -24 & -11 & -26 \\ -4 & 41 & 14 & 54 \\ 4 & 14 & 6 & 16 \end{bmatrix}_{3\times 4}$

Ex 1.1

Q:5 $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$ Calculate AB.

Can you calculate BA?

As $AB = \begin{bmatrix} 28 & 64 \\ 6 & 0 \\ 13 & 8 \end{bmatrix}$

Determinants

Determinant can be formed only when no. of rows are equal to no. of columns.

$$A = \begin{bmatrix} 3 & 4 \\ 9 & -7 \end{bmatrix} \qquad |A| = \begin{vmatrix} 3 & 4 \\ 9 & -7 \end{vmatrix}$$

$$= 3\times(-7) - 9\times4 \Rightarrow -57$$

$$\begin{array}{cc} & \begin{array}{ccc} a_{ij} & a_{ij} & a_{ij} \end{array} \\ \text{Ex} \quad |B| = & \begin{vmatrix} 1 & -3 & 4 \\ 0 & 2 & 5 \\ -2 & 6 & 3 \end{vmatrix} \end{array} \qquad \underline{a_{11} \quad a_{12} \quad a_{13}} \qquad C_{ij} = \begin{cases} M_{ij}, & \text{if even} \\ -M_{ij}, & \text{if } i+j \text{ odd} \end{cases}$$

Cofactor

$$= 1 \underbrace{\begin{vmatrix} 2 & 5 \\ 6 & 3 \end{vmatrix}}_{\text{Minor}} - (-3) \underbrace{\begin{vmatrix} 0 & 5 \\ -2 & 3 \end{vmatrix}}_{\text{cofactor}} + 4 \begin{vmatrix} 0 & 2 \\ -2 & 6 \end{vmatrix}$$

$$= 1(6-30) + 3(0+10) + 4(0+4)$$

$$= -24 + 30 + 16 = 22$$

Ex = 1.2

① $|A| = \begin{vmatrix} 3 & 2 \\ -5 & -4 \end{vmatrix}$

② (a) $\begin{vmatrix} 1 & 2 & -3 \\ 4 & 5 & 4 \\ 3 & -2 & 1 \end{vmatrix}$

Ans -2

A- 98

Transpose of Matrix

$\Downarrow$

Interchange of rows and column $[A' \text{ or } A^T]$

$$A = \begin{bmatrix} 2 & -4 & 6 \\ 3 & 1 & 4 \end{bmatrix} \qquad A' = \begin{bmatrix} 2 & 3 \\ -4 & 1 \\ 6 & 4 \end{bmatrix}$$

Ans

Adjoint of Matrix

$\Rightarrow$ Transpose of Cofactor Matrix $[C_{ij}]^T$.

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Example **22 .** Find the adjoint of matrix **A** $= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 4 & 3 & 2 \end{bmatrix}$.

Solution . We have

$$\begin{array}{lll} C_{11} = -3 & C_{12} = 6 & C_{13} = -3 \\ C_{21} = 5 & C_{22} = -10 & C_{23} = 5 \\ C_{31} = 2 & C_{32} = -4 & C_{33} = 2 \end{array}$$

Thus,

$$\text{adj } \mathbf{A} = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & 5 & 2 \\ 6 & -10 & -4 \\ -3 & 5 & 2 \end{bmatrix}.$$

Properties of the Adjoint of a Matrix

1. *If* **A** *is a square matrix of order n, then* **A** (adj **A**) $= |\mathbf{A}|\, I_n =$ (adj **A**) **A**, where I_n is a square matrix of order n.

2. *If* **A** *is a square matrix of order n, then adj* (**A'**) = (*adj* **A**)'.

3. *If* **A** *and* **B** *are two square matrices of the same order, then adj* **(AB)** = adj **(B)** adj **(A)**.

2. Find the adjoint of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 7 & 8 & 3 \\ 5 & 9 & 2 \end{bmatrix}$.

$$\operatorname{adj} \mathbf{A} = \begin{bmatrix} -11 & 46 & -36 \\ 1 & -26 & 36 \\ 23 & 2 & -12 \end{bmatrix}$$

Inverse of a Square Matrix → Only Singular square matrix is

$A^{-1} = \frac{1}{|A|} \text{adj } A$ i.e if $|A| = 0$ inverse is not possible.

Some properties

① Non-Singular matrix $|A| \neq 0$ is invertible

② $(A^{-1})^{-1} = A$

③ $(A')^{-1} = (A^{-1})'$

④ $(AB)^{-1} = B^{-1}A^{-1}$ (Reversal law)

Ex-1.3

Q6) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. Compute A^{-1}

$|A| = 0(2-3) - 1(1-9) + 2(1-6) \Rightarrow 8-10 \Rightarrow -2 \neq 0$

So A^{-1} exist

$C_{11} = -1$ $C_{12} = +8$ $C_{13} = -5$

$C_{21} = +1$ $C_{22} = -6$ $C_{23} = +3$

$C_{31} = -1$ $C_{32} = 2$ $C_{33} = -1$

$A^{-1} = \frac{1}{|A|} \text{adj} A$

$A^{-1} = -\frac{1}{2} \begin{vmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{vmatrix}$ Ans/

Q5 $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{bmatrix}$. find A^{-1}

$\frac{-1}{8} \begin{bmatrix} 20 & -16 & 4 \\ -10 & 6 & -2 \\ -2 & 2 & -2 \end{bmatrix}$

System of Linear Equations

$$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = b_2$$

$$\vdots$$

$$A = \begin{bmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

we need to find value of x_1 & x_2 .....

