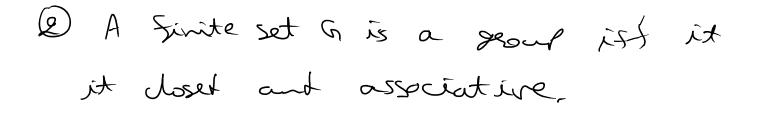
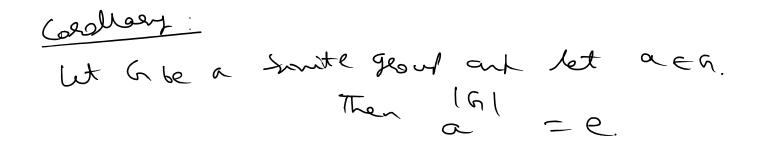
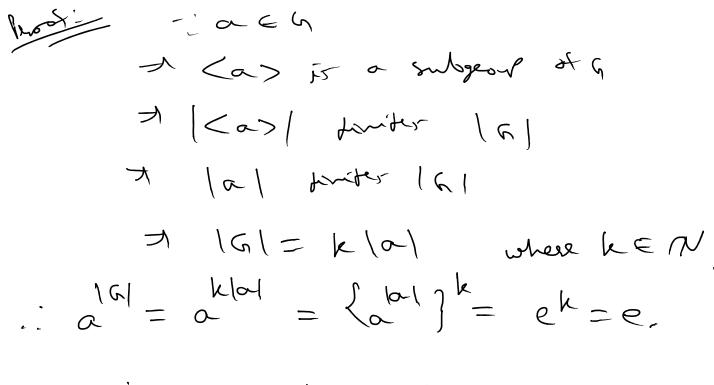
06 October 2020 09:39
Collary George of frime Dedees are Cyclic
\mathcal{C}
lest i let 6 be a geoup and IG = P(kine)
Now, let a E G and a Fe,
Then <a> is a subgeoup of G.
=) [<a>] divites [G]
) <a> diviter p
) $ \langle a \rangle = ol p $
$\forall \langle a \rangle = P \qquad (: a \neq e)$
$\exists \langle a \rangle = G $
J 0 = 16 J h is cyclic.
Note: DA georg & of order is cyclic if and only if
G has an element of order n.
ie. it Jack s.t.

i.e.
$$j \neq j = a \in G$$
 s.t.
 $|G| = |a| = m$, then
 $a \neq G = \langle a \rangle$.







Coedloez: Format's little Theolen:

For Every integer a and every percept,

$$a^{p} \equiv a \pmod{p}$$
,
 $e^{p} \equiv a \pmod{p}$,
 $e^{p} \equiv a \pmod{p}$,
 $e^{p} \equiv a \pmod{p}$,
 $e^{p} \equiv a (\cosh p)$,
 $f^{p} \equiv a (\cosh p) \equiv p$,
 $f^{p} \equiv a (\cosh p) \equiv p$,
 $e^{p} \equiv a (\cosh p)$,
 $e^{p} \equiv a (\mod p)$,

Gese-II! ged
$$(a, p) = 1$$
.
In this are pXa .

By the division algorithm.

$$\alpha = Pm + 2$$
, where $x = 1, 2, ..., p - 1$.
 $\overline{i - e}$, $x \in V(p)$.

Now we know that U(4) is a gray under multiplication modulo 12

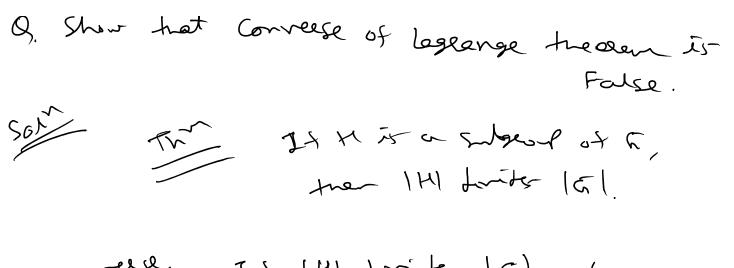
and
$$x \in U(p)$$
 $|U(p)| \equiv 1 \pmod{p}$
 $\Rightarrow x = 1 \pmod{p}$
 $\Rightarrow x^{p-1} \equiv 1 \pmod{p}$
 $\Rightarrow x^{p-1} \equiv 1 \pmod{p}$

