# Unit 4 : Electromagnetic Induction

Electromagnetic Induction or Induction is a process in which a conductor is put in a particular position and magnetic field keeps varying or magnetic field is stationary and a conductor is moving. This produces a Voltage or EMF (Electromotive Force) across the electrical conductor. Michael Faraday discovered Law of Induction in 1830.

Ques : When a credit card is swiped through a card reader, the information coded in a magnetic pattern on the back of the card is transmitted to the card holder's bank. Why is it necessary to swipe the card rather than holding it motionless in the card reader's slot? Can moving objects produce Electric currents?

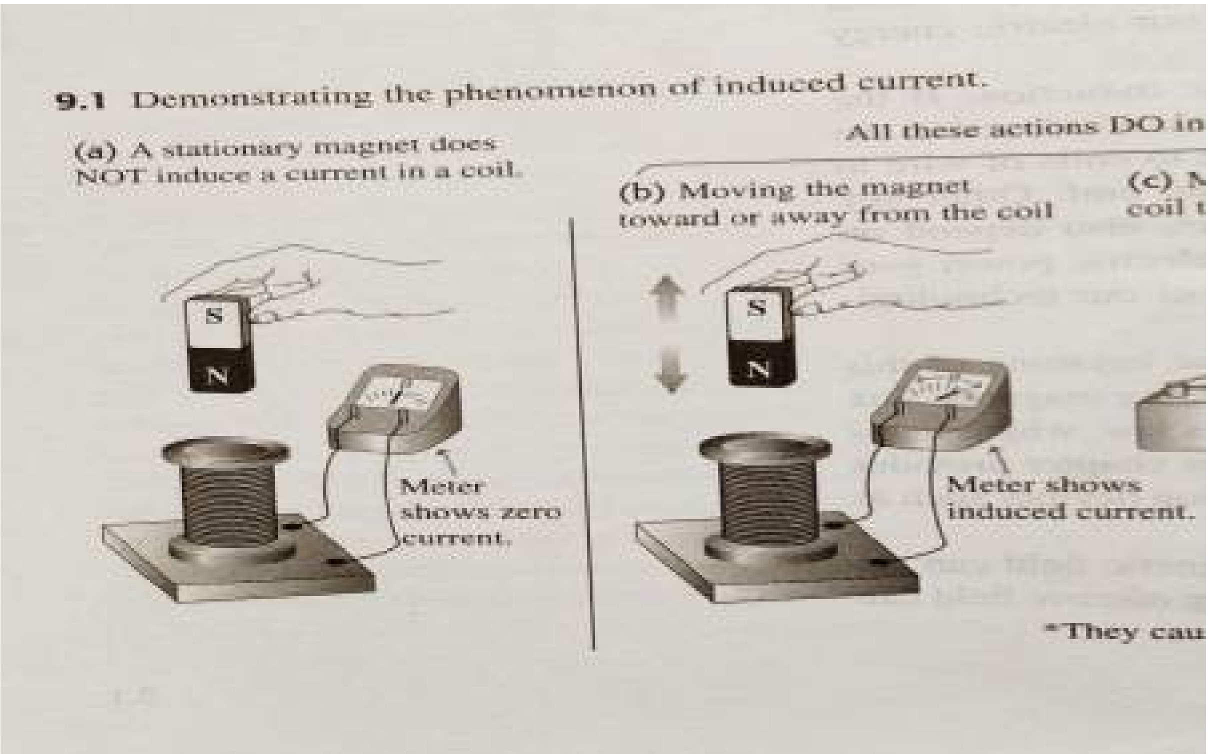
Electricity and Magnetism are intimately connected

**Faraday's Laws of Electromagnetic Induction (EMI)**

It is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction. The phenomenon of production of induced emf and hence induced current due to a change of magnetic flux linked with a closed circuit is called Electromagnetic Induction. This emf / current **exists as long as flux is changing** and not otherwise. Hence, EMI means inducing Electricity by Magnetism.