Methods

Matrix Inverse Method

$(AX = B)$

$$X = A^{-1}B$$

Cramer's Rule [Determinant Rule]

Apply only if $D \neq 0$

$$x_1 = \frac{D_1}{D} \;..\; x_2 = \frac{D_2}{D} \; \dots \text{ and so on}$$

Example **27.** Using matrix inversion method, solve the following system of equations for x, y and z.

$$x + y + z = 5; \quad 2x + y - z = 2; \quad 2x - y + z = 2$$

Solution . The given system can be expressed in the matrix form as

$$\mathbf{AX = B},$$

where $\quad \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad$ and $\quad \mathbf{B} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$

Now $\quad |\mathbf{A}| = 1\,(1-1) - 1\,(2+2) + 1\,(-2-2) = 0 - 4 - 4 = -8 \neq 0.$

Since $|\mathbf{A}| \neq 0$, $\mathbf{A}^{-1}$ exists and the system has a unique solution given by $\mathbf{X = A^{-1}\,B}$. It can be easily verified that

$$\mathbf{A}^{-1} = -\frac{1}{8}\begin{bmatrix} 0 & -2 & -2 \\ -4 & -1 & 3 \\ -4 & 3 & -1 \end{bmatrix}$$

Thus, $\quad \mathbf{X = A^{-1}\,B} = -\dfrac{1}{8}\begin{bmatrix} 0 & -2 & -2 \\ -4 & -1 & 3 \\ -4 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} = -\dfrac{1}{8}\begin{bmatrix} -8 \\ -16 \\ -16 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Q: (iv) $2x - y - z = 7$

$= \underline{\underline{1.4}}$ $3x + y - z = 7$

$x + y - z = 3$

As. $x = 2$ $z = -2$

$y = -1$

Cramer's Rule

Ex

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

$$D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix} \Rightarrow 5(50) + 7(-33) + 1(36) = 55$$

$$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix} \Rightarrow 11(50) + 7\overset{(-90+7)}{(\cancel{-15+7})} + 1(86)$$
$$\Rightarrow 550 - 581 + 86 \Rightarrow 55$$

$$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} \quad 5(-90+7) - 11(-36+3) + 1(42-45)$$
$$\Rightarrow -415 + 363 - 3 \Rightarrow -55$$

$$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix} \Rightarrow 5(-56+30) + 7(42-45) + 11(12+24)$$
$$\Rightarrow 5(-86) - 21 + 11(36)$$
$$\Rightarrow -430 - 21 + 396$$
$$\Rightarrow -55$$

$$x = \frac{D_1}{D} \Rightarrow \frac{55}{55} = 1$$
$$y = \frac{D_2}{D} = \frac{-55}{55} = -1$$
$$z = \frac{D_3}{D} \Rightarrow \frac{-55}{5} = -1$$

Ans/.

Solve the following systems of equations using Cramer's rule :

(*i*) $5x - 7y + z = 11;\quad 6x - 8y - z = 15;\quad 3x + 2y - 6z = 7$

(*ii*) $x + 2y - 2z = -7;\quad 2x - y + z = 6;\quad x - y - 3z = -3$

(*iii*) $6x + y - 3z = 5;\quad x + 3y - 2z = 5;\quad 2x + y + 4z = 8$

(*iv*) $2x - 3y - 4z = 29;\quad -2x + 5y - z = -15;\quad 3x - y + 5z = -11$

CHAPTER

2

APPLICATIONS OF MATRICES TO BUSINESS AND ECONOMICS

$$\mathbf{A+B} = \begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \end{bmatrix}$$

Example **2 .** The sales figure for two car dealers during January showed that Dealer A sold 5 deluxe, 3 premium and 4 standard cars, while Dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the 2-month period of January-February revealed that Dealer A sold 8 deluxe, 7 premium and 6 standard cars. In the same 2-month period, Dealer B sold 10 deluxe, 5 premium and 7 standard cars.

Write 2×3 matrices summarizing sales data for January and the 2-month period for each dealer. Hence find the sales in February for each dealer.

***Solution* .** The sales for the month of January can be represented by the matrix

$$\mathbf{A} = \begin{array}{c} \\ \textit{Dealer A} \\ \textit{Dealer B} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Deluxe} & \textit{Premium} & \textit{Standard} \end{array} \\ \begin{bmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{bmatrix} \end{array}$$

Each row of **A** gives the number of each model sold by a given dealer. The sales for the 2-month period can be represented by the matrix

$$\mathbf{B} = \begin{bmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{bmatrix}$$

The sales for the month of February is equal to the sales for the 2-month period of January-February minus the sales for the month of January. Thus the sales in February is

$$\mathbf{B-A} = \begin{bmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 3 & 3 & 4 \end{bmatrix}.$$

***Example* 5 .** A transport company uses 3 types of trucks T_1, T_2 and T_3, to transport 3 types of vehicles V_1, V_2 and V_3. The capacity of each truck in terms of 3 types of vehicles is given below :

	V_1	V_2	V_3
T_1	1	3	2
T_2	2	2	3
T_3	3	2	2

Using matrix method, find the number of vehicles of each type which can be transported if company has 10, 20 and 30 trucks of each type respectively.

