## **EXERCISE 7.1**

1. (a) Explain the concept of skewness. Draw the sketch of a skewed frequency distribution and show the position (Delhi Univ. B.Com., 1997) of the mean, median and mode when the distribution is asymmetric.

(b) Explain the concept of positive and negative skewness.

(c) Show graphically the positions of mean, median and mode in a positively and negatively skewed series.

[Delhi Univ. B.Com. (Pass), 1998]

[Delhi Univ. B.Com. (Hons.) 2002]

## 2. Comment on the following :

(a) In a symmetrical distribution, we have mean = median  $\neq$  mode.

(b) "A series representing U-shaped curve is symmetrical." Comment.

[Delhi Univ. B.Com. (Pass), 2001] 3. (a) The values of mean and median are 30 and 40 respectively in a frequency distribution. Is the distribution (new 2002) [Delhi Univ. B. Com. (Pass), 2002] skewed? If yes, state the direction of skewness.

Ans. Mean < Median, the distribution is negatively skewed.

(b) The mean for a symmetrical distribution is 50.6. Find the values of median and mode. [C.S. (Foundation), June 2001]

Ans. Median = Mode = 50.6.

4. (a) Define Pearson's	measure of s	kewness. Wl	hat is the c	lifference t	oetween rela	tive measur	C and the abrol	
(b) From the following	data find out k	arl Pearson					e and the absolu	ute
(b) Measurement	1	o contreation	s co-effici	ent of skcw	ness :			
Frequency	1	0	11	12	13	14	15	
Frequency .		2	4	10	8	5	15	
Ans. 0.3478.							,	
5. Calculate the Pearson	's coefficient	of skewness	from the f	ollowing :				
Wages (Rs.)	: 0–	-10	10-20	20-	_30	20 40	10	
No. of Workers	: 1	5	20		0	25	40-50	
<b>Ans.</b> 0.1845.				2	•	25	10	
6. Calculate Karl Pearso	n's coefficien	t of skewnes	s from the	following	data and ern	loin its size		
Wages : 70-80	) 80-90	90-100	100-1	10 110	120 120	iam its sign	llicance :	
No. of Persons: 12	18	35	42	50	120 120-	-130 130-	-140 14015	50
			12	50	י 45 [גרע	) 2 hillein Da	0 8	
Ans. $M = \text{Rs. } 110.43$ , M	$I_0 = R_{s} \cdot 116.1$	5 0-Pc 1	7 7 6 81	0.001.6	[Dell	ni Univ. B.C	.om. (Hons.), 20	<i>1</i> 00]
7 What is Positive and	Nogotive Class	0, 0 = Ks. 1	1.20, 3k =	- 0.3316.				
table :	regative skew	wness ? Calc	ulate Karl	Pearson's	coefficient o	f skewness	from the follow	ing
Wages (in Rs) 50	) 60 60	70					7 3	Ū
Wages (III NS.)	15	—/0    70		80—90	90—100	100-11	0 110-120	O
	15	18	17	30	40	20	10	
					[De	lhi Univ. B.	Com. (Pass), 19	996]
Ans. Mean = $85.8$ , Mod	$e = 93.33, \sigma =$	= 16·99, <i>Sk</i> (H	Carl Pearso	(n) = -0.44	ł			
8. From the following d	ata of age of e	mployees, ca	alculate co	efficient of	skewness ar	nd comment	on the result	
Age below (yrs.)	: 25	30	35	40	45	50	55	
No. of employees	: 8	20	40	65	80	92	100	
						[Delh	i Univ. MBA. 19	9971
Ans. $\bar{x} = 37.25$ yrs.; M	o = 36.67 yrs.	; $\sigma = 16.99$	yrs.; Sk (k	Carl Pearson	n) = 0.07	ਂ ਬੰੱ		
9. Calculate Karl Pearso	n's Coefficier	t of skewnes	ss from the	following	data			
Class	Fre	auency		Clas	-			
4060	+	25		10 15		Frequency		
30-40	*	15		1013		6		
20-30		12		3_5			4	
15-20		8		<u> </u>		3		
Hint. Since classes are	funequal ma	mitudes we	estimate t	ha value of			2	
	n unequal ma	gintudes, we	cstimate t	a (Massa	mode by us	ing :		
Mo = 3Md - 2M	=	•	Sk =	3 (mean –	Median)			
An- 14				σ	5 K			
Ans. Mean = $31.13$ , M	edian = $31.67$	$\sigma = 16.06,$	Sk = -0	·1.				
10. Calculate Karl Pears	on's coefficie	nt of skewne	ss from th	e following	series :			
Wt. in kgs.	Below 40	40-	-50	50-60	) 6	0-70	70 80	
No. of persons	10	1	6	18	· · ·	25	20	
Wt. in kgs.	8090	90—	100	100 and ab	ove	20	20	
no. of persons :	4	4		3				
Hint. Take the first class	s as 30—40 ar	d the last cla	ass as 100.	-110				
Ans. Mean = $62.200$ kg			135 US 100	16.0671				
11. Calculate Karl D	Mede	(5 022 1						
	Mode	= 65.833 kg	ς, σ=	10.857 Kg	3k = -0.2	155.		
Marks (abour)	Mode on's coefficie	= 65.833 kg nt of skewne	$s, \sigma =$	e following	SK = -0.2 g data :	155.		
Marks (above) : No. of students	Mode on's coefficie 0 10	e = 65.833 kg nt of skewne 0 20	$s, \sigma =$ ss from the 30	e following 40	$S_{R} = -0.2$ g data : 50	60	70 80	
Marks (above) : No. of students : Hint. Locate	Mode on's coefficie 0 10 150 14	= 65.833 kg nt of skewne 0 20 0 100	g, $\sigma =$ ss from th 30 80	e following 40 80	, SK = - 0.2 g data : 50 70	60 30	70 80 14 0	
Marks (above) : No. of students : Hint. Locate mode by Median.	Mode on's coefficie 0 10 150 14 the method of	= 65.833 kg nt of skewne 0 20 .0 100 f grouping ; 1	s, o = ss from th 30 80 two moda	e following 40 80 I classes 10	, 3k = - 0.2 g data : 50 70 020 and 5	60 30 50—60; mod	70 80 14 0 1e ill-defined. F	ïnd