## Faraday's Experiment

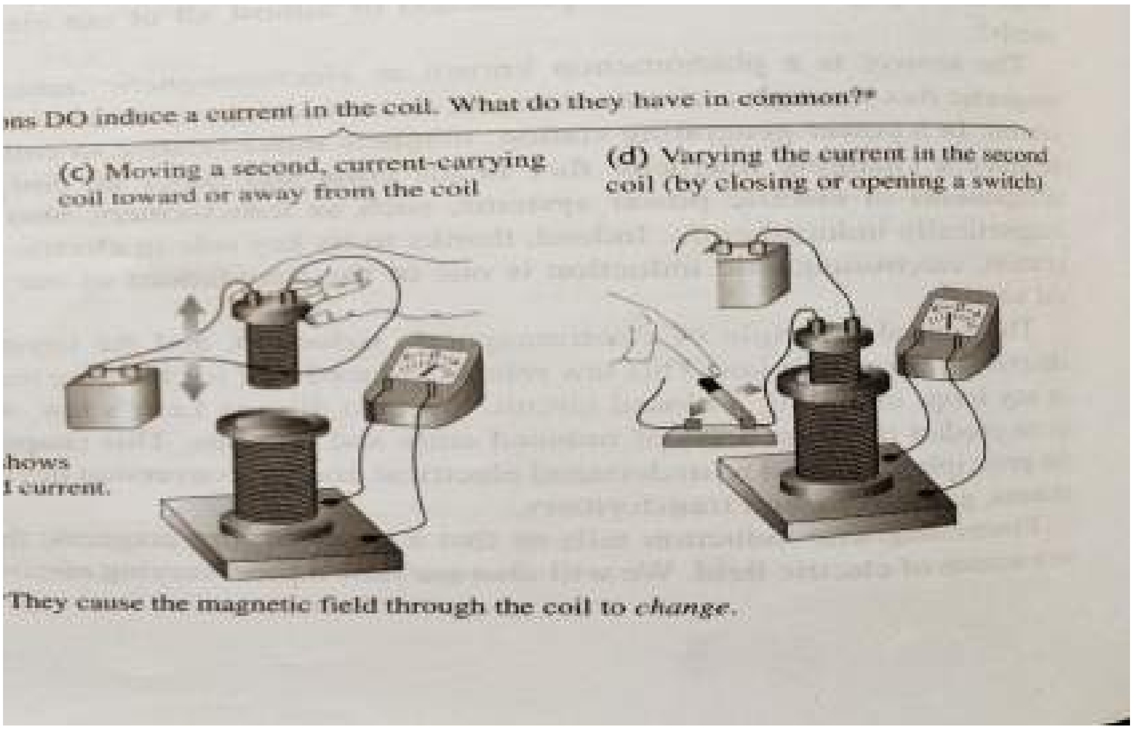
**Experiment 1:**



**Observations**

| Position of magnet | | Deflection in galvanometer |
|---|---|---|
| Magnet at rest | Coil is Stationary | No deflection in the galvanometer |
| Magnet moves towards the coil | | Deflection in galvanometer in one direction |
| Magnet is held stationary at same position (near the coil) | | No deflection in the galvanometer |
| Magnet moves away from the coil | | Deflection in galvanometer but in the opposite direction |
| Magnet is held stationary at the same position (away from the coil) | | No deflection in the galvanometer |
| Magnet is held stationary and the coil moves towards the magnet | Magnet is Stationary | Deflection in galvanometer in one direction |
| Magnet is held stationary and the coil moves away from the magnet | | Deflection in galvanometer but in the opposite direction |

