04 December 2021 11:08

Subgroup lattice:

Subgroup lattice is a diagram that includes all the subgroups of the groups and connects a subgroup H at one level to a subgroup K at a higher level with a sequence of line segments if and only if H is a proper subgroup of K.

Example: Draw a subgroup lattice for $\mathbb{Z}_{30}$!

Sol^n// The subgroups of $\mathbb{Z}_{30}$ are

$\langle 1 \rangle = \mathbb{Z}_{30} = \{0, 1, 2, \ldots, 29\}$

$\langle 2 \rangle = \{0, 2, 4, 6, \ldots, 28\}$

$\langle 3 \rangle = \{0, 3, 6, 9, \ldots, 27\}$

$\boxed{\text{factors of 30 are } 1, 2, 3, 5, 6, 10, 15, 30}$

$\langle 5 \rangle = \{0, 5, 10, 15, 20, 25\}$

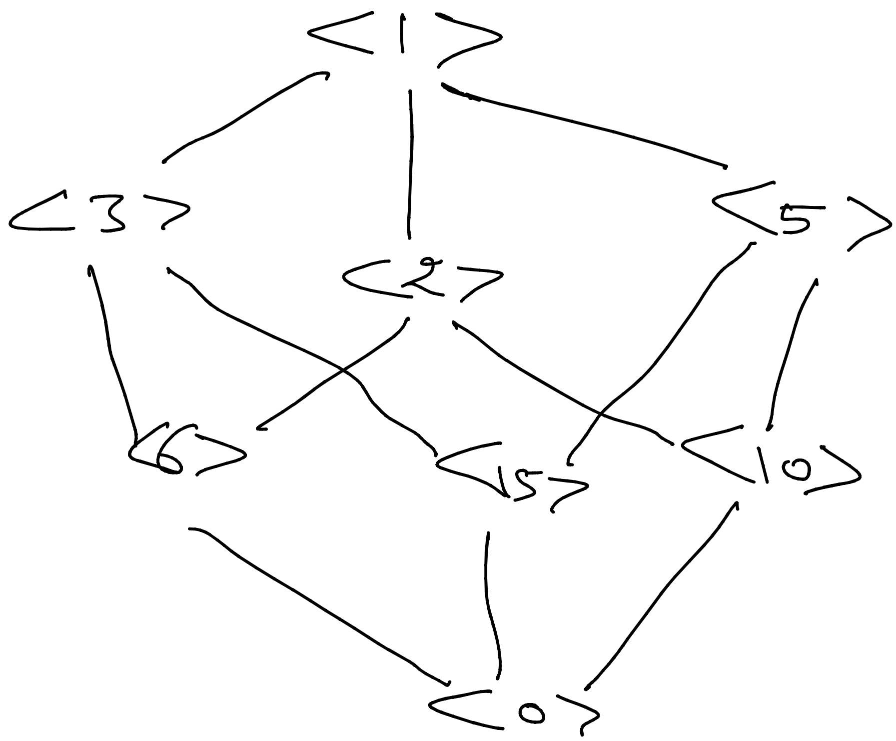
$\langle 6 \rangle = \{0, 6, 12, 18, 24\}$

$\langle 10 \rangle = \{0, 10, 20\}$

$\langle 15 \rangle = \{0, 15\}$

$\langle 30 \rangle = \langle 0 \rangle = \{0\}$.

Hence, subgroup lattice for $\mathbb{Z}_{30}$ is as follows:



Subgroup lattice for $\mathbb{Z}_{30}$.