[Delhi Univ. B.Com. (H) 1999, 2006 (SOL)]

***Solution* .** The capacity of each truck in terms of 3 types of vehicles can be represented by the matrix

$$\mathbf{X} = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \end{array} \begin{array}{c} \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix} \end{array}$$

The number of trucks of each type used by the company can be represented by the column matrix

$$\mathbf{Y} = \begin{array}{c} T_1 \\ T_2 \\ T_3 \end{array} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

The total number of vehicles of each type which can be transported is given by the matrix product

$$\mathbf{XY} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 140 \\ 130 \\ 140 \end{bmatrix}$$

Thus the number of 3 types of vehicles V_1, V_2 and V_3 which can be transported are :

$$V_1 = 140, \quad V_2 = 130 \quad \text{and} \quad V_3 = 140$$

Example **8 .** Mr. A went to a market to purchase 3 kg of sugar, 10 kg of wheat and 1 kg of salt. In a shop near to Mr. A residence, these commodities are priced at ₹ 20, ₹ 10 and ₹ 8 per kg whereas in the local market these commodities are priced at ₹ 15, ₹ 8 and ₹ 6 per kg respectively. If cost of travelling to local market is ₹ 25, find the net savings of Mr. A using matrix multiplication. **[*Delhi Univ. B.Com. (H) 2011*]**

***Solution* .** The quantity of sugar, wheat and salt purchased by Mr. A can be represented by the matrix

$$\mathbf{Q} = \begin{array}{c} \\ A \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Sugar} & \textit{Wheat} & \textit{Salt} \end{array} \\ \begin{bmatrix} 3 & \quad 10 \quad & 1 \end{bmatrix} \end{array}$$

The prices of these commodities in the nearby shop and the local market can be represented by the matrix

$$\mathbf{P} = \begin{array}{c} \\ \textit{Sugar} \\ \textit{Wheat} \\ \textit{Salt} \end{array}\begin{array}{c} \begin{array}{cc} \textit{Shop} & \textit{Local Market} \end{array} \\ \begin{bmatrix} 20 & 15 \\ 10 & 8 \\ 8 & 6 \end{bmatrix} \end{array}$$

The total cost of purchasing at two different places is given by

$$\mathbf{QP} = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix}\begin{bmatrix} 20 & 15 \\ 10 & 8 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 168 & 131 \end{bmatrix}$$

$\therefore$ Cost of purchasing from a nearby shop = ₹ 168

and cost of purchasing from the local market = ₹ 131

However, cost of travelling to local market = ₹ 25

$\therefore$ Net savings of Mr. A = $168 - 131 - 25$ = ₹12 .

$$
\mathbf{A} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array}
\begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{array}
$$

Using matrix notations, find

(*i*) The total requirement of each material if the firm produces 100 units of each product.

(*ii*) The per unit cost of production of each product if the per unit costs of materials M_1, M_2 and M_3 are ₹ 5, ₹ 10 and ₹ 5 respectively, and

(*iii*) The total cost of production if the firm produces 200 units of each product.

[*Delhi Univ. B.Com. (H) 1998*]

Solution . (*i*) The total requirement of each material if the firm produces 100 units of each product is given by the matrix product

$$
\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array}
\begin{array}{c} \begin{array}{ccc} P_1 & P_2 & P_3 \end{array} \\ \begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 5 & 2 \end{bmatrix} \end{array}
\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}
\begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}
=
\begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}
\begin{bmatrix} 800 \\ 900 \\ 800 \end{bmatrix}
$$

Thus the firm requires 800 units of material M_1, 900 units of material M_2 and 800 units of material M_3 to produce 100 units of each product.

(*ii*) The per unit cost of production of each product is given by the matrix product

$$
\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array}
\begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{array}
\begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}
\begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}
=
\begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}
\begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix}
$$

Thus the per unit costs of three products P_1, P_2 and P_3 are ₹ 45, ₹ 65 and ₹ 60 respectively.

(*iii*) The total cost of production if the firm produces 200 units of each product is given by

$$
\begin{bmatrix} 200 & 200 & 200 \end{bmatrix} \begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix} = \begin{bmatrix} 34{,}000 \end{bmatrix}
$$

Thus the total cost of production is ₹ 34,000.

4. A company has two plants. Plant 1 is capable of producing 5 items of A, 10 items of B and 3 items of C per one hour of operation. Plant 2 is capable of producing 5 items of A, 6 items of B and 6 items of C per one hour of operation. Express this information in a 3×2 matrix. Using matrix multiplication, determine the total number of items A, B and C produced if Plant 1 is operated for 10 hours and Plant 2 is operated for 5 hours.