**Experiment 2 :**




**Observations**

There are two circuits in the above picture. The one connected to a battery is primary circuit and the other one connected to galvanometer is secondary circuit

| | | Deflection in galvanometer of secondary circuit |
|---|---|---|
| The moment Primary circuit is closed. Tapping key is | Both the Circuits are stationary | Deflection in galvanometer in one direction |

| | | |
|---|---|---|
| pressed. Current is increasing in primary circuit. | | |
| Primary circuit is kept pressed and steady current flows through primary coil | | No deflection in the galvanometer |
| The moment Primary circuit is opened. Tapping key is released. Current is decreasing in primary circuit. | | Deflection in galvanometer but in the opposite direction |
| Primary circuit is kept opened. Current is 0. | | No deflection in the galvanometer |
| Primary Circuit is moved towards / away from Stationary Secondary Circuit | Primary Circuit is Closed | Deflection in galvanometer in one direction / opposite direction respectively |
| Secondary Circuit is moved towards / away from Stationary Primary Circuit | | Deflection in galvanometer in one direction / opposite direction respectively |
| Both the circuits move towards / away from each other | | Deflection in galvanometer in one direction / opposite direction respectively |

**Conclusion:**

From this experiment, Faraday concluded that whenever there is relative motion between a conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage (induced emf) across a coil or induced current is developed in the secondary circuit. It is also seen that the faster the change in the magnetic field, the greater will be the induced EMF or voltage in the coil. This phenomenon is Electromagnetic Induction.

Michael Faraday formulated two laws on the basis of the above experiments. These laws are called Faraday's laws of electromagnetic induction. This can be stated as follows

1. When the magnetic flux linked with a closed circuit changes, an emf ( and hence a current) is induced in it which lasts as long as the change in flux is taking place.
2. This induced emf is equal to the negative rate of change of magnetic flux φ
   i.e $e = -\frac{d\varphi}{dt}$

**How To Increase EMF Induced in a Coil ?**

By increasing the number of turns in the coil i.e N

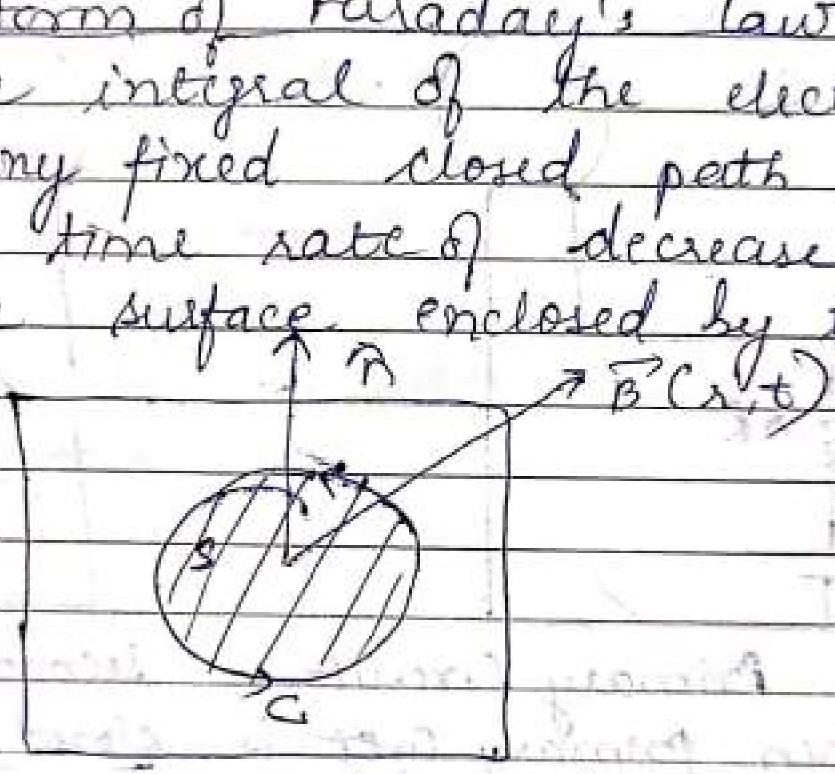
By increasing magnetic field strength i.e B surrounding the coil- Mathematically, if magnetic field increases, flux increases and if flux increases emf induced will also get increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force pass through the coil and hence there will be more emf induced.

