I.A. ASSIGNMENT

Dated : 30/03/2020

STATISTICAL MECHANICS

B.Sc(H) PHYSICS - VI SEMESTER

QUES 1. (i)Consider a system of $N >> 1$ weakly interacting identical particles (neglect spin), with each particle having possible non-degenerate states with energies ε, 2ε, 3ε,.., *n*ε,... where *n* is a natural number. What is the ground state energy of the system if the particles are (a) Bosons (b) Fermions?

(ii) Let v_x, v_y, v_z denote the x, y and z components of velocity of an ideal gas particle. the gas is at absolute temperature *T*, to what power of *T* is the average of the product $v_x{}_x^2$ v_y^2 v_z^2 proportional to?

(iii)For a two state system, the first excited state of a particle is 1.5 eV above the ground state. The ground state is doubly degenerate while the first excited state has a four-fold degeneracy. If the system is heated to a temperature of 7000 ^{0}K, what fraction of atoms are in the excited state?

(iv)The Earth's average temperature is 290 ^{0}K. Make an approximate sketch of the intensity of blackbody radiation emitted by the Earth vs wavelength of the emitted radiation. You can use the fact that the Sun emits maximum radiation at 6000 Å, while its surface temperature is 5500 ^{0}K.

(v)A system of *N* spin-1/2 particles, each with magnetic moment *m*, is in presence of a magnetic field *B*. Initially, all the magnetic dipoles are oriented along the direction of the magnetic field. If the energy of the system is increased by $\Delta E = + mB$, what is the change in entropy of the system?

(vi)A mixed 'Bosonic' system consists of N_R number of red balls, and N_B number of blue balls. Any number of these can be placed in any of the g number of boxes. Find the number of distinct microstates of the system.

(vii)Consider an isolated system of ten particles undergoing random collisions, and confined to a cubical box of volume *V*. Compare the probabilities that all the particles are in the left half vs five particles are in the left half. Is this in contradiction with the principle of equal a-priori probabilities?

QUES 2 (a) Consider a system of $N >> 1$ weakly interacting spins in an external magnetic field B. Each spin can be parallel or antiparallel to B, with the energy of the spin equal to $+mB$ if the spin is antiparallel, and $-mB$ if it is parallel. The system is in thermal equilibrium at temperature T. Calculate the partition function of the system and from it, the mean magnetization of the system. Precisely state the condition (as a mathematical inequality) under which the magnetization of the system is proportional to the magnetic field.

(b) Consider a system of four non-interacting distinguishable particles, with energy of each particle restricted to values $\varepsilon = 0$, ε_0, $2\varepsilon_0$, $3\varepsilon_0$,. Let the energy of the system be $E = 3\varepsilon_0$. Given that the system is in equilibrium, explicitly enumerate the microstates of the system corresponding to this macrostate, and determine the thermodynamic probability. What is the probability that a given particle has energy ε_0? Given that particle 1 has energy ε_0, what is the probability that particle 2 has energy 0?

(c) Consider an isolated system of $3N >> 1$ distinct particles, each of which can be in one of two states with energies 0 and ε respectively. Give the range of energies for which the system has a negative temperature. The system is separated into two sub-systems A and B, system A having N particles and system B having $2N$ particles. Subsystem A has energy $E_A = 3N\varepsilon/8$ and B has energy $E_B = 7N\varepsilon/4$. Using the above result, specify which of these subsystems has a positive temperature, and which has a negative temperature. The two subsystems are now allowed to exchange energy, keeping the total energy constant. Once equilibrium is attained, is the temperature of the two systems positive or negative?

QUES 3 (a) Consider a system of N monoatomic classical gas molecules at temperature T and occupying a cubical box of volume V. Using the Maxwell-Boltzmann distribution, find an expression for the normalized probability that (i) a particle has momentum between (p_x, p_y, p_z) and $(p_x+dp_x, p_y+dp_y, p_z+dp_z)$ (ii) position between (x, y, z) and $(x+dx, y+dy, z+dz)$. Assume that the origin is at one corner of the box.