Sol

$$A = \begin{array}{c c} & \begin{array}{cc} \text{I} & \text{II} \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 5 & 5 \\ 10 & 6 \\ 3 & 6 \end{bmatrix} \end{array} \qquad B = \begin{array}{c c} \begin{array}{c} \text{I} \\ \text{II} \end{array} & \begin{bmatrix} 10 \\ 5 \end{bmatrix} \end{array}$$

Total number of item

$$AB = \begin{bmatrix} 50+25 \\ 100+30 \\ 30+30 \end{bmatrix} \Rightarrow \begin{array}{cc} \begin{bmatrix} 75 \\ 130 \\ 60 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

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. In a certain city, there are 5 colleges and 20 schools. Each school has 3 peons, 1 clerk and 1 head clerk, whereas a college has 5 peons, 3 clerks, 1 head clerk and an additional staff as a caretaker. The monthly salary of each of them is as follows :

Peon– ₹ 1100, Clerk– ₹ 1700, Head Clerk– ₹ 3000 and Caretaker– ₹ 2500

Using matrix method, find

(*i*) the total number of posts of each kind in schools and colleges taken together,

(*ii*) the total monthly salary bill of each school and college, and

(*iii*) the total monthly salary bill of all the schools and colleges taken together.

Sol

$$A = \begin{array}{l} \\ \text{Head Clerk} \\ \text{Cashier} \\ \text{Clerk} \\ \text{Peon} \\ \text{Superintend } \cancel{\text{typist}} \\ \text{Typist} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Division} & \text{Dist.} & \text{zones} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$B = \begin{array}{l} \text{Div} \\ \text{Dist} \\ \text{zone} \end{array} \begin{bmatrix} 200 \\ 5 \\ 30 \end{bmatrix}$$

$$AB = \begin{bmatrix} 200+5+30 \\ 200+5+30 \\ 200+15+60 \\ 200+10+60 \\ 5 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 235 \\ 235 \\ 275 \\ 270 \\ 5 \\ 5 \end{bmatrix}$$

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(b) Total basic montly salary bill of each kind of office

Sol

$$A = \begin{array}{c} HC \\ Ca \\ C \\ B \\ S \\ T \end{array} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 2000 & 1750 & 1500 & 1000 & 5000 & 1500 \end{bmatrix}$$

$$CA = \begin{bmatrix} 2000 + 1750 + 1500 + 1000 & 16750 & 8750 \end{bmatrix}$$

$$\begin{array}{c} \\ CA = \end{array} \begin{array}{c} \begin{array}{ccc} Z & D & Dis^t \end{array} \\ \begin{bmatrix} 6250 & 16750 & 8750 \end{bmatrix} \end{array}$$

(c) Total salary in all office together

$$\begin{bmatrix} 2000 & 1750 & 1500 & 1000 & 5000 & 1500 \end{bmatrix} \begin{bmatrix} 235 \\ 235 \\ 275 \\ 270 \\ 5 \\ 5 \end{bmatrix}$$

$$\Rightarrow \left[570000 + \quad + 270000 + 25000 + 7500 \right.$$

Sol

$$A = \begin{array}{cc} \text{School} & \text{College} \\ [\,20 & 5\,] \end{array}$$

$$B = \begin{array}{c|cccc} & \text{Peon} & C & HC & \text{Caretaker} \\ S & 3 & 1 & 1 & 0 \\ C & 5 & 3 & 1 & 1 \end{array}$$

(i) Total no. of post $= A \times B$

$$\begin{bmatrix} 20 & 5 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 & 0 \\ 5 & 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 85 & 35 & 25 & 5 \end{bmatrix}$$

(ii) Monthly salary bill of each school & college

$$C = \begin{bmatrix} 1100 \\ 1700 \\ 3000 \\ 2500 \end{bmatrix}$$

$$\Rightarrow B \times C \Rightarrow \begin{bmatrix} 3 & 1 & 1 & 0 \\ 5 & 3 & 1 & 1 \end{bmatrix}_{2\times 4} \begin{bmatrix} 1100 \\ 1700 \\ 3000 \\ 2500 \end{bmatrix}_{4\times 1}$$

$$= \begin{bmatrix} 8000 \\ 16100 \end{bmatrix} \quad 1\cancel{6100} \text{ Rs/-}$$

iii) Total salary school + college.

$$\begin{bmatrix} 20 & 5 \end{bmatrix} \begin{bmatrix} 8000 \\ 16100 \end{bmatrix} \Rightarrow \begin{bmatrix} 240500 \end{bmatrix} \text{ Rs/-}$$

EXERCISE 2.2

1. An amount of ₹ 10,000 is put into three investments at the rate of interest of 10, 12 and 15 per cent per annum. The combined income is ₹ 1310 and the combined income of the first and second investment is ₹ 190 short of the income from the third. Find the investment in each using determinant method (Cramer's rule). [Delhi Univ. B.Com. (H) 1984]

CR = α·L

Solution of linear equation

① Let x, y and z are investment @ 10%, 12% and 15%.