By increasing the speed of the relative motion between the coil and the magnet – If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

$dt$

## Integral form of Faraday's law

The line integral of the electric intensity around any fixed closed path is equal to the time rate of decrease of flux through the surface enclosed by the curve




Consider a surface S bounded by a closed loop C and located in a region in which there is a time-varying magnetic field

Magnetic Flux in the surface S due to $\vec{B}$ is
$$\phi_B = \int_S \vec{B} \cdot d\vec{s}$$

Due to time varying magnetic field, a current flows in the closed loop C, which creates an $\vec{E}$. The emf correspondi to

$$e = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{s}$$

$$\boxed{e = \oint_C \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}}$$

Differential Form of Faraday's law

A space derivates of E at a particular point is equal to the time rate of change of $\vec{B}$ at that point.

$$\oint_C \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Stoke's Theorem

$$\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

It is valid over any arbitrary surface

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Note:

1. Differential form of Faraday's laws of Induction tells that the electric field is created not only by electric charges but

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2) also by a varying magnetic field.

Imp **Electrostatic**

point charges $\vec{\nabla} \times \vec{E} = 0$

$\oint \vec{E} \cdot d\vec{l} = 0 \rightarrow$ conservative nature of $E$ as a scalar potⁿ $= -\vec{\nabla} V$

Varying B $\Rightarrow$ electric field E

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{\nabla} \times \vec{E} = 0$ when $\frac{\partial \vec{B}}{\partial t} = 0$, or $\vec{B}$ is con

$$\oint_C \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \neq 0$$

E is not path independent

Hence E is non-conservative field.
so Electric field produced by a time varying magnetic field is NOT conservative

**What is Lenz's Law?**

Lenz's law of electromagnetic induction states that the direction of the current induced in a conductor by a changing magnetic field (as per Faraday's law of electromagnetic induction) is such that the magnetic field created by the induced current opposes the initial changing magnetic field which produced it. The direction of this current flow is given by Fleming's right hand rule.

**Explanation** : If the flux increases in a circuit, the induced emf tends to cause current in such a direction so as to decrease the flux. In general, the cause of the current may be due to the motion of the conductor in a magnetic field or change of the magnetic field itself.

Rule to determine the direction of induced current due to the motion of the conductor in a perpendicular magnetic field :

**Fleming's right hand rule** : If we stretch the thumb, the fore finger and the central finger in the mutually perpendicular directions and if the fore finger points in the direction of the magnetic field, thumb in the direction of the motion of conductor then the central finger points in the direction of the induced current / emf in the conductor.

**Self Inductance of a Coil**

Phenomenon of producing an induced emf in a circuit due to changes in the circuit itself is known as Self Inductance. The induced emf is known as self-induced emf.

Note : The negative sign in e = -dφ/dt indicates that voltage induced opposes the change in current through the coil per unit time ( di/dt ).

$$e = \frac{-d\varphi}{dt}$$

$$\varphi \; \alpha \; I \;\; implies \; \varphi = LI$$

$$L = \frac{\varphi}{I}$$

$$which \; is \; a \; constant \; called \; as \; Coefficient \; of \; Self \; Inductance \; or \; Self \; Inductance.$$

$$If \; there \; are \; N \; TURNS \; in \; the \; coil, then \; L = N\frac{\varphi}{I}$$

$$e = \frac{-d\varphi}{dt} = \frac{-d(LI)}{dt} = -L\frac{dI}{dt}$$

$$implies \; magnitude \; L = \frac{e}{dI/_{dt}}$$

Self Inductance of a circuit or I Henry is numerically equal to the induced emf of 1Volt when the current in it is changing at a unit rate i.e 1A /sec .

$$Units \; of \; L = \frac{Volts}{Amp/_{sec}} = VsA^{-1} = Henry = Web \; Amp^{-1}$$

**Physical Significance of Self Inductance**

It is actually a measure of an inductors "resistance" to the change of the current flowing through the circuit and the larger is its value in Henries, the lower will be the rate of current change. It plays the same role in electric circuit as the mass / moment of inertia in mechanical motion. When the circuit is closed, self-inductance induces an emf so as to slow down the growth of current. When circuit is opened, self-inductance induces an emf so as to slow down the decay of current. This law validates the law of conservation of energy.