 $\Delta s. Sk = 3(M - Md)/\sigma = -0.6622.$ 

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 7.9

## BUSINESS STATISTICS

0.69

12. The daily expenditure of 100 fa	milies is given below :			
Deith Expanditure $0-2$	0 20-40	4060	60	30 80-100
Daily Expenditure . 13	?	27	7	16
No. of Families	to the Karl Pears	on coefficient of s	kewness.	
If the mode of the distribution is 44	, calculate the Kall I call	co 90 is 10		
Ans. Frequency for the class 20-4	0 is 25 and for the class	60-80 is 19.		
$Sk$ (Karl Pearson) = $\frac{50}{21}$	$\frac{-44}{5\cdot 3} = 0.24.$			
12 The following facts are gathere	d before and after an ind	istrial dispute :		
13. The following facts are gathere		Before disp	oute	After dispute
			515	Aco Aco
No. of workers employed		Rs 49	.50	P. 10.70
Mean wages		Ro. 57	.80	Na. 52/0
Median wages		KS. 52		Ks. 50.00
Variance of wages		( <b>Rs</b> .)* 121	-00	(Rs.) <sup>2</sup> 144-00
Compare the position before and a	fter the dispute in respect	of		
(a) total wages (b) m(	dal wages (c) stand	lard deviation,	and	(d) skewness.
(a) total wages, $(b)$ inc	Mai Wages; (c) stat	Refore dispute		After dispute
Ans.	т	25 492.50		Rs 26 849.75
(i) Total wages	г	(S. 25,492.50		De 44.50
(ii) Modal wages		KS. 39.40		No. 44-50
(iii) C.V.		22.22		22.14

14. You are given below the following details relating to the wages is respect of two factories from which it is concluded that the skewness and variability are same in both the factories.

	-	Factory A	Factory B
Arithmetic Mea	in :	50	45
Mode	:	45	50
Variance	:	100	100

- 0.90

Point out the mistake or the wrong inference in the above statement.

Ans. C.V. (A) = 20, C.V. (B) = 22.2; Sk(A) = +0.5, Sk(B) = -0.5.

15. The sum of 20 observations is 300 and its sum of squares is 5,000 and median is 15. Find the coefficient of skewness and coefficient of variation.

**Ans.** Sk = 0, C.V.  $= \frac{5}{15} \times 100 = 33.3$ .

16. For a group of 10 items  $\Sigma X = 452$ ,  $\Sigma X^2 = 24,270$  and Mode = 43.7. Find the Pearsonian coefficient of skewness.

**Ans.** Sk = 0.08.

(iv) Skewness

17. If the mode and mean of a moderately asymmetrical series are respectively 16 inches and 15.6 inches, would be its most probable median?

Ans. Median = 15.73 inches.

18. In a slightly skew distribution the arithmetic mean is Rs. 45 and the median is Rs. 48. Find the approximate value of mode.

Ans. Mode = Rs. 54.

