$$\frac{\partial}{\partial t}|\psi|^2 = \frac{\partial}{\partial t}(\psi^* \psi)$$

$$= \psi^* \frac{\partial \psi}{\partial t} + \frac{\partial \psi^*}{\partial t}\psi$$

Usin TDS.E; $i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$

$$-i\hbar \frac{\partial \psi^*}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2}$$

$$\Rightarrow \frac{\partial}{\partial t}|\psi|^2 = \psi^* \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} - \frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\psi$$

$$\Rightarrow \frac{i\hbar}{2m}\frac{\partial}{\partial x}\left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi\right)$$

$\psi(x) \rightarrow 0$ as $x \rightarrow \pm\infty$

$$\Rightarrow \frac{d}{dt}\int_{-\infty}^{+\infty} |\psi(x,t)|^2 dx = 0$$

i.e. $\psi(x,t)$ is constant & time ind. & its wave fun is normalised for all future times.

WAVE PACKET (Do it from Register. (

Statistical Interpretation of Wave function & Conservation of probability.

The statistical interpretation of wave fun. associated with a particle was given by Max Born i.e. If a particle is descri by wave function $\psi(\vec{r},t)$ then probabl of finding the particle at time t, in volume element $d\tau = dx\,dy\,dz$ about point $\vec{r}$ is given by:

$P(x,t)\,dx = \psi^*(x,t)\,\psi(x,t)\,dx = |\psi|^2 dx$

$$\boxed{P(x,t) = |\psi(x,t)|^2}$$

The quantity $P(x,t)$ is known as Position probablity density.
Since probablity of finding the particle in space at time t is unity, the wave function chosen are to be normalised wave funⁿ.

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 dx = 1$$

As the time changes the probablity of finding the particle somewhere must remain conserved. i.e. the above integral should be time independent.
(Proof done on Page-3)
So from this we can that probablity of finding a particle is conserved in time.

Conservation of Probablity & Hermiticity of Hamiltonian operator.

By conservation of prob. we can see that Hamiltonian is a hermitian operator.

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x,t)$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$-i\hbar \frac{\partial \psi^*}{\partial t} = (H\psi)^*$$

$$\frac{\partial}{\partial t}\int \psi^*\psi\,dx = \int\left(\psi^*\frac{\partial \psi}{\partial t} + \psi\frac{\partial \psi^*}{\partial t}\right)dx$$

$$= \frac{1}{i\hbar}\int [\psi^* (H\psi) - \psi (H\psi)^*] dx$$

$$\because \frac{\partial}{\partial t}\int \psi^* \psi \, dx = 0$$

$$\Rightarrow \int \psi^* (H\psi)\, dx = \int (H\psi)^* \psi \, dx$$

which proves H is Hermitian.

Q. Which of the following are acceptable wave function over indicated interval?

a) $e^{-\frac{x^2}{2}}$; $-\infty \le x \le +\infty$

b.) e^{-ix} ; $0 \le x \le 2\pi$

c.) $x^2 e^{-2\pi i x}$; $0 \le x \le \infty$

d.) $x e^{-x}$; $0 \le x \le \infty$

EXPECTATION VALUES (Of Dynamical Variables) (5)

In QM a particle is represented by a wave function which can be obtained by solving S.E. & contains all available information about particle. Since ψ has probabilistic interpretation, so exact information about particle cannot be obtained. Instead when we have large number of particles each of which is in state $\psi(x,t)$ & we measure the position of each of these particles in separate expt. all at same time t, we will obtain a set of diff. results for all expt. Thus we calculate the average or mean value of those measurement. Average value of dynamical variables are called expectation values.

For discrete set of element, Mean value is given by

$$\bar{x} = \Sigma x P_x$$

But all the possible values of x are distributed continuously over the entire range of real number so $\Sigma \rightarrow \int$ & P_x is replaced by $P(x)\,dx$ where $P(x)$ is probability density. In QM

$$P(x) = |\psi|^2 = \psi^* \psi$$

$$\therefore \langle x \rangle = \int_{-\infty}^{+\infty} x |\psi|^2 dx$$

$$= \int_{-\infty}^{+\infty} \psi^* \, x \, \psi \, dx$$

For any function $f(x)$

$$\langle f \rangle = \int_{-\infty}^{+\infty} \psi^* f(x) \psi \, dx$$

* Expectation values of physical quantities a[illegible] always real. Expectation value is a f[illegible] only of time because space coordinates have been integrated out.

The results of various set of expts. scatt[illegible] about the expectation value so we [illegible] calculate standard deviation.

