

Classification of second order PDE:-

①

Now, Consider equations of the type

$$R \frac{\partial^2 z}{\partial x^2} + S \frac{\partial^2 z}{\partial x \partial y} + T \frac{\partial^2 z}{\partial y^2} + f(x, y, z, p, q) = 0$$

When $R, S, & T$ are functions of $x & y$.

①

Case (I):- If $S^2 - 4RT > 0$, eqⁿ ① is hyperbolic.

Case (II):- If $S^2 - 4RT = 0$, eqⁿ ① is parabolic.

Case (III):- If $S^2 - 4RT < 0$, eqⁿ ① is elliptic.

Examples:- ① $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$

② $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$

③ $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$

exercises

For each of the differential equations in Exercises 1–10 find a solution which contains two arbitrary functions. In each case determine whether the equation is hyperbolic, parabolic, or elliptic.

1. $\frac{\partial^2 u}{\partial x^2} - 7 \frac{\partial^2 u}{\partial x \partial y} + 6 \frac{\partial^2 u}{\partial y^2} = 0.$

2. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 6 \frac{\partial^2 u}{\partial y^2} = 0.$

3. $\frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + 9 \frac{\partial^2 u}{\partial y^2} = 0.$

4. $4 \frac{\partial^2 u}{\partial x^2} - 4 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$

5. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

6. $2 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = 0.$

7. $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} = 0.$

8. $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} = 0.$

9. $8 \frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} = 0.$

10. $2 \frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} = 0.$