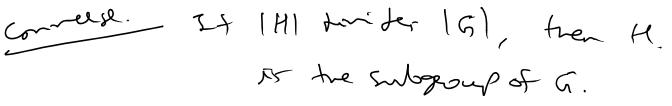
Now for
$$e^{n}(D)$$
, $a = Prifl
 $\Rightarrow a \equiv x \pmod{p}$
 $\Rightarrow a^{p-1} \equiv x^{p-1} \pmod{p}$
 $\Rightarrow a^{p-1} \equiv 1 \pmod{p}$ (using $e^{n}(a)$
 $\Rightarrow a^{p-1} \equiv 1 \pmod{p}$
 $\Rightarrow a^{p-1} a \equiv a + 1 \pmod{p}$
 $\Rightarrow a^{p} \equiv a \pmod{p}$$

Second statement of Fearat's little that
Let a be any prtegee and
$$pX = pX = a$$
, then
 $a^{p-1} \equiv 1 \pmod{p}$
when we purite $a^{p-1} + y + p$, then
semainder is 1.

•

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From the is A_{4} -1 Attendations show of degree 4. $|A_{4}| = \frac{4!}{2} = \frac{29}{2} = |2| |A_{1}| = \frac{N!}{2}$ Gaydey-table of A_{4} . A q hos eight elements of order 3.

Nor we will show that 6/12 but Aqhan No subgeoup of order 6.

Inder of a subgedief: The inder of a subgeoup H in a geo. the number of fistinct left (a fight cosets of I in G. It is knoted by [G: H]oe [G: H] |G:H| = |G|If G AT Smith, then Suppose mot H is a subgroup of Aq |H| = 6a be any element of office Aq. 3 in let The left cosets of Kose H, a Hand a? H. Nor, |G:H| = |G| = |G| = |2] = -12-: 11 her when 2 in Aq. atmost two of the caset II, a Hand all are fistinct.

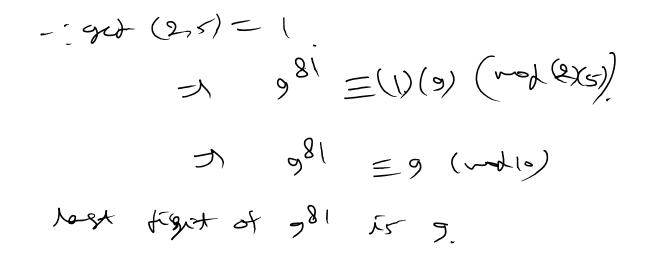
TTAH=H > AEH

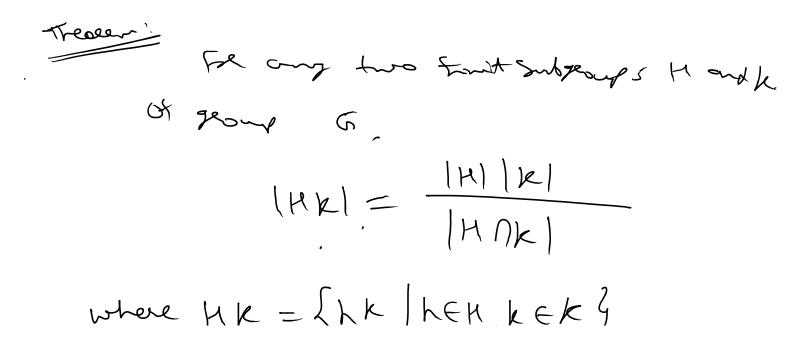
IT aN = at I H = at I a EH If ME of H J at = of H J aH = H J AEH. - a is on adouted a element of elder 3 in Aq. and mere are eight elements of order I in Aq Thurs, a subgeout of sefer 6 would have to contain eight elements of alder? which is a contradiction. Aq has no subgeour at order 6. Q find last fight of $9^{81}/\chi = 9$ and 72. $1/\chi = 72$ $\begin{array}{l} & & & \\ & &$