Inductors are devices that can store their energy in the form of a magnetic field. Inductors are made from individual loops of wire combined to produce a coil and if the number of loops within the coil are increased, then for the same amount of current flowing through the coil, the magnetic flux will also increase. So by increasing the number of loops or turns within a coil, increases the coils inductance. Then the relationship between self-inductance, ( L ) and the number of turns, ( N ) and for a simple single layered coil can be given as:

Self Inductance of a Coil

$$L = N\frac{\Phi}{I}$$

It is also defined as the magnetic flux linkage, ( NΦ ) per unit current.

The self-inductance of a coil depends upon the characteristics like shape, size, length, number of turns etc. of the coil. It also depends on the medium around it.

## Self-Inductance in Solenoid

If the inner core of a long solenoid coil of length l with N number of turns is hollow, then the magnetic induction within its core will be given as:

$$B = \mu_o H = \mu_o \frac{N.I}{\ell}$$

Then by substituting these expressions in the equation above for Inductance will give us:

$$L = N\frac{\Phi}{I} = N\frac{B.A}{I} = N\frac{\mu_o.N.I}{\ell.I}.A$$

$$L = \mu_o \frac{N^2.A}{\ell}$$

## Energy associated with an inductor

When a current flows in a circuit, the self inductance opposes the growth of it $\therefore$ work is done by the source or battery to establish the current. This work done is stored as Pot$^n$ egy.

Let $e$ be the emf at any instant $t$.

$$e = -L\frac{dI}{dt}$$

To add a charge to the conductor, work is done by the battery, the power supplied to the inductor

$$P = -eI$$

$$P = LI\frac{dI}{dt}$$

Total work done in passing a charge $I_0$

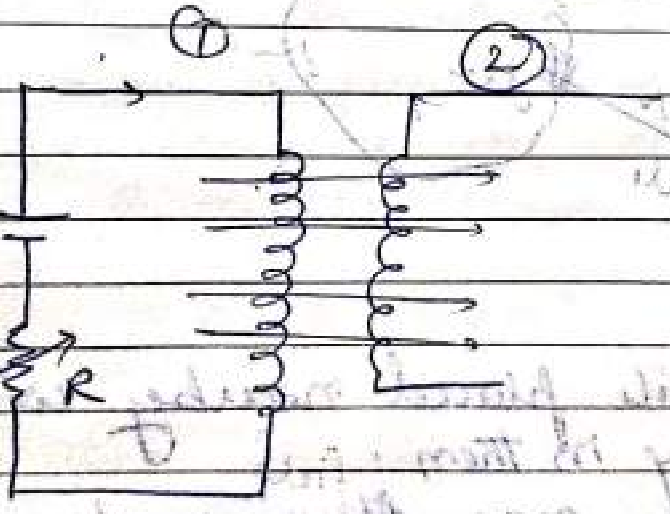
$$W = \int_0^{I_0} P\,dt = \int_0^{I_0} LI\frac{dI}{\cancel{dt}} \times \cancel{dt}$$

$$\boxed{W = \frac{1}{2}LI_0^2}$$

Coefficient of self Inductance is defined as twice the work done in establishing magnetic flux associated with the current of 1 unit in the circuit.

$$\boxed{L = \frac{2W}{I_0^2}}$$

Mutual Inductance when 2 coils ~~placed and~~ ~~on~~ a circuit are placed & closed by a path of the flux produced by one is linked with the other coil. Any change of flux in one circuit will produce an induced emf in the second. The phenomenon of producing an induced emf in a circuit due to changes in another circuit close by is known as mutual inductance.




consider 2 coils in which coil 1 is connected to a battery. Some of the magnetic lines of coil 1 will intersect with coil 2 which is placed nearby

$\phi_{21}$ = mag. flux of coil 2 due to current in coil 1 ($I_1$).