Q. Draw subgroup lattice for $\mathbb{Z}_{p^2 q}$, where p & q are primes

Sol// subgroups of $\mathbb{Z}_{p^2 q}$ are :

$$\langle 1 \rangle = \{0, 1, 2, \dots, p^2 q - 1\} = \mathbb{Z}_{p^2 q}$$

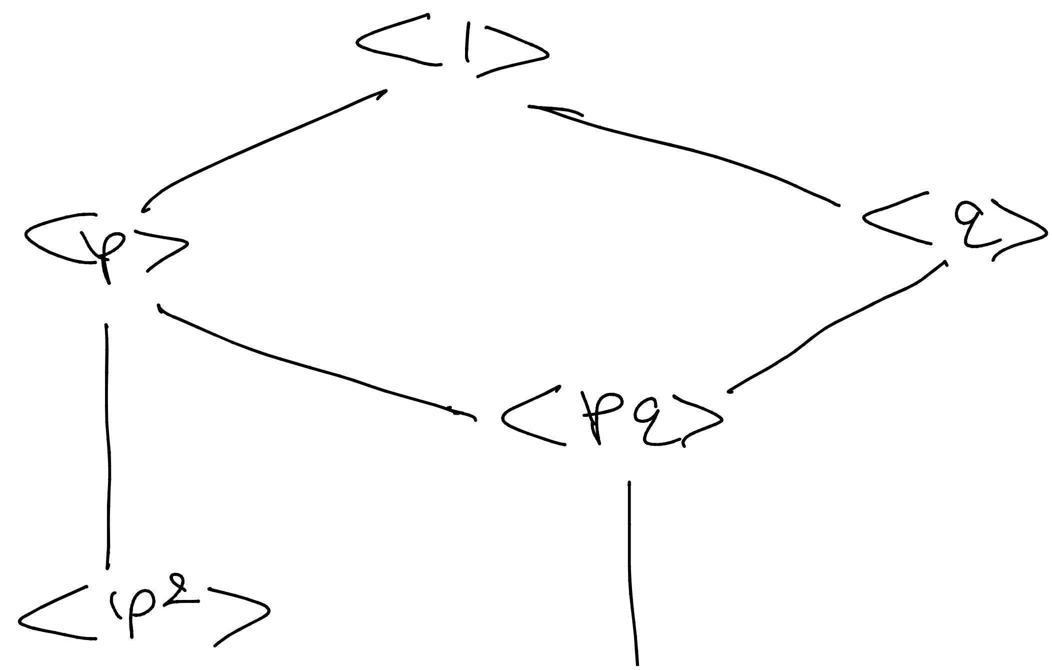
$$\langle p \rangle = \{0, p, 2p, 3p, \dots, (pq-1)p\}$$

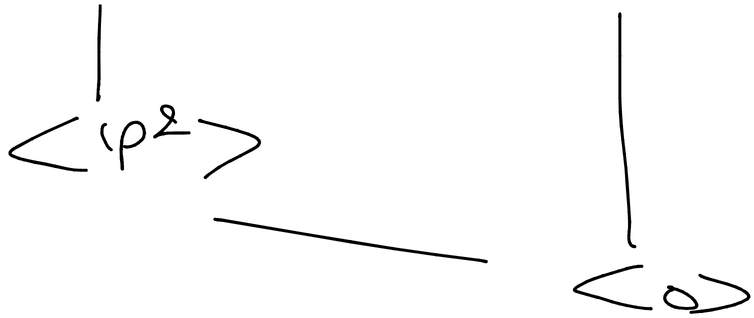
$$\langle q \rangle = \{0, q, 2q, 3q, \dots, (p^2-1)q\}$$

$$\langle pq \rangle = \{0, pq, 2pq, \dots, (p-1)pq\}$$

$$\langle p^2 q \rangle = \langle 0 \rangle = \{0\}.$$

Hence, subgroup lattice for $\mathbb{Z}_{p^2 q}$ is as follows:





Subgroup Lattice for $\mathbb{Z}_{p^2 q}$.

Subring Lattice:

Subring lattice is a diagram that includes all the subrings of the ring and connects a subring S at one level to a subring T at a higher level with a sequence of line segments if and only if S is a proper subring of T.

Q. Draw Subring lattice for $\mathbb{Z}_8$?

Sol// $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$.