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Example: Suppose one-step transition

probability matrix is given by

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

If initial distribution $\alpha_0 = 0.4$. $\alpha_1 = 0.6$.

find pmf of X_4

Sol^n $\alpha_0 = 0.4 \Rightarrow P(X_0 = 0) = 0.4$

$\alpha_1 = 0.6 \Rightarrow P(X_0 = 1) = 0.6$

$P(X_4 = 0) = ?$ $P(X_4 = 1) = ?$

$$P(X_4 = 0) = \sum_{i=0}^{1} P(X_4 = 0 \mid X_0 = i) P(X_0 = i)$$

$$= P(X_4 = 0 \mid X_0 = 0) P(X_0 = 0)$$

$$+ P(X_4=0 \mid X_0=1)\, P(X_0=1)$$

$$= P_{00}^{(4)}(0.4) + P_{10}^{(4)}(0.6) \quad \text{—— } (1)$$

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 0.61 & 0.39 \\ 0.52 & 0.48 \end{bmatrix}$$

$$P^4 = P^2 \cdot P^2 = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.5749 & 0.4251 \\ 1 & 0.5668 & 0.4332 \end{array}$$

$$P_{00}^{(4)} = 0.5749, \quad P_{10}^{(4)} = 0.5668$$

using these in eqn (1), we have.

$$P(X_4=0) = (0.4)(0.5749) + (0.6)(0.5668)$$

$$\Rightarrow P(X_4=0) = 0.5700$$

$$\therefore P(X_4=1) = 1 - 0.5700 = 0.4300$$

$\therefore P(X_4=1) = 1-0.5700 = 0.4300$

pmf is $P(X_4=x) = \begin{cases} 0.5700, & \text{if } x=0 \\ 0.4300, & \text{if } x=1 \\ 0, & \text{elsewhere} \end{cases}$

<u>Example:</u> Suppose that we want to determine the probability that a Markov chain enters any of a specified set of states A by time n.

One way to accomplish this is

$$P(X_{n+1}=j \mid X_n=i) = \begin{cases} 1, & \text{if } i\in A,\ j=i \\ 0, & \text{if } i\in A,\ j\neq i \end{cases}$$

That is, transform all states in A into absorbing states which once entered can never be left.

~~Example:~~ A Pensioner receives 2000 \$ at the beginning of each month. The amount of money he needs to spend during a month is independent of the amount he has and is equal to i with Probability P_i $(i=1,2,3,4)$ & $\sum P_i = 1$. If the Pensioner has more than 3000 \$ at the end of a month, he gives the amount greater than 3000 \$ to his son. If, after receiving his Payment at the beginning of a month, the Pensioner has a capital of 5000, what is the Probability that his capital is ever 1000 or less at any time within the following four months.