$$x + y + z = 10000$$

$$10x + 12y + 15z = 131000$$

$$\frac{10x}{100} + \frac{12y}{100} = \frac{15z}{100} - 190 \Rightarrow 10x + 12y - 15z = -19000$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 10 & 12 & 15 \\ 10 & 12 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 131000 \\ -19000 \end{bmatrix}$$

$$D = -60$$

$$D_1 = \begin{vmatrix} 10000 & 1 & 1 \\ 131000 & 12 & 15 \\ -19000 & 12 & -15 \end{vmatrix} \Rightarrow -120000$$

$$D_2 = \begin{vmatrix} 1 & 10000 & 1 \\ 10 & 131000 & 15 \\ 10 & -19000 & -15 \end{vmatrix} \rightarrow -180000$$

$$D_3 = \begin{vmatrix} 1 & 1 & 10000 \\ 10 & 12 & 131000 \\ 10 & 12 & -19000 \end{vmatrix} \Rightarrow -300000$$

$$x = \frac{D_1}{D} \Rightarrow \frac{\cancel{-}120000}{\cancel{-}60} \quad y = \frac{D_2}{D} \Rightarrow 3000 \quad z = \frac{D_3}{D} \Rightarrow 5000$$

$$\Rightarrow 2000$$

6. The prices of three commodities X, Y and Z are, x, y and z per unit respectively. A purchases 4 units of Z and sells 3 units of X and 5 units of Y. B purchases 3 unit of Y and sells 2 units of X and 1 unit of Z. C purchases 1 unit of X and sells 4 units of Y and 6 units of Z. In the process, A, B, C earn ₹ 6000, ₹ 5000 and ₹ 13,000 respectively. Using matrices, find the prices per unit of the three commodities (note that selling the units is positive earning and buying the units is negative earning).

$Q6 \Rightarrow$ Prize of 3 commodities X, Y, Z are x, y and z
(Sale (+)ve and purchase (−)ve)

$$-4z + 3x + 5y = 6000$$
$$-3y + 2x + 1z = 5000$$
$$-1x + 4y + 6z = 13000$$

$$\Leftrightarrow \quad 3x + 5y - 4z = 6000$$
$$2x - 3y + 1z = 5000$$
$$-1x + 4y + 6z = 13000$$

$$\begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$$A \quad X = B$$

$$X = A^{-1} B$$

$$|A| = -151$$

$$A^{-1} = \frac{1}{-151} \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$X = \frac{-1}{151} \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix} \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix} \quad Ans/-$$

capacity. [Delhi Univ. B.Com. (H) 1988]

11. A salesman has the following record of sales during three weeks for the three items X, Y and Z which have different rates of commission.

Weeks	*Units sold*			*Total Commission (in ₹)*
	X	Y	Z	
I	40	30	20	270
II	50	50	40	450
III	60	30	10	260

Find out, using matrix method, the rates of commission on items X, Y and Z.

[*Delhi Univ. B.Com. (H) 1988*]

Sol 11

$$40x + 30y + 20z = 270$$
$$50x + 50y + 40z = 450$$
$$60x + 30y + 10z = 260$$

$$\begin{bmatrix} 40 & 30 & 20 \\ 50 & 50 & 40 \\ 60 & 30 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 270 \\ 450 \\ 260 \end{bmatrix}$$

$$D = -1000$$

$$D_1 = \begin{vmatrix} 270 & 30 & 20 \\ 450 & 50 & 40 \\ 260 & 30 & 10 \end{vmatrix} = -2000$$

$$D_2 = \begin{vmatrix} 40 & 270 & 20 \\ 50 & 450 & 40 \\ 60 & 260 & 10 \end{vmatrix} = -3000$$

$$D_3 = \begin{vmatrix} 40 & 30 & 270 \\ 50 & 50 & 450 \\ 60 & 30 & 260 \end{vmatrix} = -5000$$

$x = \frac{-2000}{-1000}$ 2 $\quad y = 3 \quad z = 5$

Sol 19 Let the daily Production of P_1 and P_2 are x & y.

$$\frac{x}{8} + \frac{y}{10} = 33 \qquad 5x + 4y = 1320$$

$$\Leftrightarrow$$

$$\frac{x}{12} + \frac{y}{12} = 25 \qquad 12x + 12y = 25$$

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19. A firm produces two products P_1 and P_2 passing through two machines M_1 and M_2 before completion. M_1 can produce either 8 units of P_1 or 10 units of P_2 per hour. M_2 can produce 12 units of either product per hour. Using matrix notations, determine:

(*i*) Production of P_1 and P_2 if time available on two machines is 33 hours and 25 hours respectively.

(*ii*) Per unit cost of production if cost of operating per hour on two machines is ₹ 400 and ₹ 300 respectively.

(*iii*) Total cost of production. **[*Delhi Univ. B.Com. (H) 2013*]**

20. A firm produces three products P_1, P_2 and P_3 processed on four machines M_1, M_2, M_3 and

matrix algebra. *[Delhi Univ. B.Com. (H) 1994]*

24. A firm has two service departments, S_1, S_2 and three production departments, P_1, P_2 and P_3. The direct cost of each department and the percentage of the total cost of each department allocated to various departments are given below :

Service Department	*Percentages to be Allocated to Departments*				
	S_1	S_2	P_1	P_2	P_3
S_1	0	10	20	40	30
S_2	20	0	50	10	20
Direct Cost (in ₹)	98,000	1,17,600	14,00,000	21,00,000	6,40,000

(*i*) Express the relevant simultaneous equations for determining the total cost (direct plus allocated) of each service department in matrix forms;

(*ii*) Determine the total cost of each service department using matrix method; and

(*iii*) Verify that the sum of the sevice costs allocated to the production departments equals the sum of the direct costs of the service departments.

