

PHYS 3324 Lab

Millikan Oil Drop Experiment:

Demonstration of the Quantization of Charge

Background reading

Read the introduction below before answering the Prelab Questions

Prelab Questions

1. A typical oil drop in this experiment will have a radius of $0.7 \mu\text{m} = 0.7 \times 10^{-6} \text{ m}$. Using equation 1 in the introduction, determine the time it takes for an oil drop of this radius to fall through a distance of 1 mm. (note that the velocity of the falling drop v_f is a constant in this case due the combined effects of gravity and the viscosity of the air).
2. Assume that the drop in question 1 (radius of $0.7 \mu\text{m}$) is now between plates of a parallel plate capacitor with a potential difference of $V=500 \text{ V}$ and a separation of $6 \text{ mm} = 6 \times 10^{-3} \text{ m}$. Assume that the upper plate is positive, so a negatively charged oil drop will rise. Assume that the magnitude of the charge on the drop is $2e$, where $e=1.60 \times 10^{-19} \text{ C}$ is the fundamental electric charge. Use equation 2 in the introduction to determine the time it takes for this oil drop to rise through a distance of 1 mm in this electric field. (note that you will need to use the velocity of the falling drop v_f that you computed in question 1 as you solve this problem).

Introduction

The electronic charge, or electrical charge carried by an electron, is a fundamental constant in physics. During the years 1909 to 1913, R.A. Millikan used the oil-drop experiment to demonstrate the discreteness, or singleness of value, of the electronic charge, and to make the first accurate measurement of the value of this constant. In that experiment, a small charged drop of oil is observed in a closed chamber between two horizontal parallel plates. Such a chamber is shown in Figure 1. By measuring the velocity of the drop under gravity and its velocity of rise when the plates are at a high electrical potential difference, data is obtained from which the residual electrical charge on the drop may be computed.

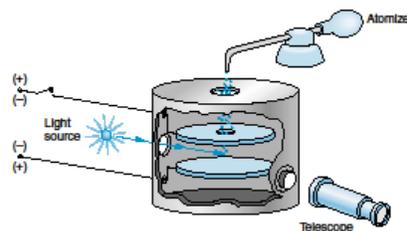


Figure 1: Schematic diagram of a Millikan oil-drop apparatus.

The oil drops in the electric field between the plates are subject to three different forces: gravitational, electric and viscous (frictional or “drag” force between the air and oil). By analyzing these various forces, an expression can be derived which will enable measurement of the charge on the oil drop and determination of the unit charge on the electron.

If m is the mass of the drop (sphere) under observation between the plates, the gravitational force on the drop is $F_g = mg$. In the absence of any electric field, the only other force that acts is the viscous retarding force between the oil and the air; this force is given by $F_d = kv = 6\pi\eta Rv$, where η is the coefficient of viscosity of the fluid (air) and R is the radius of the drop. The velocity of the drop quickly increases to the constant “terminal velocity” where these two forces are equal in magnitude $F_g = F_d$. This is shown in the left figure of Figure 2. So we have:

$$F_g = mg = F_d = 6\pi\eta Rv_f$$

The mass of the spherical oil drop can be written in terms of the density ρ and radius R as $m = (4/3)\pi R^3\rho$. So, by measuring the fall velocity of the drop v_f we can determine the radius of the drop by manipulating the above expression to give:

$$R = \sqrt{\frac{9\eta v_f}{2\rho g}} \quad (1)$$

For parallel plates, a uniform electric field $E = V/d$ can be generated by applying an electrical potential difference V across plates separated by a distance d . We will assume that the electric field is applied in such a direction that the force on a negatively charged particle will act upward (see right side of Figure 2). The combination of the three forces (gravity, viscous, and electrical) will cause the drop to rise with a constant velocity v_r . The three forces combine to give zero net force:

$$mg - qE + 6\pi\eta Rv_r = 0$$

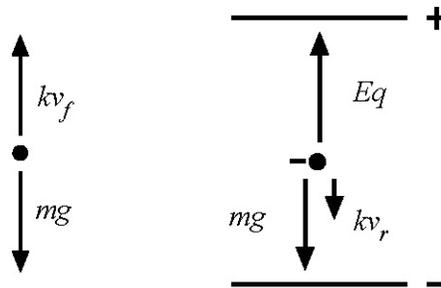


Figure 2: Left: No electric field; only gravitational and viscous forces act on the oil drop. Right: Electric field applied; gravitational, viscous, and electrical forces act on the oil drop.

