Quantitative Techniques for Management
[DUAL]

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Writing the Dual of an LPP

Consider a normal / Standard LPP :-

Maximize $Z = C_1 x_1 + C_2 x_2 + C_3 x_3$.

Subject to

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \leq b_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 \leq b_3.$$

$$x_1, x_2\ x_3 \geq 0$$

$\Downarrow$ Its Dual will be:

Minimize $Z = b_1 y_1 + b_2 y_2 + b_3 y_3$.

Subject to :-

$$a_{11} y_1 + a_{21} y_2 + a_{31} y_3 \geq c_1$$

$$a_{12} y_1 + a_{22} y_2 + a_{32} y_3 \geq c_2$$

$$a_{13} y_1 + a_{23} y_2 + a_{33} y_3 \geq c_3.$$

$$y_1, y_2, y_3 \geq 0.$$

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Thus, you can observe the following

PRIMAL		DUAL
Maximization	OBJECTIVE	MINIMIZATION
c_j's	Coefficients	b_i
b_i's	RHS.	c_j
$x_i \geq 0$	variables	$y_i \geq 0$.
$\leq$	Constraints	$\geq$
m variables & n contraints	Variables vs. Constraints	n variables & m constraints

Study of both Primal & Dual gives a more wholistic view of the problem.

Solution of one can be read from the solution of the other with the help of the shadow prices.

Formulation of the Dual:-

① Body matrix of a_{ij}'s in the Dual is the Transpose of the Primal.

② Constraints b_i's of Primal become objective function c_j's of the Dual

③ Objective c_j's of Primal become

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b_i's of the Dual

(4) There are as many variables in the dual as the no. of constraints in the Primal.

Finding the solution of the dual from the solution of the Primal (without solving Dual)

(1) Value of objective function of both is the same (optimal value)

(2) The optimal value of dual variables can be obtained from the $c_j - z_j$ row values of S_1, S_2, S_3 i.e. shadow prices (ignoring the sign)