$\phi_{21} \propto I_1$

$\phi_{21} = M_{21} I_1$

$M_{21}$ = mutual inductance

unit = W/A or H

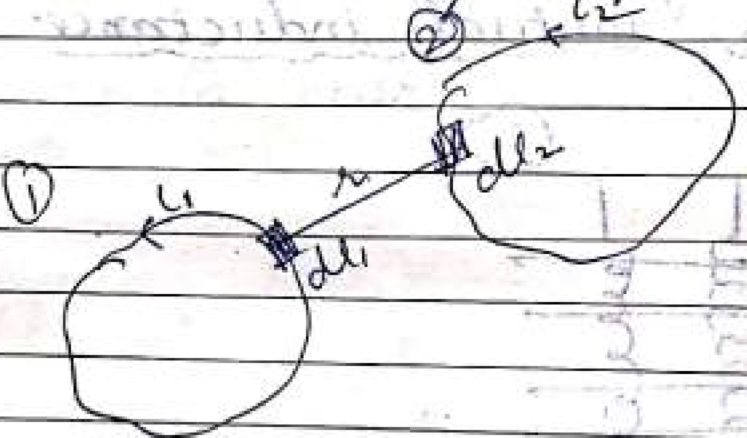
→ It depends on shape, size & no. of turns of the coil

→ It also depends on medium in which they are placed.

Induced emf on coil 2

$$e = -\frac{d\phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$$

Theorem for Mutual Inductance of 2 current circuits / Reciprocity theorem (Rel?)




Consider 2 coils placed nearby with currents $i_1, i_2$ flowing in them.

Let $B_1$ be the mag. field produced by coil 1 due to current $i_1$.

Flux induced in coil 2 due to $B_1$

$$\phi_2 = \int_{S_2} \vec{B_1} \cdot d\vec{S_2}$$

$$\vec{B_1} = \vec{\nabla} \times \vec{A_1}$$

$$\phi_2 = \int_{S_2} (\vec{\nabla} \times \vec{A_1}) \cdot ds_2$$

$\vec{A_1}$ = magnetic vector potential

Stoke's Theorem

$$\phi_2 = \oint_{C_2} \vec{A_1} \cdot \vec{dl_2}$$

$$\vec{A_1} = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{i_1 \vec{dl_1}}{r}$$

$$\phi_2 = \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{i_1 \vec{dl_1} \cdot \vec{dl_2}}{r}$$

$$= \underbrace{\left[\frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{\vec{dl_1} \cdot \vec{dl_2}}{r}\right]}_{M_{21}} i_1$$

$$\phi_2 = M_{21}\, i_1$$

Let $B_2$ be the magnetic field due to current $i_2$ flowing in coil 2.

Flux induced in coil 1 due to $B_2$

$$\phi_1 = \int_{S_1} \vec{B_2} \cdot \vec{ds_1}$$

$$= \int_{S_1} (\vec{\nabla} \times \vec{A_2}) \cdot \vec{ds_1}$$

$$\phi_1 = \int_{C_1} \vec{A_2} \cdot \vec{dl_1}$$

$$\vec{A_2} = \frac{\mu_0}{4\pi} \oint_{C_2} \frac{i_2 \vec{dl_2}}{r}$$

$$\phi_1 = \oint_{C_1} \frac{\mu_0}{4\pi} \oint_{C_2} \frac{i_2 \vec{dl_2} \cdot \vec{dl_1}}{r}$$

$$\phi_1 = \underbrace{\left[\frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\frac{\vec{dl_2}\cdot\vec{dl_1}}{r}\right]}_{M_{12}} i_2$$

$$\phi_1 = M_{12}\, i_2$$

∵ $dl_1$ & $dl_2$ are independent integration variables, the order of integration can be changed

i.e. $\vec{dl_2}\cdot\vec{dl_1} = \vec{dl_2}\cdot\vec{dl_1}$

$$\boxed{M_{21} = M_{12}} = \frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\frac{\vec{dl_2}\cdot\vec{dl_1}}{r}$$

Reciprocity Relⁿ.