The above equation can be combined with equation 1 for the radius of the drop to give this expression for the charge on the drop (in terms of the measured fall and rise velocities – v_f and v_r):

$$q = (v_f + v_r) \frac{\sqrt{v_f}}{V} \eta^{3/2} \frac{18\pi d}{\sqrt{2\rho g}} \quad (2)$$

For a given oil drop, you will measure the fall time (T_f) in the absence of the electric field and the rise time (T_r) in the presence of the electric field over a fixed distance s . The fall and rise velocities can then be determined from $v_f = s/T_f$ and $v_r = s/T_r$ and the charge on the drop can be computed from Equation 2 above. (Note: all the quantities in equation 2 are magnitudes, meaning they are all positive quantities. The signs were taken account of in the derivation of the equation.)

Useful constants and data about our apparatus:

d = separation of electric field capacitor plates = 6 mm = 6×10^{-3} m

ρ = density of the oil = 874 kg/m³

η = coefficient of viscosity of air = 1.81×10^{-5} Ns/m²

g = acceleration due to gravity = 9.81 m/s²

e = accepted value of magnitude of the electron's charge = 1.602×10^{-19} C

As always, when you are doing computations in this lab, it is very important to use a consistent set of units. Use SI units – meter for length, kilogram for mass, seconds for time, newtons for force, coulombs for charge, and volts for electric potential difference.

Experimental Goal

The goal of this experiment is to use the Millikan Oil Drop Apparatus to do the following:

- Demonstrate that electric charge only comes in discrete units – “the quantization of charge”
- Measure the intrinsic charge of the electron (the smallest discrete unit of charge) e

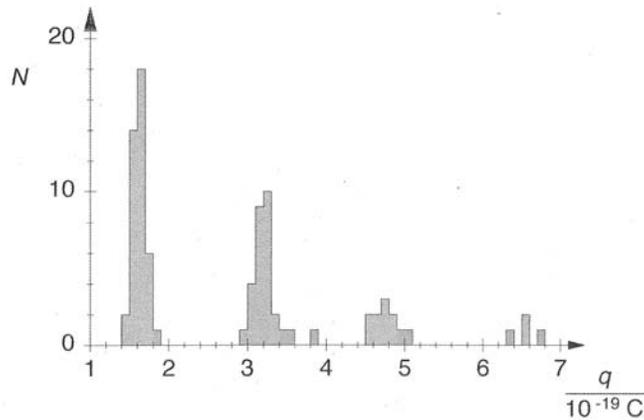


Figure 3: Desired output of the experiment – a histogram of the results of repeated measurements of the excess charge on oil drops. The histogram will clearly show the quantization of charge and allow a measurement of the elementary charge e .

Apparatus

First familiarize yourself with the apparatus. A diagram and picture of the apparatus as it should be set up for you are shown in Figure 4.