Sol 19 Let the daily Production of P_1 and P_2 are x & y.

$$\frac{x}{8} + \frac{y}{10} = 33 \qquad 5x + 4y = 1320$$

$$\Leftrightarrow \qquad 1\cancel{2}x + 1\cancel{2}y = \cancel{300}\ 25$$

$$\frac{x}{12} + \frac{y}{12} = 25$$

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(i) $\begin{bmatrix} 5 & 4 \\ 12 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1320 \\ 300 \end{bmatrix}$

$|A| = 12$

$X = A^{-1}B$

$\cancel{= \frac{1}{12} \begin{bmatrix} 12 & -4 \\ -12 & 5 \end{bmatrix} \begin{bmatrix} 1320 \\ 300 \end{bmatrix}} \Rightarrow \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 1320 \\ 300 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 120 \\ 180 \end{bmatrix}$

(ii) Production cost per unit of

$$\begin{array}{c c} & \begin{array}{cc} M_1 & M_2 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \end{array} & \begin{bmatrix} 1/8 & 1/12 \\ 1/10 & 1/12 \end{bmatrix} \end{array} \begin{bmatrix} 400 \\ 300 \end{bmatrix} = \begin{bmatrix} 75 \\ 65 \end{bmatrix} \text{Rs/-}$$

(iii) $\begin{bmatrix} 120 & 180 \end{bmatrix} \begin{bmatrix} 75 \\ 65 \end{bmatrix} = 20700$ Rs/-

---

24) Let x denotes the total cost of the service department S_1 and y total cost of S_2

$x = 98000 + 0.20y \qquad \Leftrightarrow \qquad x - 0.2y = 98000$

$y = 117600 + 0.10x \qquad\qquad -0.1x - y = 117600$

(i) $\begin{bmatrix} 01 & -0.2 \\ -0.1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 98000 \\ 117600 \end{bmatrix}$

(ii) $X = A^{-1} B$

$$= \frac{1}{0.98}\begin{bmatrix} 1 & 0.2 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 98000 \\ 117600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1.02 & 0.204 \\ 0.102 & 1.02 \end{bmatrix} \begin{bmatrix} 98000 \\ 117600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 124000 \\ 130000 \end{bmatrix} \text{ ₹/-}$$

(iii) Sum of the service cost allocated to the production dept is given by

$$\begin{bmatrix} 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 124000 \\ 130000 \end{bmatrix} + \begin{bmatrix} 0.4 & 0.1 \end{bmatrix} \begin{bmatrix} 124000 \\ 130000 \end{bmatrix} + \begin{bmatrix} 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 124000 \\ 130000 \end{bmatrix}$$

$$\Rightarrow 89800 + 62600 + 63200$$

$$\Rightarrow 215600$$

which is same as the sum of direct cost of service dept

$$(98000 + 117600) \Rightarrow 215600$$

Leontief Input - Output Model

⇓

Given by Wassily Leontief (Noble Prize, 1973)

⇓

Based on Assumption that whatever produced is consumed

Not in Syllabus

Closed Model *

Production is Consumed internally

Open Model

internal + external bodies.

Economy "N" Industries.

Eg

	Steel	Construction	Transportation
Steel	I_{11}	I_{12}	I_{13}
Construction	I_{21}	I_{22}	I_{23}
Transport			

Note Output of one industry is used by other industries as well as Industry Itself.

Consumer

Producers.	I_1	I_2	I_3	....	I_n	Final Demand	Total Output
(kg) I_1	6	10	12	...	8	14	50 ✓
(litres) I_2	8	2	11		3	9	33 ✓
(gram) I_3	2	1	2		4	2	11
⋮							
($) I_n	5	3	2		1	5	16
Total Consumption	×						

Ex

Consupⁿ

	I_1	I_2	final DD	Output
I_1	5	5	10	30
I_2	3	17	5	25

Total output of I_1 = 30
" " " I_2 = 25

Input-Output Coefficient } ⇒ To produce one unit of output how much unit of others Industry are required

Let say

5 unit of I_1 = 30 output

$\frac{5}{30}$ Input = 1 output

Same 3 unit of I_2 = 30 output of I_1

$\frac{3}{30}$ Input = 1 output

	I_1	I_2
I_1	$\frac{5}{30}$	$\frac{5}{25}$
I_2	$\frac{3}{30}$	$\frac{17}{25}$

⟹ Input - Output coefficient

⇓ or

Technical Matrix

$$A = \begin{bmatrix} \frac{5}{30} & \frac{5}{25} \\ \frac{3}{30} & \frac{17}{25} \end{bmatrix}$$

⇒ for producing one unit of output
Input required from other industry.

Industry

Self | other Industries | Household & Govt.