Ac:- The emf induced in each loop increases not only due to the change in magnetic flux created by the currents in other loops but also due to the variation of magnetic flux created by the current in the same loop.

For coil 1

$$\phi_1 = \phi_{11} + \phi_{12}$$

$\phi_{11}$ = Flux due to current flowing in itself.

$\phi_{12}$ = flux due to current flowing in coil 2.

IIly for coil 2

$$\phi_2 = \phi_{22} + \phi_{21}$$

$\phi_{22}$ = flux due to current flowing in itself

$\phi_{21}$ = flux due to current flowing in coil 1.

emf in coil 1 is $-\frac{d\phi_1}{dt}$

$$= -\frac{d}{dt}(\phi_{11} + \phi_{12})$$

emf in coil 2 is $-\frac{d\phi_2}{dt}$

$$= -\frac{d}{dt}(\phi_{22} + \phi_{21})$$

Q.

Coefficients of Coupling of the two Circuits

Coefficient of coupling of 2 circuits is the mutual inductance between them in terms of the self inductance of each circuit alone. This is the case of maximum flux linkage when all the flux through 1 circuit links with the other.

Consider 2 coils of equal area (but not necessarily of equal no. of turns) should be placed next to one another.

let $N_1$ be no. of turns in coil 1

$N_2$ " " " " " 2

$i_1$ current in coil 1

$i_2$ " " " 2

let $\phi_{12}$ be flux per unit turn in coil 1 due to current $i_2$ in coil 2.

$\phi_{21}$ be flux per turn in coil 2 due to current $i_1$ in coil 1

Total Flux in coil 1 due to current $i_2$ in coil 2 = $N_1\phi_{12}$

This flux $\propto i_2$

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$$N_1 \phi_{12} \propto i_2$$

$$N_1 \phi_{12} = M_{12} i_2$$

$$M_{12} = \frac{N_1 \phi_{12}}{i_2} \quad (1)$$

Total flux in coil 2 due to current $i_1$ in coil 1 $= N_2 \phi_{2-1}$

This flux $\propto i_1$

$$N_2 \phi_{21} \propto i_1$$

$$N_2 \phi_{21} = M_{21} i_1$$

$$M_{21} = \frac{N_2 \phi_{21}}{i_1} \quad (2)$$

Self inductance of coil 1

$$N_1 \phi_1 \propto i_1$$

$$N_1 \phi_1 = L_1 i_1$$

$$L_1 = \frac{N_1 \phi_1}{i_1} \quad (3)$$

Self inductance of coil 2

$$N_2 \phi_2 \propto i_2$$

$$N_2 \phi_2 = L_2 i_2$$

$$L_2 = \frac{N_2 \phi_2}{i_2} \quad (4)$$

Condition for Maximum flux Linkage

Flux produced by coil 1 is equal (or totally intersected with coil 2) to flux in coil 2 (because of current flowing in coil 1)

$$L_1 = M_{21}$$

$$\frac{N_1 \phi_1}{i_1} = \frac{N_2 \phi_{21}}{i_1}$$

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Consider for 1 turn

$$\frac{\phi_1}{i_1} = \frac{\phi_{21}}{i_1}$$

$$\boxed{\phi_1 = \phi_{21}}$$

Flux produced in coil 2 is equal to the flux in coil 1 (because of current in coil 2)