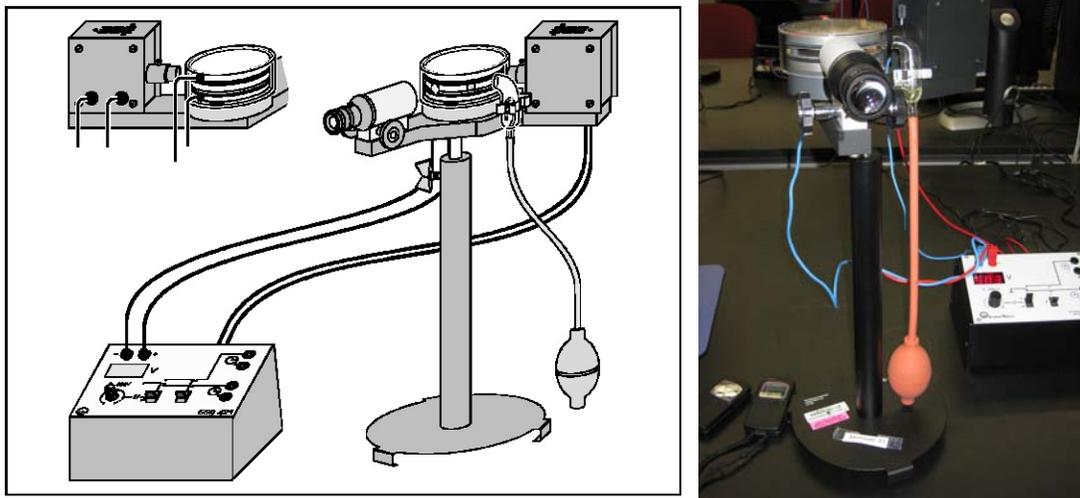


Figure 4: The Leybold Didactic Millikan Oil Drop Apparatus

Take a look at the apparatus and identify the following pieces:

- Two stopwatches: you will use these to measure the fall and rise time of each drop you study. The right button starts and stops the stop watch. The left button resets it.
- The Millikan power supply: this has no on/off switch. To bring it on plug in the connector from the power adapter to the 12 V in hole in the back. This supply does two things. It supplies the power to the light (in the black housing on the stand that illuminates the oil drop chamber), and it applies the electric potential difference to the capacitor plates. There are two switches; put the switch labeled “*t*” in the down position and leave it that way for the whole lab – you won’t be using it.
- Take a look at the stand. You should see the Millikan oil drop chamber with its lucite housing (to reduce the effect of air currents). It has two metal capacitor plates separated by about 6 mm. There are also two small holes on the right hand side where oil drops can be admitted. Right by them is the atomizer, which is attached to a rubber bulb. By squeezing hard on the bulb, you force oil up through a capillary tube in the atomizer and a mist of oil droplet “spheres” is created.
- You should also see that there is a lamp on the right hand side (powered by the power supply) that shines light into the chamber. Note also that the chamber has black tape in the back – that is the dark background that you will view the bright oil droplets against. Finally, note that there is a telescope that you will use to actually view the droplets – more on that later.
- Verify that the connections from the power supply to the capacitor plates are hooked up so that the positive side (red lead) is on the top and the ground side (blue lead) is on the bottom.
- Turn the “U” switch to on and dial the voltage knob over its full range (0 – 600 V). Then set it to 500 V (within $\sim \pm 5$ V or so). Leave it at that setting for the remainder of the lab. Turn the U knob off for now.

Procedure

1. First you will get use to viewing the oil drops and observe their general behavior. Look into the eyepiece. You should be able to see a vertical micrometer scale and a black background (the tape at the back of the chamber). The eyepiece has knob you can turn to adjust the focus on the micrometer scale; optimize it for your eye. Now squeeze on the atomizer rubber bulb. You may need to adjust the glass oil holder so its output is centered over the small holes on the right side of the chamber. You can vary the distance of this output from the side of the chamber to try to optimize the number of droplets you get in the chamber at a given time. You should be able to see small bright droplets in the chamber now. You can adjust the objective lens on the microscope using the rotary knob on its two sides. This will focus on different parts of the chamber, so you will see some droplets come into focus while others go out of focus. Finally, if needed you can adjust the position of the light bulb relative to the chamber by loosening the screw on top that

holds it in place. We tried to optimize its position beforehand, but feel free to adjust it if you want to try to vary the lighting of the interior of the chamber.

2. Before doing anything quantitative, notice some general features of the behavior of the droplet. First, it is important to note that the microscope's objective lens **inverts** the image. So if a drop is falling, you will see it rise in your image. Conversely, if a drop is rising you will see it falling in your image.

