The Postulates of Quantum Mechanics :- The following are the postulates of quantum mechanics

1) The physical state of a system at time t is described by wave function $\psi(x,t)$, ~~this function is single valued, continuous & finite throughout the space.~~

2.) The wave function $\psi(x,t)$ & its first & second derivatives $\partial\psi(x,t)/\partial x$ & $\partial^2\psi(x,t)/\partial x^2$ are continuous, finite & single valued for all values of x. Also the wave function $\psi(x,t)$ is normalized i.e.

$$\int_{-\infty}^{\infty} \psi^*(x,t)\, \psi(x,t)\, dx = 1$$

where ψ^* is complex conjugate of ψ formed by replacing i with $-i$ wherever it occurs in the function ψ $(i = \sqrt{-1})$

3.) A physically observable quantity can be represented by Hermitian operator. An operator $\hat{A}$ is said to be Hermitian if it satisfies the following condition:

$$\int \psi_i^* \hat{A} \psi_j \, dx = \int \psi_j (\hat{A}\psi_i)^* dx$$

where ψ_i & ψ_j are the wave functions representing the physical state of quantum system i.e. a particle, an atom or a molecule.

4.) The allowed values of an observable A are the eigen values a_i in the operator eqn.

$$\hat{A}\psi_i = a_i \psi_i$$

The above eqn. is known as eigen value eqn. where a_i is an eigen value.

5.) The average value (or, the expectation value) $\langle A \rangle$ of an observable A, corresponding to operator $\hat{A}$ is obtained by relation

$$\langle A \rangle = \frac{\int_{-\infty}^{\infty} \psi^* \hat{A} \psi \, dx}{\int_{-\infty}^{\infty} \psi^* \psi \, dx}$$

[If ψ is normalized then $\int \psi^*\psi\, dx = 1$]

6.) To each observable quantity in classical mechanics, like, position, velocity, momentum, energy etc. there corresponds a certain mathematical operator in quantum mechanics the nature of which depends upon the classical expression for the observable quantity. Some classical mechanical observable & their corresponding quantum mechanical operators are given in following Table

Classical variable (observables)		Operators	
Name	Symbol	Symbol	Operation
Position	x	$\hat{x}$	Multiplication by x
Position squared	x^2	$\hat{x}^2$	Multiplication by x^2
Momentum	p_x	$\hat{p}_x$	$-i\frac{h}{2\pi}\frac{\partial}{\partial x} = -i\hbar\frac{\partial}{\partial x}$
	p	$\hat{p} = -i\hbar\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]$	(taking derivative w.r.t. x & multiply by $-i\hbar$)
Momentum squared	p_x^2	$\hat{p}_x^2$	$-\frac{h^2}{4\pi^2}\frac{\partial^2}{\partial x^2} = -\hbar^2\frac{\partial^2}{\partial x^2}$ (taking double derivative w.r.t. x & multiply by $-\hbar^2$)
Kinetic Energy	$T_x = \frac{p_x^2}{2m}$	$\hat{T}_x$	$-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
	K	$\hat{K} = \frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$	
Potential energy	$V(x)$	$\hat{V}(x)$	Multiplication by $\hat{V}(x)$
Total energy	$E = T_x + V(x)$	$\hat{H}$ $-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] + V(x,y,z)$	$-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2} + V(x)$

1.7.0

Num:- Show that e^{ax} is an eigen function of the operator d/dx & find the corresponding eigen value. Show that e^{ax^2} is not an eigenfunction of d/dx

Sol:- In first case $\psi = e^{ax}$ $\therefore \frac{d\psi}{dx} = \frac{d}{dx}(e^{ax}) = ae^{ax} = a\psi$

Hence e^{ax} is an eigen function of d/dx & its eigen value is a

In II^nd case $\psi = e^{ax^2}$

$\therefore \frac{d\psi}{dx} = \frac{d}{dx}(e^{ax^2}) = \cancel{a \cdot e^{ax^2}} \; \cancel{\frac{d}{dx}(ax^2)} \; e^{ax^2}\frac{d}{dx}(ax^2)$

$= 2ax e^{ax^2} = 2ax\psi$

Thus the result obtained is not a constant factor multiplied with ψ but a variable factor $2ax$ multiplied with ψ. Hence e^{ax^2} is not an eigenfunction of d/dx

Num:- Justify whether the function $\cos ax$ is an eigen function of a) d/dx b) d^2/dx^2. What is the corresponding eigenvalue if any?

Ans:— a) Here $\psi = \cos ax$ $\therefore \frac{d\psi}{dx} = \frac{d}{dx}(\cos ax)$ (3)

$= -a\sin ax$

Hence $\cos ax$ is not an eigenfunction of d/dx

b) $\frac{d^2\psi}{dx^2} = \frac{d^2}{dx^2}(\cos ax) = \frac{d}{dx}\left(\frac{d}{dx}\cos ax\right)$

$= \frac{d}{dx}(-a\sin ax) = -a^2\cos ax = -a^2\psi$

Hence $\cos ax$ is an eigen function of d^2/dx^2 with eigen value $= -a^2$

Num:— Find commutators of the operators for momentum & position, the two conjugate properties of Heisenberg's Uncertainty principle.

Ans:— $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$, $\hat{x} = x$

Let $\psi(x)$ be the operand

$\therefore [\hat{p}_x, \hat{x}]\psi = [p_x x - x p_x]\psi$

$= \left[-i\hbar\frac{\partial}{\partial x}\cdot x - x\left(-i\hbar\frac{\partial}{\partial x}\right)\right]\psi$

$= -i\hbar\frac{\partial}{\partial x}(x\psi) - x(-i)\hbar\frac{\partial\psi}{\partial x}$

$= -i\hbar\psi - \cancel{i\hbar x\frac{\partial\psi}{\partial x}} + \cancel{i x\hbar\frac{\partial\psi}{\partial x}} = -i\hbar\psi$

or $[\hat{p}_x, \hat{x}] = -i\hbar$

Hence momentum & position do not commute therefore they cannot be measured simultaneously