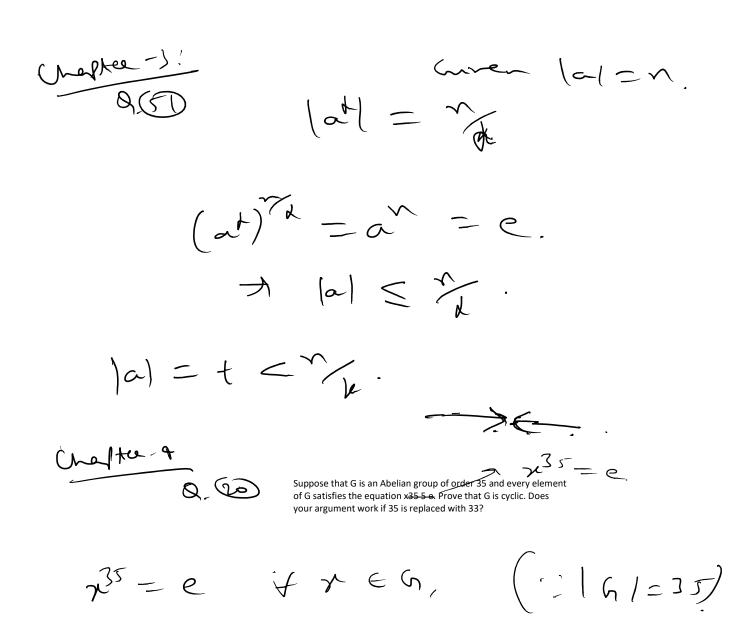
$$5 = 9 \pmod{2}$$

$$5 = 9 \pmod{2}$$

$$5 = 1 (1000 \text{ Merce } 2^{p-1} \equiv 1 (1000 \text{ Me$$







To Prove: Ghas an element of oe tee 35. $\left(a^{\left(G\right) }=e\right)$ -: 161=35 & ate: & ates lal la be. 5 er 7 er 35. (- 35 = e.) [Tal finites [G] > cooles.) Now, assume that 6 hop vo element of oater 35. $\phi(d)$ In a finite group, the number of elements of order d is a multiple <u>of f(el)</u>. ho. if elements of order 5 is a multiple of $\phi(5) = 4$. · AX34 J all Monidentity elements-of trave not of edges 5. I all nonibertity elements of G are not of erfer 7. . · 6 × 34 Nor, When elements of cedeer 5 and 7.

Wt a EG + laI=T. ~+ b E G & / b = 7. (Usure lighty then abe G 4 |ab| = 75. (ab) $5 \neq e$ (ab) $7 \neq e$ Ghas an element of el fel ✐ aychic. 7 6 25 $\langle a \rangle \leq C(a /$ For any element a in any group G, prove that ket is a subgroup of C(a) (the centralizer of a). a, be <a> > a a [e <a] If d is a positive integer, $d \ge 2$, and d divides n, show that the number Q 🐼 of elements of order d in Dn is (d). How many elements of order 2 does Dn have? $|D_n| = 2N$. 50 In Dr. there are not atron and reflection Cach respection is of server 2 fotution Kigo is the only estati that have alter 2

But files is the element of Dn, if files EDn. rightitions of Dn from a cyclic group. i no. of potations of orfer of is \$(4) no, at reflections of alder 1 is 0. (dtg) A no. of elements of sedee him Drif \$(d). let for tenote the set of fotations of Dr. |Fn| = N.& R. is cyclic.

If 2/n i.e. nor even, then no op elements of offer 2 is $\varphi(2) = 1$, nearly

It 2Xn, i.e.n it oft, then no of elevents of ceder 2 in Rn is O.