Internal Consump^n + Final Demand = Total output produced.

$$AX + D = X$$

$$D = X - AX$$

$$D = (I - A)X$$

$$X = (I-A)^{-1} D$$

The Matrix $(I-A)$ is called Leontief Matrix

However, This is applicable only when $I-A$ is viable

i.e, $|I-A| > 0$ {Hawkins Simon Condition}

Ex - 2.3

Sol 1

$$A = \begin{bmatrix} \frac{300}{1000} & \frac{600}{2000} \\ \frac{400}{1000} & \frac{1200}{2000} \end{bmatrix} \Rightarrow D = \begin{bmatrix} 200 \\ 800 \end{bmatrix}$$

$$X = (I-A)^{-1} D$$

$$A = \begin{bmatrix} 3/10 & 3/10 \\ 2/5 & 3/5 \end{bmatrix}$$

$$[I-A] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3/10 & 3/10 \\ 2/5 & 3/5 \end{bmatrix} \Rightarrow \begin{bmatrix} 7/10 & -3/10 \\ -2/5 & 2/5 \end{bmatrix}$$

$$|I-A| = \frac{14}{50} - \frac{6}{50} \Rightarrow \frac{8}{50} \neq 0$$

$$(I-A)^{-1} = \frac{1}{|I-A|} \operatorname{adj}(I-A)$$

$$= \frac{50}{8} \begin{bmatrix} 2/5 & 3/10 \\ 2/5 & 7/10 \end{bmatrix}$$

$$X = \frac{50}{8} \begin{bmatrix} 2/5 & 3/10 \\ 2/5 & 7/10 \end{bmatrix} \begin{bmatrix} 200 \\ 800 \end{bmatrix}$$

$$\Rightarrow \frac{50}{8} \begin{bmatrix} 80 + 240 \\ 80 + 560 \end{bmatrix}$$

$$\Rightarrow \frac{50}{8} \begin{bmatrix} 320 \\ 640 \end{bmatrix} \Rightarrow \begin{bmatrix} 2000 \\ 4000 \end{bmatrix}$$ Ans/-

Sol 4 ⇒

$$A = \begin{bmatrix} 1/4 & 1/4 & 1/3 \\ 1/4 & 1/2 & 1/5 \\ 1/4 & 1/4 & 1/3 \end{bmatrix} \qquad D = \begin{bmatrix} 60 \\ 40 \\ 60 \end{bmatrix}$$

$$I-A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/4 & 1/4 & 1/3 \\ 1/4 & 1/2 & 1/5 \\ 1/4 & 1/4 & 1/3 \end{bmatrix}$$

$$I-A \Rightarrow \begin{bmatrix} 3/4 & -1/4 & -1/3 \\ -1/4 & 1/2 & -1/5 \\ -1/4 & -1/4 & 2/3 \end{bmatrix}$$

$$|I-A| = 0.095833$$

$$(I-A)^{-1} = \frac{1}{0.0958} \begin{bmatrix} 2/7 & 1/4 & 2/9 \\ 2/9 & 3/7 & 1/4 \\ 1/5 & 1/4 & 1/3 \end{bmatrix}$$

$$X = (I-A)^{-1} D$$

$$= \frac{1}{0.0958} \begin{bmatrix} 2/7 & 1/4 & 2/9 \\ 2/9 & 3/7 & 1/4 \\ 1/5 & 1/4 & 1/3 \end{bmatrix} \begin{bmatrix} 60 \\ 40 \\ 60 \end{bmatrix}$$

$$\Rightarrow \frac{1}{0.095833} \begin{bmatrix} 40 \\ 43.667 \\ 40 \end{bmatrix} \Rightarrow \begin{bmatrix} 417.391 \\ 455.652 \\ 417.391 \end{bmatrix} \text{ Ans/-}$$

Equilibrium Price

Let consider an economy produce only two goods X and Y with one Primary Input Labour (l).

$$A = \begin{array}{cc} X & Y \end{array}$$
$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} \Rightarrow$ unit of X and
$a_{21} \Rightarrow$ unit of Y needed to produce a unit of X.

Let suppose labour is used as primary unit and l_1 units and l_2 units of labour are needed to produce each unit of X & Y.

	X	Y	find DD	TR
X	$a_{11} p_1$	$a_{12} p_1$	$d_1 p_1$	p_1
Y	$a_{21} p_2$	$a_{22} p_2$	$d_2 p_2$	p_2
Primary Input	$l_1 w$	$l_2 w$		Lw
Total cost	$a_{11} p_1 + a_{21} p_2 + l_1 w$	Same		

Let eq^n price of X & Y are p_1, p_2 and price of input w per unit

At equilibrium TR = TC

$$a_{11} p_1 + a_{21} p_2 + l_1 w = p_1$$
$$a_{12} p_1 + a_{22} p_2 + l_2 w = p_2$$

$\Leftrightarrow$

$$(1-a_{11}) p_1 - a_{21} p_2 = l_1 w$$
$$-a_{21} p_1 + (1-a_{22}) p_2 = l_2 w$$

$$\begin{bmatrix} 1-a_{11} & -a_{21} \\ -a_{12} & 1-a_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} l_1 w \\ l_2 w \end{bmatrix}$$

$$(I-A)' \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} l_1 w \\ l_2 w \end{bmatrix} \Leftrightarrow \boxed{[(I-A)^{-1}]^t \begin{bmatrix} l_1 w \\ l_2 w \end{bmatrix}}$$