- First look at the droplets with no voltage applied to the capacitor plates. You should see them mostly rising (although some large ones may be nearly stationary). Since they appear to be rising, they are actually falling under the influence of gravity.
- Now turn on the voltage to the capacitor plates (500 V). You will see some drops slow down and some will actually reverse direction (meaning they are actually rising). These are droplets with an excess negative charge that are attracted to the top plate of the capacitor. So they either slow the rate of fall of the drop or reverse it and start rising if the residual negative charge is large enough.
- On the other hand, you will see some droplets actually start to fall faster (or rise faster as you actually see it). These are positively charged drops that are attracted electrically to the bottom plate of the capacitor (ie. the force on them is the same direction as the gravitational force).

3. Now you are ready to do some quantitative measurements. The first thing to note is the calibration of the scale. The objective lens of the microscope has a magnification of 2.00 and the eyepiece lens has a magnification of 10. So that means that the smallest division you see on the micrometer scale corresponds to a length of .05 mm. The distance between major scale divisions (which each have 10 minor divisions between them) is 0.5 mm. For convenience, all of your measurements should be done over the same vertical fall distance – 20 minor scale divisions (which is the same as two major scale divisions). That corresponds to a real vertical distance that the drop falls through of 1.0 mm. You won't need to worry in detail about this number until you do your report.

4. For the quantitative part of the lab, you will just focus on negatively charged drops. Those are the ones that appear to be rising (actually falling) with 0V applied, and then when you apply 500 V, they reverse direction and appear to be falling (actually rising). For each droplet you select, you want to measure the time it takes to fall through 20 minor scale divisions and the time it takes to rise through 20 minor scale divisions. For your report, you will use those two numbers to determine the radius of each drop and the charge on it.

5. Prepare a table with columns for "Fall time" and "Rise time". To be clear – fall time is the time with 0 V applied, so you will actually see the drop rising. Rise time is the time with 500 V applied, so you will actually see the drop falling. You are going to want to get a fall time and a rise time for a total of 30 drops. One partner should view the drops

while the other runs the stopwatches. Each partner should get a chance to view drops (for 2 partners each should view 15 apiece, for 3 partners each should view 10 apiece).

6. Here is the recommended procedure for taking data for a drop:
 - Set the U switch to ON (for 500 V applied)
 - Squeeze the bulb to put a puff of drops in the chamber
 - Locate a drop that appears to be going down (actually it is rising) and focus the objective lens on it
 - Flip the switch to 0 V and 500 V back and forth to make sure you really have a drop that is reversing direction and that you have isolated on it.
 - Now that you have “control” of the drop, set U to 0V and measure the time it takes to fall (it will appear to be rising) through 20 minor scale divisions (2 major scale divisions).
 - Then flip the switch to on to apply 500 V and measure the time it takes to rise (it will appear to be falling) through 20 minor scale divisions (2 major scale divisions).
 - Record your two times, reset the stopwatches, put another puff of droplets in the chamber, and look for a new drop to measure.

7. Here are some typical ranges of times you should expect to use as a guide:
 - Fall times: Fall times can be in the range of 5 – 40 seconds, with the bigger drops falling faster. Most of the fall times will be in the range 20-30 seconds, but if you occasionally see a fast one (5-10 seconds) try to take data on it so you have looked at a range of droplets.
 - Rise time: Rise times can be seen over a somewhat wider range 5 – 70 seconds; once again try to get a range of values. The faster rise times typically correspond to drops with more charge on them.

8. Before you leave lab, make sure you have a table with 30 values of fall and rise times for 30 different oil drops.

Report

Make sure your report for this lab includes all of the following:

1. Introduction

2. **Qualitative description of droplet behavior:** During the lab (in procedure step 2) you made some qualitative observations of the behavior of the droplets. Briefly describe what you observed when:

- you viewed the droplets with no electric field applied ($V = 0$ volts)
- you viewed the droplets with an electric field applied ($V = 500$ volts); you should have seen drops going both directions – explain that behavior

