Matrix Notation
or
Array Notation

$$\begin{bmatrix} 1 & 2 & 3 \\ \alpha(1) & \alpha(2) & \alpha(3) \end{bmatrix}$$

$$\alpha = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

β is the another permutation of A defined as
$\beta(1) = 3, \quad \beta(2) = 2, \quad \beta(3) = 1$

$$\therefore \beta = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

Permutation Group:

A permutation group of a set A is the set of all permutations of A that forms a group under the operation composition.

Composition of permutations:

Let α and β be two permutations of set $A = \{1, 2, 3\}$ defined as

$$\alpha = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$\alpha\beta = ?$ $\quad \alpha\beta(1) = ?$. $\alpha\beta(2) = ?$. $\alpha\beta(3) = ?$.

$\alpha\beta = \alpha \circ \beta \qquad \alpha\beta(1) = \alpha\{\beta(1)\}$

$$\alpha\beta(1) = \alpha\{\beta(1)\} = \alpha(3) = 2$$

$$\alpha\beta(2) = \alpha\{\beta(2)\} = \alpha(2) = 3$$

$$\alpha\beta(3) = \alpha\{\beta(3)\} = \alpha(1) = 1$$

$$\therefore \alpha\beta = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\alpha\beta = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\therefore \alpha\beta = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

Similarly,

$$\beta\alpha = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

$$\Rightarrow \beta\alpha = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\therefore \alpha\beta \neq \beta\alpha$$

$\Rightarrow$ Composition of permutations is not commutative.