Solution 14

$$A = \begin{bmatrix} 16/40 & 20/80 \\ 8/40 & 40/80 \end{bmatrix} \qquad D = \begin{bmatrix} 18 \\ 44 \end{bmatrix}$$

$$\cancel{A}\, X = (I-A)^{-1} D$$

$$A = \begin{bmatrix} 2/5 & 1/4 \\ 1/5 & 1/2 \end{bmatrix}$$

$$(I-A) = \begin{bmatrix} 3/5 & -1/4 \\ -1/5 & 1/2 \end{bmatrix}$$

$$|I-A| = \frac{3}{10} - \frac{1}{20} \Rightarrow \frac{5}{20} > 0$$

$$(I-A)^{-1} = \frac{20}{5} \begin{bmatrix} 1/2 & 1/4 \\ 1/5 & 3/5 \end{bmatrix}$$

$$X = \frac{20}{5} \begin{bmatrix} 1/2 & 1/4 \\ 1/5 & 3/5 \end{bmatrix} \begin{bmatrix} 18 \\ 44 \end{bmatrix}$$

$$\Rightarrow \frac{20}{5} \begin{bmatrix} 9+11 \\ 150/5 \end{bmatrix} \Rightarrow \frac{20}{5} \begin{bmatrix} \overset{4}{20} \\ 30^{6} \end{bmatrix}$$

Gross output $X \Rightarrow \begin{bmatrix} 80 \\ 120 \end{bmatrix}$ Ans/

(ii) Total labour required

$$[l_1 \quad l_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$l_1 = \frac{80}{40} = 2$ unit of l_1

$l_2 = \frac{120}{80}$ $\frac{3}{2}$ unit of l_2

$$[2 \quad 3/2] \begin{bmatrix} 80 \\ 120 \end{bmatrix} \Rightarrow 160 + 180$$

$$\Rightarrow 340 \text{ Rs/-}$$

(iii) Total value additions, if wages is ₹40 per day.

$$[x_1 \quad x_2] \begin{bmatrix} l_1 w \\ l_2 w \end{bmatrix}$$

$$[80 \quad 120] \begin{bmatrix} 2 \times 40 \\ \frac{3}{2} \times 40 \end{bmatrix} \Rightarrow [80 \quad 120] \begin{bmatrix} 80 \\ 60 \end{bmatrix}$$

$$\Rightarrow 6400 + 7200 \Rightarrow 13600 \text{ Rs/-}$$

(iii) Equilibrium prices

$$((I-A)^{-1})^t \begin{bmatrix} l_1 w \\ l_2 w \end{bmatrix}$$

$$\Rightarrow \frac{20}{5} \begin{bmatrix} 1/2 & 1/5 \\ 1/4 & 3/5 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \end{bmatrix}$$

$$\Rightarrow \frac{20}{5} \begin{bmatrix} 52 \\ 20 + 36 \end{bmatrix} \Rightarrow \begin{bmatrix} 208 \\ 224 \end{bmatrix} \text{ Rs/-}$$

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Sol 16

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{bmatrix} \qquad D = \begin{bmatrix} 10 \\ 5 \\ 6 \end{bmatrix}$$

$$(I-A) = \begin{bmatrix} 0.8 & -0.3 & -0.2 \\ -0.4 & 0.9 & -0.2 \\ -0.1 & -0.3 & 0.8 \end{bmatrix}$$

$$|I-A| = 0.384 > 0$$

$$(I-A)^{-1} = \frac{1}{0.384} \begin{bmatrix} 0.66 & 0.3 & 0.24 \\ 0.34 & 0.62 & 0.24 \\ 0.21 & 0.27 & 0.6 \end{bmatrix}$$

$$X = \frac{1}{0.384} \begin{bmatrix} 0.66 & 0.3 & 0.24 \\ 0.34 & 0.62 & 0.24 \\ 0.21 & 0.27 & 0.6 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 6 \end{bmatrix}$$

$$\Rightarrow \frac{1}{0.384} \begin{bmatrix} 9.54 \\ 7.94 \\ 7.05 \end{bmatrix} \Rightarrow \begin{bmatrix} 24.84 \\ 20.67 \\ 18.36 \end{bmatrix} \quad \text{Ans/-}$$

Equilibrium price (Two primary inputs)

$$\boxed{\left[(I-A)^{-1}\right]^{t} \begin{bmatrix} l_1 w + Lr \\ l_2 w + Lr \end{bmatrix}}$$

$$\Rightarrow \frac{1}{0.384} \begin{bmatrix} 0.66 & 0.34 & 0.21 \\ 0.3 & 0.62 & 0.27 \\ 0.24 & 0.24 & 0.6 \end{bmatrix} \begin{bmatrix} 20+30 \\ 5+20 \\ 10+50 \end{bmatrix}$$

$$\Rightarrow \frac{1}{0.384} \begin{bmatrix} 54.1 \\ 46.7 \\ 54 \end{bmatrix} \Rightarrow \begin{bmatrix} 140.89 \\ 121.61 \\ 140.63 \end{bmatrix} \quad \text{Ans/-}$$