3. **Equations:** Prepare the equations you will need to analyze your data. For each of your 30 droplet measurements, you should have recorded the “fall time” T_f (time to fall under gravity alone with no electric field applied – $V = 0$ volts) and the “rise time” T_r (time for negatively charged particles to rise in an applied electric field with $V = 500$ volts). These times were to be measured over a fixed distance of 20 minor scale divisions. The calibration of the minor scale divisions was .05 mm per division (note: an earlier version of the lab writeup had an incorrect number for this). So over 20 divisions a distance of $1 \text{ mm} = 1.0 \times 10^{-3} \text{ m}$ was covered. (This is the “fixed distance” referred to as s on page 3 in the introduction to this lab). Start with equations (1) and (2) in the introduction to get the following two formulae in a form that will be useful to you:

- Radius: Plug in all the known constants to determine a formula for the radius of the droplets that only involves the fall time T_f and a numerical constant.
- Charge: Plug in all the known constants to determine a formula for the charge (normalized to the accepted charge) q/e that only involves the fall and rise times T_f and T_r and a numerical constant.

4. **Data and calculations table:** Make a single table for your 30 droplets with the following columns:

- Fall time in seconds
- Rise time in seconds
- Calculated value of the radius of the drop (in units of micrometers = $1 \mu\text{m} = 1.0 \times 10^{-6} \text{ m}$)
- Calculated value of the charge (divided by the accepted value of the charge e): q/e
- “Corrected” value of the charge. It turns out that there are deviations from Stoke’s law (see introduction page 2) for the viscous retarding force between the droplets and the air when the droplets become as small as the ones we are using in this experiment. That is because the size of the droplets starts to become comparable to the mean free path between the air atoms. There is an empirical correction to to correct for this effect known as “Cunningham’s law”. It was applied by Millikan to his own data. It leads to a formula for the corrected value of the charge based on the observed radius of the drop. It is given as:

$$q_{corrected} = \frac{q}{\left(1 + \frac{.07776 \mu\text{m}}{R}\right)^2}$$

where R is the radius of the drop. Apply this correction factor to each of your 30 drops and put the result in a column of “Corrected q/e ”

Before exporting this table from your spreadsheet program, make sure to sort it based on this last column, so your data is sorted from the smallest value of “Correct q/e ” to the largest.

5. **Plots:** The best way to display your data is to make histograms of it. Histograms are plots of the frequency of occurrence of a particular quantity. You can generate histograms using any plotting package you prefer, but if you use Excel, you can find instructions for how to do it at:
<http://www.ncsu.edu/labwrite/res/gt/gt-bar-home.html#cb1>

For your 30 drops, make a histogram of the droplet radii and of your values of “Corrected” q/e .

6. **Extracted value of the fundamental charge:** Look at your “Corrected q/e ” values in your table and in your histogram. If your data was taken properly, you should see clusters of data near 1, 2, 3 (and maybe even 4, although that is rare) times the fundamental charge. Make an estimate of where the “dividing lines” are between these groups, and then for each group compute the average, standard deviation, and standard error of the mean. Record this information in a small table that includes the average, standard error of the mean, standard deviation, and number of points in that group. Now take the value of the charge from each group, divide by the number of charges (1,2,3, ...) so you report an average value of the charge (and its error) as determined from each group. To determine your final value of the charge you need to take the weighted average of these numbers (and determine its error).

7. **Analysis with more data:** On the “Calendar of Sessions” page of the lab website there is a link to “Millikan Data”. Download the Excel file there. It has some data taken using our apparatus with a large number of drops (159 in all). Do the same analysis as you did on the other data (but you don’t need to put in a complete data table for this one). Put the following into your report for this data:

- Histogram of the droplet radii
- Histogram of the corrected values of the charge (q/e)
- Group the data into charge groups and then go through the same steps from item 6 for this data; include the small summary table and final value of the charge (q/e) and its error

8. **Conclusion:** Make sure your conclusion includes your conclusions about the goals:
- Demonstration of charge quantization
 - Measurement of the intrinsic unit of charge – report both the final result from your 30 drop measurement and the result from the larger 159 drop data set
 - If your q/e value is not equal to 1 within your measured errors, then make a comment about why not. There is a systematic error that we did not include here. Do you know what it is?