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N}}$$

$$\Sigma(x_i - \bar{x})^2 = \frac{\Sigma(x_i)^2}{N} - 2\bar{x}\frac{\Sigma x_i}{N} + \frac{(\bar{x})^2}{N}$$

$$= \bar{x^2} - 2\bar{x}\bar{x} + (\bar{x})^2$$

$$= \bar{x^2} - (\bar{x})^2$$

$$\text{So, } \sigma = \sqrt{\bar{x^2} - (\bar{x})^2}$$

In $\sigma = \Delta x$ = uncertainity in positio[illegible] & $\bar{x} = \langle x \rangle$;

So, $\boxed{\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}}$

How to obtain exp. value for quantit[illegible] which are function of mom. & positio[illegible] for eg. Energy.

For this we use operator representation

$$\hat{p} = -i\hbar \nabla$$

$$\hat{p}^2 = -\hbar^2 \nabla^2$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

Consider classical expression for energy

$$E = \frac{p^2}{2m} + V$$

Using correspondance principle,

$$\langle E \rangle = \left\langle \frac{p^2}{2m} \right\rangle + \langle V \rangle \quad (1)$$

$$\left\langle i\hbar \frac{\partial}{\partial t} \right\rangle = \left\langle \frac{-\hbar^2}{2m} \nabla^2 \right\rangle + \langle V \rangle \quad (2)$$

This eqn must be consistent with S.E.

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi$$

Multiply by ψ^* on left & integrating

$$\int \psi^* \left(i\hbar \frac{\partial}{\partial t}\right) \psi \, d\tau = \int \psi^* \left(\frac{-\hbar^2}{2m}\nabla^2\right) \psi \, d\tau + \int \psi^* V \psi \, d\tau \quad (3)$$

Eqn (2) & (3) are consistent if expectation value is defined in general case with operator acting on ψ & multiplied by ψ^* on left. we then have

$$\langle E \rangle = \int \psi^* \, i\hbar \frac{\partial \psi}{\partial t} \, d\tau$$

$$\langle p \rangle = \int \psi^* (-i\hbar) \nabla \psi \, d\tau$$

Generalising above, suppose we have dynamical state of particle described by normalised wave fun $\psi(r,t)$ Let $A(r,p,t)$ be dynamical variable, we obtain operator $\hat{A}(r, -i\hbar \nabla, t)$ by performing sub. $p \rightarrow -i\hbar \nabla$ & then calculating exp. value of $\hat{A}$ as

$$\langle A \rangle = \int \psi^*(r,t) \, \hat{A}(r, -i\hbar\nabla, t) \, \psi(r,t) \, d\tau.$$

Since exp. value is always real so,

$$\langle A\rangle^* = \langle A\rangle,$$

From this we get that operator

$\hat{A}$ must satisfy

$$\int \psi^* \hat{A} \psi \, d\tau = \int (\hat{A}\psi)^* \psi$$

* Thus operator associated with dynam. quantity must be Hermitian.

* Probability conservation & hermiticity of hamiltonian. (Page - (4))

Operator Formulation.

1.) For any physically measurable quantity i.e. dynamical variable, there is associated an operator A.

2.) The E.V. of A are the only possible results of measurement of the quantity

3.) The eigenfun^n of ~~the system~~ A represent the states of particle in which the value of quantity a is preciesly known.

4.) If the wave function of the system before the measurement of a happens to be a eigenfunction of A, the result of measurement is sure to be the corresponding eigenvalue

5.) If the wavefunction of particle before measurement is not an eigenfunction of A, result of measurement cannot be predicted beforehand with certainity.

6.) If the ~~a~~ measurement gives a result a_0, the wavefunction just after the measurement be an eigenfunction of A corresponding to eigenvalue irrespective of what it was before measurement. This principle is called "collapse of wave function in measurement".

7.) Any wavefunction $\psi(x)$ can be written as linear combination of eigenfunction of A. If the eigenvalue a_i of A are discrete and $\phi_i(x)$ rep. eigenfunction of A, the wave function $\psi(x)$ can be written as

$$\psi(x) = \sum_i c_i \phi_i(x)$$

8.) If the eigenvalues a of A are continuous & $\phi(x,a)$ rep. eigenfunction of A, the wave function $\psi(x)$ can be written as

$$\psi(x) = \int_{-\infty}^{+\infty} c(a)\,\phi(x,a)\,da.$$

9.) The probability of getting particular value in the measurement of a is related to coefficient of expansion c_i or $c(a)$

$p = -i\hbar \frac{\partial}{\partial x}$
$p = mv$
$v = \frac{p}{m}$

* Expectation value of the velocity.

Since velocity $= \frac{dx}{dt}$, so $\langle v \rangle = \frac{d\langle x \rangle}{dt}$

$$\frac{d\langle x\rangle}{dt} = \int x \frac{\partial}{\partial t} |\psi|^2 dx \qquad x$$

$$= \frac{i\hbar}{2m} \int \frac{x}{\mathrm{I}} \frac{\partial}{\partial x}\Big(\underbrace{\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi}_{\mathrm{II}}\Big) dx$$

By integrating once we get

$$\boxed{\frac{d\langle x\rangle}{dt} = -\frac{i\hbar}{m}\int \psi^* \frac{\partial \psi}{\partial x}\, dx}$$