19. In a frequency distribution, Karl Pearson's coefficient of skewness revealed that the distribution was skewed w

the left to an extent of 0.6. Its mean value was less than its modal value by 4.8. What was the standard deviation? Ans.  $\sigma = 8$ 

**20.** In a distribution mean = 65, median = 70 and the coefficient of skewness is -0.6. Find mode and coefficient of tion. (Assume that the distribution is variation. (Assume that the distribution is moderately asymmetrical.)

**Ans.** Mode = 80, C.V. = 38.46

21. In a certain distribution the following results were obtained :

Arithmetic Mean (X) = 45

Median 
$$= 48$$

Coefficient of skewness = -0.4The person who gave you this data, failed to give you S.D. (Standard Deviation). You are required to estimate it the help of the above data. [Delhi Univ. B. Com. (Hons.), 1997] with the help of the above data.

Ans.  $\sigma = 22.5$ .

## SKEWNESS AND KURTOSIS

22. Karl Pearson's measure of skewness of a distribution is 0.5. The median and mode of the distribution are respectively, 42 and 32.

Find: (i) Mean, (ii) the S.D., (iii) the coefficient of variation. [Delhi Univ. B.A. (Econ. Hons.), 2000] Ans. (i) 47, (ii) 30, (iii) 63-83.

23. Karl Pearson's coefficient of skewness of a distribution is 0.32. Its standard deviation is 6.5 and mean is 29.6. Find the mode and median of the distribution.

If the mode of the above is 24.8, what will be the standard deviation ? [Delhi Univ. B.Com. (Hons.), 1998]

Ans. Mode = 27.52, Median = 28.91,  $\sigma = 15$ 

24. Karl Pearson's coefficient of skewness of a distribution is +0.40. Its standard deviation is 8 and mean is 30. Find the mode and median of the distribution

Ans. Mode = 26.8. Median = 28.93

25. The median, mode and coefficient of skewness for a certain distribution are respectively 17-4, 15-3 and 0-35. Calculate the coefficient of variation.

Ans. Mean = 18.45; C.V. = 48.78.

26. From a moderately skewed distribution of retail prices for men's shirts, it is found that the mean price is Rs. 20 and the median price is Rs. 17. If the coefficient of variation is 20%, find the Pearsonian coefficient of skewness of the distribution. [Delhi Univ. B. Com. (Pass), 2002]

Ans. Skewness (Karl Pearson) = 2.25.

27. (a) A frequency distribution is positively skewed. The mean of the distribution is :

Greater than the mode, Less than the mode, Equal to the mode, None of these.

Tick the correct answer.

(b) In a moderatly skewed distribution the values of mean and median are 5 and 6 respectively. The value of mode in such a situation is approximately equal to....

(i) 8 (iii) 16 (ii) 11 Ans. (a) M > Mo, (b) (i) : 8.

28. In a moderately asymmetrical distribution :

(i) The mode and median are 300 and 240 respectively. Find the value of mean.

[C.S. (Foundation), Dec. 2001] (ii) Mean = 200 and Mode = 150. Find the value of median.

Ans. (i) Mean = 210, (ii) Median = 183.33.

29. Which group is more skewed?

(i) Mean = 22; Median = 24; s.d. = 10. (ii) Mean = 22; Median = 25; s.d. = 12. Ans. Sk(i) = -0.60; Sk(ii) = -0.75. Group (ii) is more skewed to the 'eft.

30. What is the relationship between mean, mode and median ? What is the condition under which this relationship holds? Locate graphically the position of the three measures in the case of both negatively as well as positively skewed distribution.

Ans. Mode = 3 Median - 2 Mean, for a moderately asymmetrical distribution.

7.2.3. Bowley's Coefficient of Skewness. Prof. A.L. Bowley's coefficient of skewness is based on the quartiles and is given by :

$$Sk = \frac{(Q_3 - Md) - (Md - Q_1)}{(Q_3 - Md) + (Md - Q_1)}$$
(7.8)

$$Sk = \frac{Q_3 + Q_1 - 2Md}{Q_3 - Q_1}$$
(7.9)

Remarks 1. Bowley's coefficient of skewness is also known as Quartile coefficient of skewness and is <sup>cspecially</sup> useful in situations where quartiles and median are used viz., :

(i) When the mode is ill-defined and extreme observations are present in the data.

(ii) When the distribution has open end classes or unequal class intervals

In these situations, Pearson's coefficient of skewness cannot be used.

2. From (7.8), we observe that :

3

$$Sk = 0$$
, if  $O_3 - Md = Md - O_3$ 

(iv) None of these.

(7.10)