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ex 1 $(\mathbb{N}, \leq)$ chain.
greatest elt does not exist
least elt $= 1$

ex 2 $P = \mathcal{P}(A)$ (A = any set)
$(\mathcal{P}(A), \subseteq)$ is a Poset
greatest elt $= A$
least elt $= \emptyset$.

ex3 P = positive divisors of 54
$= \{1, 2, 3, 6, 9, 18, 27, 54\}$
$(P, '|')$ is a Poset.
greatest element $= 54$
least element $= 1$.

ex4 $P = \{1, 2, 3, 6, 9, 18, 27\}$.
$a \leq b$ iff $a \mid b$.
$(P, \leq)$ is a Poset.
greatest element does not exist
least element $= 1$.

Defn Maximal element
Let $(P, \leq)$ be a poset.
An element $c \in P$ is called a maximal element of P if
$c \leq x$ for some $x \in P \Rightarrow c = x$.

Defn Minimal element
Let $(P, \leq)$ be a Poset
An element $d \in P$ is called a minimal element of P if
$x \leq d$ for some $x \in P \Rightarrow x = d$.

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ex $P = \{1,2,3,6,9,18,27\}$
$(P, '|')$ is a Poset.
It has 2 Maximal elements 18, 27
Minimal element is Unique $= 1$.

ex $P = \{1,2,3,4,5\}$
Let $\leq$ be discrete Order on P.
Maximal elements $= 1,2,3,4,5$
Minimal elts $= 1,2,3,4,5$.

Result Every greatest element is maximal.
Pf let $(P, \leq)$ be a poset & let
$a =$ the greatest element of P.
let $x \in P$ s.t. $a \leq x$. —— (1)
Since $x \in P$ & $a =$ greatest element
$\therefore x \leq a$ —— (2)
By (1), (2) & antisymmetry,
$a = x$
$\therefore a =$ maximal element of P.

Result Every least element is minimal.
(Prove)

Defn let $(P, \leq)$ be a poset.
Let $Q \subseteq P$.
1) $a \in P$ is called an Upper bound of Q if $x \leq a \ \forall x \in Q$.
2) $b \in P$ is called a lower bound of Q if $b \leq x \ \forall x \in Q$.
3) Least upper bound (supremum)
If $Q^u =$ set of all upper bds of Q in P
Then least element of Q^u, if it exists,

is called the least upper bd of Q or Sup Q.

4) Greatest lower bound (Infimum)
If Q^L = set of all lower bds of Q in P, then greatest element of Q^L, if it ~~is called~~ exists, is called the greatest lower bound of Q or Inf Q.

ex $P = \{1,2,3,6,9,18,27\}$
$(P, '|')$ is a Poset.

Let $Q = \{27, 18\} \subseteq P$
$Q^L = \{9, 3, 1\}$
greatest elt of $Q^L = 9$
$= glb$ of Q
$= \inf Q$.

$Q^u = \emptyset$.
$\therefore$ Sup Q does not exist.