## Heisenberg's Uncertainty Principle :

It is impossible to determine precisely and simultaneously the value of both the members of a pair of physical variables which describe the motion of an atomic system.

$$\Delta x \Delta p \geq \hbar/2$$

## Energy Time Uncertainty Relation

Consider a free particle of mass m moving with velocity $v$.

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Uncertainty in energy

$$\Delta E = \frac{2p\Delta p}{2m} = \frac{p\Delta p}{m} = \frac{mv\Delta p}{m} = v\Delta p$$

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta E = \frac{\Delta x}{\Delta t}\Delta p$$

$$\Delta E \Delta t = \Delta x \Delta p$$

Uncertainty relation in terms of position & momentum

$$\Delta x \Delta p \geq \hbar/2$$

$$\boxed{\Delta E \Delta t \geq \hbar/2}$$


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## Group velocity & wave velocity :-

$E = h\nu \quad E = mc^2$

$\nu = \frac{mc^2}{h}.$

The velocity of Propogation of the wave, called Phase velocity.

$$v_p = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{mv}.$$

$$v_p = \frac{c^2}{v} = \text{De Broglie wave velocity (Phase velocity)}$$

The velocity with which a slow varying envelope or Packet due to group of wave travel in a medium.

## Phase velocity

The phase velocity $v_p$ of a monochromatic wave is the velocity with which a definite Phase of the wave, such as its crest or trough, is propagated in a medium.

$$y = A\cos(\omega t - kx)$$

$$\omega = 2\pi\nu \qquad k = 2\pi/\lambda$$

$$v_p = \nu\lambda = \omega/k$$

$$\lambda = h/mv$$

$$h\nu = mc^2 \text{ or } \nu = mc^2/h$$

$$v_p = \nu\lambda = \frac{mc^2}{h}\frac{h}{mv} = \frac{c^2}{v}$$

De Broglie wave velocity $v_p$ must be greater than c.

## Group velocity :-

The velocity with which the centre of wave group maxmimum amplitude moves is called the group velocity of the wave group



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## Expression for Group velocity :-

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos[(\omega+\Delta\omega)t - (k+\Delta k)x]$$

$$y = y_1 + y_2$$

$$= A\cos(\omega t - kx) + A\cos[(\omega+\Delta\omega)t - (k+\Delta k)x]$$

$$= 2A\cos\tfrac{1}{2}[(2\omega+\Delta\omega)t - (2k+\Delta k)x]\cos\tfrac{1}{2}(\Delta\omega t - \Delta k x)$$

$2\omega + \Delta\omega \simeq 2\omega$, $\quad 2k + \Delta k \simeq 2k$.

$$y = 2A\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)\cos(\omega t - kx)$$

$$v_g = \frac{\Delta\omega}{\Delta k} \qquad v_g = \frac{d\omega}{dk}$$

## Group velocity of de Broglie wave :-

$$v_g = \frac{d\omega}{dk}. \qquad \omega = 2\pi\nu = \frac{2\pi mc^2}{h}.$$

$$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1-v^2/c^2}} \quad \text{and} \quad k = \frac{2\pi}{\lambda}$$

$$= \frac{2\pi mv}{h} = \frac{2\pi m_0 v}{h\sqrt{1-v^2/c^2}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\sqrt[3]{1-v^2/c^2}}, \quad \frac{dk}{dv} = \frac{2\pi m_0}{h(1-v^2/c^2)^{3/2}}$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = v$$

$$\boxed{v_g = v}$$

De Broglie wave group associated with a moving Particle travels with the same velocity as the Particle.


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