(b) A model for an anisotropic crystalline solid visualizes each atom of the solid as an independent classical harmonic oscillator with different frequencies of oscillations in different directions. In suitable coordinates, the expression for energy of an atom is

$$\mathbf{E} = \frac{1}{2m}(\mathbf{p_x}^2 + \mathbf{p_y}^2 + \mathbf{p_z}^2) + \frac{1}{2m}(\omega_x^2\, \mathbf{x}^2 + \omega_y^2\, \mathbf{y}^2 + \omega_z^2\, \mathbf{z}^2)$$

where ω_x, ω_y and ω_z are all different. Calculate the mean energy per atom and the molar specific heat of the system. Is the result for the molar specific heat different from that of an isotropic oscillator, for which ω_x, ω_y and ω_z are all same? Comment.

(c) N monoatomic atoms adsorbed on a surface are free to move on this surface. To a good approximation, the system can be treated as a two-dimensional classical ideal gas constrained to occupy an area A. The system is at temperature T. Calculate the partition function of the system. Using this, find an expression for the heat capacity of one mole of the system. If one mole of this system were instead confined to a three-dimensional volume, will the amount of heat energy required to raise the temperature of the system by 1 ^{0}K be more, or less?

QUES 4 (a) Consider electromagnetic radiation enclosed in a cubical cavity of volume V and temperature T. Show that the average number density of photons is proportional to T^3. Given that the system is confined to volume V which is finite, is there a lower limit to the frequency of radiation below which the Planck distribution is not valid? Comment.

(b) Electromagnetic radiation at temperature T_0 fills a cavity of volume V_0. The cavity is thermally insulated, and its volume increased quasi-statically to eight times the original volume. What thermodynamic function is conserved? What is the final temperature of radiation?

(b) A nuclear explosion produces a temperature of about 10^6 K over a sphere of diameter 10 cm. Calculate the intensity of this radiation at a distance of 1 km and the wavelength corresponding to which the intensity is maximum.

QUES 5 (a) Consider a system of N weakly interacting spin-1/2 Fermions occupying volume V and at temperature T. Deduce an expression for the mean occupation number for a given energy level as a function of temperature and sketch a plot of the mean occupation vs energy at non-zero temperature $T << T_F$ where T_F is the Fermi temperature. Based on this plot, give a qualitative argument why the molar specific heat of the system is not constant, but proportional to the ratio T/T_F.

(b) A simple model of a neutron star assumes that it is a system consisting of neutrons with number density $n \sim 10^{45}$ m^{-3} and temperature 10^6 ^{0}K. Show that the neutron gas is strongly degenerate. Analyze if the neutrons are non-relativistic, relativistic or ultra-relativistic. The rest mass of a neutron is 1.7×10^{-27}kg and its rest mass energy is about 10^3 MeV.

(c) Calculate the mean speed and the root mean squared speed of a free electron gas at 0^0K, consisting of N electrons confined to volume V. What do you expect the mean value of any one component of velocity of the electron? Explain.

QUES 6 (a) Consider a system of N weakly interacting spin-zero Bosons confined to volume V and at temperature T. Assuming that the system is strongly degenerate, show that the heat capacity at constant volume is proportional to $T^{3/2}$. Qualitatively, relate this to the fraction of Bosons in the ground state of the system.

(b) Consider a system consisting of two non-interacting particles in equilibrium with a heat bath at temperature T. Each particle can be in one of three states with energies 0, ε_1 and ε_2. Find the partition function for the system assuming that the particles are (i) distinct (ii) Bosons. For the two cases, compare the probability that both particles are in the ground state.

(c) Bose-Einstein condensation occurs in a system of weakly interacting particles at temperature 2.17 ^{0}K at a certain density. If the density of the system is 1000 times smaller, at what temperature will the transition take place? In which of the two isotopes of Potassium, Potassium-40 (which has 19 protons and 21 neutrons) and Potassium-41 (which has 19 protons and 22 neutrons) will such a 'condensation' take place? Give reasons.

NOTE : ALL THE STUDENTS ARE REQUIRED TO COMPLETE THIS ASSIGNMENT LATEST BY **07.04.2020** AND SEND ME THE PDF OF HANNDWRITTEN ASSIGNMENT MENTIONING THE NAME AND ROLL NUMBER OF INDIVIDUAL .

EMAIL-ID FOR THE SAME IS **:** contact_diya_15@yahoo.co.in

Kindly mention the subject of mail as statistical mechanics assignment.