Anisotropic media, Double refraction, Malus law Retardation Plates, Production and analysis of polarized waves.

1. In what respect does an electrically anisotropic medium differ from an isotropic medium? Write the expression for the permittivity tensor and comment on its nature.
2. Prove that in a crystalline medium the displacement vectors $\overrightarrow{D_1}$ and $\overrightarrow{D_2}$ associated with the two modes of propagation are normal to each other.
3. For a given direction of wave vector $\vec{k}$, obtain expressions for the two phase velocities in a uniaxial crystal in terms of its principal velocities.
4. For an electromagnetic wave propagating through an anisotropic medium , show that the angle between vectors D and E is the same as between the pointing vectors P and the wave propagation vector k.
5. Distinguish between isotropic and anisotropic medium. Assuming the symmetry of dielectric tensor, discuss the propagation of e.m waves in an anisotropic medium and drawn a diagram showing the relative orientations of $\vec{E}$, $\vec{H}$, $\vec{k}$, and $\vec{P}$ vectors.
6. Starting from Fresnel's formula, show that an e.m. wave , in general has two different velocities, for a given direction of propagation in an uniaxial crystal and hence define optic axis.
7. Derive Fresnel's law for the phase velocities in an electrically anisotropic medium. How does this law leads to the phenomenon of double refraction?
8. Show that in an electrically anisotropic dielectric medium the permittivity tensor is symmetric.
9. Show the in an anisotropic medium, energy is not propagated entirely in the direction of propagation of electromagnetic wave.

Applied questions / problems

1 A beam of linearly polarized light is changed into circularly polarized light by passing it through a slice of crystal of thickness 0.006 cm Calculate the difference in the refractive indices of the O and E rays in the crystal assuming this to be the minimum thickness that will produce the above effect. The wave length of light is $6 \times 10^{-5} cm$.

2 For a given wavelength of light 1mm of quartz cut perpendicular to the optic axis rotates the plane of polarization by 20^{o}. Find for what thickness will no light of this wavelength be transmitted when quartz piece is interposed between a pair of parallel nicols.

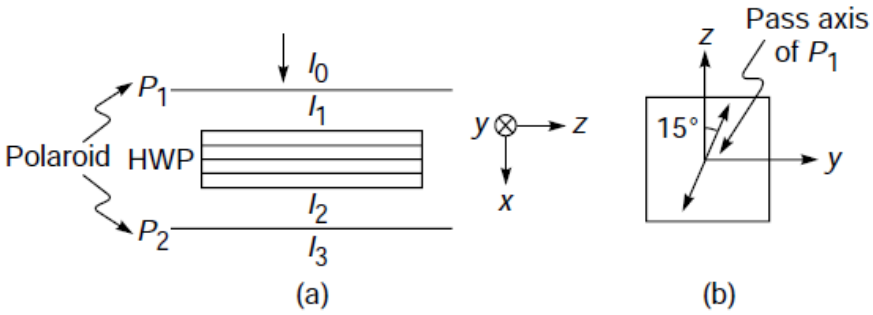
3 Distinguish between a left-handed and a right-handed elliptically polarized light. Suggest a method for converting left-handed into right-handed elliptically polarized light

4 A beam of elliptically polarized light is incident normally on a quarter wave plate. Show that the light emerging from the plate is in general, elliptically polarised. What is the state of polarization of the emergent wave if the optic axis of the plate is along the major axis of the ellipse?

5 A plane polarized light of wavelength 5890A is incident on a plate of a doubly refracting uniaxial crystal cut with faces parallel to optic axis. The crystal has a refractive index of 1.658 for ordinary ray and 1.446 for extra ordinary ray. What type of crystal is this? Calculate the least thickness of the plate for which the ordinary extraordinary rays, on emergence from the plate, recombine to form a plane polarize light.

6 Determine the state of polarization when the x and y components of electric field are given by the following equations : (a) $E_x = E_o\cos(\omega t + kz)$, $E_y = \frac{1}{\sqrt{2}}\mathrm{E_o}\cos(\omega t + kz + \pi)$ (b) $E_x = E_o\sin(\omega t + kz) E_y = \mathrm{E_o}\cos(\omega t + kz)$ (c) $E_x = E_o\sin(kz - \omega t + \frac{\pi}{3})$, $\mathrm{E_o}\sin(kz - \omega t - \frac{\pi}{6})$ (d) $E_x = E_o\sin(kz - \omega t + \frac{\pi}{4}) E_y = \frac{1}{\sqrt{2}}\mathrm{E_o}\sin(kz - \omega t)$

7 A half wave plate (HWP) is introduced between two crossed Polaroids P_1 and P_2. The optics axis makes an angle of 15° with the pass axis of P_1 as shown in Fig. 1(a) and (b). If the unpolarized beam of intensity I_o is normally incident on P_1 and if I_1, I_2 and I_3 are the intensities after P_1, after HWP, and after P_2, respectively, then calculate I_1/I_0, I_2/I_0 and I_3/I_0.



In Fig 1 (a) half wave plate (HWP) is shown between two crossed Polaroids P_1 and P_2. The optics axis makes an angle of 15° with the pass axis of P_1 (Fig. 1 (b)). If the unpolarized beam of intensity I_o is normally incident on P_1 and if I_1, I_2 and I_3 are the intensities after P_1, after HWP, and after P_2, respectively. Compute the ratio of I_1/I_0, I_2/I_0 and I_3/I_0.

13. Consider a linearly polarised beam travelling in free space

$E_x = \frac{1}{2}E_o\cos(\omega t - \mathbf{K.r})$,

$E_y = \frac{1}{\sqrt{2}}E_o\cos(\omega t - \mathbf{K.r})$,

$E_z = -\frac{1}{2}E_o\cos(\omega t - \mathbf{K.r})$

where $\mathbf{K}=\frac{k}{\sqrt{2}}(\hat{x}+\hat{z})$. Show that if the above wave is incident on a dielectric, then the reflected wave is also linearly polarised with the electric field rotated by 90°.