20 January 2022 13:12

Q. Find all the values of x that satisfy the inequality $|x-2| \leq x+1$. Sketch graphs.

Soln// Here we have two cases.

(i) $x < 2$ and (ii) $x \geq 2$

Case - I: $x < 2$

In this case, $|x-2| \leq x+1$

$$\Rightarrow -(x-2) \leq x+1$$
$$\Rightarrow -x+2 \leq x+1$$
$$\Rightarrow x+x \geq 2-1$$
$$\Rightarrow 2x \geq 1$$
$$\Rightarrow x \geq 1/2$$

In this case, the values of x satisfying the inequality is given by $\{x : x \geq 1/2\} \cap \{x : x < 2\}$

$$= \{x : \tfrac{1}{2} \leq x < 2\}$$

Case - II) When $(x-2) \geq 0$

In 1st case, $|x-2| \leq x+1$

$$\Rightarrow x-2 \leq x+1$$
$$\Rightarrow x-x \leq 1+2$$
$$\Rightarrow 0 \leq 3$$

It is always true.

Thus, every value of x satisfy the given inequality in this case.

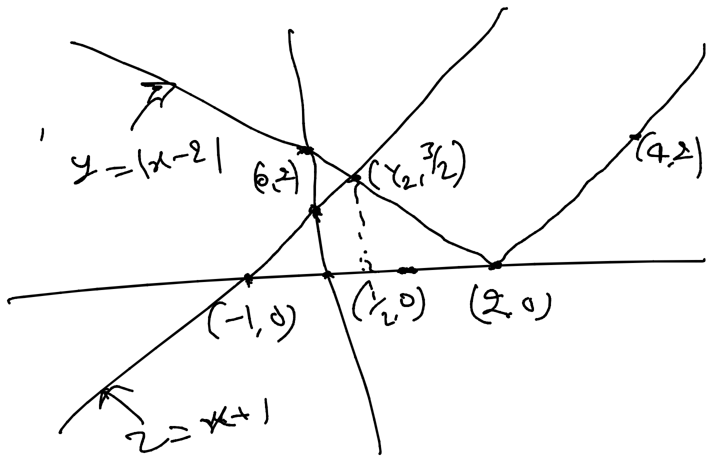
The value of x satisfying the inequality is $\{x : x \geq 2\}$.

From Case I & II the value of x satisfying the inequality $|x-2| \leq x+1$ is given by: $\{x : \frac{1}{2} \leq x < 2\} \cup \{x : x \geq 2\}$
$= \{x \in R : x \geq 1/2\}$.

$$y = |x-2| = \begin{cases} -x+2, & x<2 \\ x-2, & x \geq 2 \end{cases}$$

$z = x+1$

$-x+2 = x+1$
$\Rightarrow x = 1/2$

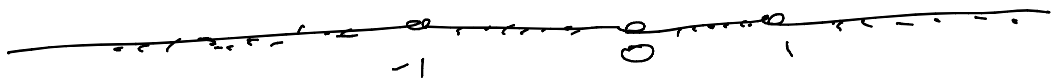


From the graph, we can see that the graph of $z = x+1$ is above than the graph of $y = |x-2|$ for $\frac{1}{2} \leq x < \infty$.

$\therefore$ The values of x, satisfying the inequality is given by $\{x \in R : x \geq 1/2\}$.

The line:

$\{x \in R : -\infty < x < \infty\}$



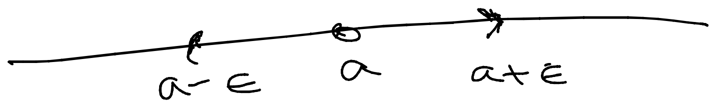
Def^{n}: Let $a \in \mathbb{R}$ and $\epsilon > 0$. Then the ϵ-neighbourhood of a is the set

$$V_\epsilon(a) = \{x \in \mathbb{R} : |x-a| < \epsilon\}.$$

$|x-a| < \epsilon$

$\Rightarrow -\epsilon < x-a < \epsilon$

$\Rightarrow a-\epsilon < x < a+\epsilon$



$a=1,\ \epsilon = 0.01,$

ϵ-neighbourhood $= \{x : |x-1| < 0.01\}$

$= \{x : 1-0.01 < x < 1+0.01\}$

$= \{x : 0.99 < x < 1.01\}$

$= (0.99, 1.01)$

If $a=1,\ \epsilon = 0.001$

then $V_\epsilon(a) = (0.999, 1.001)$

Theorem: Let $a \in \mathbb{R}$. If x belongs to the neighbourhood $V_\epsilon(a)$ for every $\epsilon > 0$, then $x = a$.

Proof:

$$V_\epsilon(a) = \{y : |y-a| < \epsilon\}$$

$\because x \in V_\epsilon(a) \quad \forall\ \epsilon > 0$

$\Rightarrow |x-a| < \epsilon \quad \forall\ \epsilon > 0$

$\Rightarrow |x-a| = 0 \quad (\because |x-a| \not< 0)$.

$\Rightarrow \quad x = a.$

Examples:

① Let $U = \{x : 0 < x < 1\}$.

every element of U has some ϵ-neighbourhood of it contained in U.

$a = 0.001, \quad \epsilon = 0.0001$

$V_\epsilon(a) = (a-\epsilon, a+\epsilon) = (0.0009, 0.0011) \subseteq U$

② Let $W = \{x : 0 \le x \le 1\}$.

Then $V_\epsilon(0)$ and $V_\epsilon(1)$ are not contained in W for any value of $\epsilon > 0$.

Upper Bound and Lower Bound:

Let S be a nonempty subset of $\mathbb{R}$.

(a) The set S is called bounded above if there exist a real number u such that

$s \le u \quad \forall s \in S$.

$\exists\, s$

This number u is called upper bound of S.

Example: $S = \{x : 0 < x < 1\}$.