$$L_2 = M_{12}$$

$$\frac{N_2 \phi_2}{i_2} = \frac{N_1 \phi_{12}}{i_2}$$

For 1 turn

$$\frac{\phi_2}{i_2} = \frac{\phi_{12}}{i_2}$$

$$\boxed{\phi_2 = \phi_{12}}$$

From Reciprocity relⁿ

$$M_{12} = M_{21} = M \text{ (say)}$$

$$M_{12} M_{21} = M^2 = \frac{N_1 \phi_{12}}{i_2} \frac{N_2 \phi_{21}}{i_1}$$

$$= \frac{N_1 \phi_2}{i_2} \frac{N_2 \phi_1}{i_1} = \frac{N_1 \phi_1}{i_1} \frac{N_2 \phi_2}{i_2}$$

$$M^2 = L_1 \times L_2$$

$$\boxed{M_{max} = \sqrt{L_1 L_2}}$$ Mutual Inductance b/w 2 coils

In general only a fraction of flux is linked to the one coil due to other

$$\phi_{12} = k_2 \phi_2$$

||ly $\phi_{21} = k_1 \phi_1$

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$$M_{12}M_{21} = M^2 = \frac{N_1 N_2 \, k_1 k_2 \, \phi_1 \phi_2}{i_1 i_2}$$

$$= k_1 k_2 \frac{N_1 \phi_1}{i_1} \frac{N_2 \phi_2}{i_2}$$

$$= k_1 k_2 L_1 L_2$$

$$M = \sqrt{k_1 L_1 k_2 L_2}$$

Let $k_1 k_2 = k^2$

$$\boxed{M = k\sqrt{L_1 L_2}}$$ Mutual Inductance b/w 2 coils

k = coefficient of coupling
k depends on the geometry of coils & their relative positions. In practice, coeffi of coupling is less than 1 since there will always be a certain amt of flux leakage.

Energy stored in magnetic field
Work done in establishing a current I in a conductor is $\frac{1}{2} L I^2$

$$= \frac{1}{2} I(LI) \quad (1)$$

Flux linked with Inductor

$$\phi = LI$$

$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

$$\phi = \int_S (\vec{\nabla} \times \vec{A}) \cdot \vec{ds}$$

Stokes theorem

$$\phi = \oint_c \vec{A} \cdot \vec{dl}$$

$(1) \Rightarrow \omega = \frac{1}{2} I (LI)$

$= \frac{1}{2} I \phi$

$$W = \frac{1}{2} I \oint_c \vec{A} \cdot \vec{dl}$$

Since dl is nothing but the path of flow of current, the vector sign is moved from dl to I

$$\boxed{\omega = \frac{1}{2} \oint_c \vec{A} \cdot I \vec{dl}}$$ for linear current

For volume current

$$\Rightarrow \boxed{\omega = \frac{1}{2} \int_\tau (\vec{A} \cdot \vec{J}) \, d\tau}$$ General

Ampere's Circuital law $\nabla \times B = \mu_0 J$

$\vec{J} = \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B})$

$$\omega = \frac{1}{2} \int_\tau \vec{A} \cdot (\nabla \times \vec{B}) \, d\tau$$

$$\vec{A} \cdot (\nabla \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \nabla \cdot (\vec{A} \times \vec{B})$$

$$W = \frac{1}{2\mu_0} \left[ \int_\tau \vec{B} \cdot \underbrace{(\nabla \times \vec{A})}_{\vec{B}} d\tau - \int_\tau \underbrace{\nabla \cdot (\vec{A} \times \vec{B}) \, d\tau}_{\text{G.D Theorem}} \right] \quad (1)$$

$$W = \frac{1}{2\mu_0} \left[ \int_\tau \vec{B} \cdot \vec{B} \, d\tau \int_S (\vec{A} \times \vec{B}) \cdot ds \right]$$

$$W = \frac{1}{2\mu_0} \left[ \int_\tau B^2 d\tau - \int_S (\vec{A} \times \vec{B}) \cdot ds \right]$$

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As the integration limits are increased beyond the volume of the object containing the current density J the surface integral vanishes.

$$\boxed{W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau}$$

Energy stored in a magnetic field per unit volume $= \frac{B^2}{2\mu_0}$