Problem 13: Consider an ideal Bose gas in three dimensions with energy-momentum relation $\epsilon \propto p^s$ with $s > 0$. For what range of s will the system undergo a Bose-Einstein condensation at a non-zero temperature?

Problem 14: Consider a system of non-interacting quantum particles in three dimensions with dispersion relation $\epsilon \propto k^s$ where ϵ is energy and $\vec{k}$ is the wave-vector, where s is an integer. To what power of ϵ is the density of states proportional to?

Problem 15: Consider a system of N weakly interacting non-relativistic Bosons confined to a two-dimensional region of area A. Repeating the standard analysis in three dimensions, test whether Bose-Einstein condensation occurs in this system at a non-zero temperature.

Problem 10: A cavity of volume $1 cm^3$ is filled with blackbody radiation at temperature $727°K$. What is the average number of photons in the cavity? Use can use the following result:

$$\int_0^\infty dx \frac{x^2}{e^x - 1} = 2.404$$

Problem 11: Radiation in equilibrium fills a hot enclosure. How high must the temperature of the enclosure be for the radiation pressure to be equal to one atmosphere?

Problem 12: The cosmic microwave background radiation left over from the Big Bang today fills the universe with blackbody radiation at temperature $T = 2.76°K$. What is the mean number density of photons? Use can use the following result:

$$\int_0^\infty dx \frac{x^2}{e^x - 1} = 2.404$$

Problem 13: Consider blackbody radiation in equilibrium at temperature T. Let $u(\nu)d\nu$ be the energy density of the radiation in the frequency interval ν to $\nu + d\nu$ and let $\tilde{u}(\lambda)d\lambda$ be the energy density in the wavelength interval λ to $\lambda + d\lambda$. Calculate ν_{max} and λ_{max}, where ν_{max} is the frequency corresponding to which $u(\nu)$ is maximum and λ_{max} the wavelength corresponding to which $\tilde{u}(\lambda)$ is maximum. Are these related as $\lambda_{max} = c\, \nu_{max}$? Why?

Problem 14: A gas cloud in our galaxy emits radiation at a rate of $10^{27} W$. The radiation has maximum intensity at wavelength $\lambda = 10 \mu m$. Assuming the cloud to be spherical and that it emits like a blackbody, estimate the diameter of the cloud.

Problem 15: Consider a hypothetical system of massless Bosonic particles which can be emitted and absorbed by matter, just like photons. The system is confined to a two-dimensional area A and is in equilibrium at temperature T. Performing an analysis similar to that for a three-dimensional photon gas, determine the temperature dependence of the energy density (energy per unit area) of the system.

3 Fermi Dirac and Bose Einstein Statistics

Problem 1: Seven Bosons are arranged in two compartments. The first compartment has 8 cells and the second compartment has 9 cells of equal size. What is the total number of microstates for the macrostate (3,4)?

Problem 2: Six Fermions are arranged in two compartments. The first compartment has 7 cells and the second compartment has 8 cells of equal size. What is the total number of microstates for the macrostate (2,4)?

Problem 3: Four weakly interacting particles are confined to a cubical box of volume V, with the energy of any one particle of the form

$$E = \frac{\pi^2\hbar^2}{2mV^{2/3}}\left(n_x^2 + n_y^2 + n_z^2\right)$$

where n_x, n_y and n_z are natural numbers. What is the energy of the system at absolute zero if the system is (i) Bosonic (ii) Fermionic? Ignore spin.

Problem 4: Consider a system of two weakly interacting particles. Each particle can be in one of two states with energies 0 and ϵ respectively. Calculate the partition function of the system if the system is (i) Bosonic (ii) Fermionic. Calculate the mean energy of the system as a function of temperature and its value as the temperature approaches absolute zero. Give a physical interpretation of the zero temperature result.

Problem 5: Calculate the Fermi energy for Silver, given that its density is 10.5 g/cc. The atomic mass of Silver is 108 g. Assume there is one free electron per atom.

Problem 6: Given a Fermi gas, what is the mean occupation number for a state with energy $2k_BT$ above the Fermi energy?

Problem 7: The Fermi energy of free electrons in Silver atoms at $0°K$ is 5.51 eV. What is the average energy per electron?

Problem 8: The occupation number for a system of Bosons diverges for $\epsilon = \mu$ at finite T. What phenomenon does this indicate? Does this happen for a system of photons?

Problem 9: An ideal non-relativistic Fermi gas at absolute zero has Fermi energy ϵ_F, with each particle having mass m. Calculate the mean value of v_x and v_x^2, where v_x is the x component of velocity of a particle.

Problem 10: Find an expression for the Fermi energy and the average energy per electron at $0°K$ for a free electron gas of N electrons confined to a one-dimensional region of length L.

Problem 11: Consider a system of N Bosons occupying volume V. At high enough temperature T, the system behaves like a classical idea gas, such that the pressure of the system is proportional to T. If the temperature is such that the system is strongly degenerate, given that a certain fraction of atoms is in the ground state (and does not contribute to pressure), what power of temperature do you expect the pressure to be proportionate to? Explain.

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