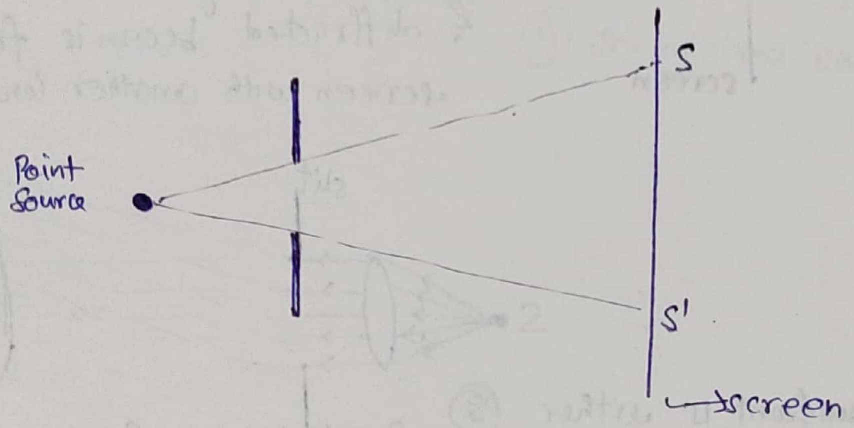
Diffraction

Suppose we placed an opaque object b/w a source of light and a screen, a sharp shadow is obtained on the screen which shows that light travels in a st. lines. If, however, the size of obstacle is small (compared to wavelength of light), there's a departure from straight line propagation & the light bends into the geometrical shadow. So "this phenomenon of bending of light waves around corners of an obstacle and their spreading into the geometrical shadow of an object (obstacle) is called diffraction.

And the resulting intensity distribution of the light if observed on the screen is called diffraction pattern.

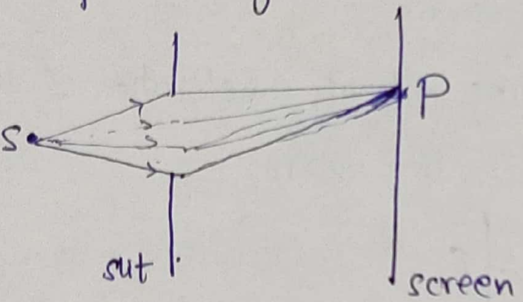


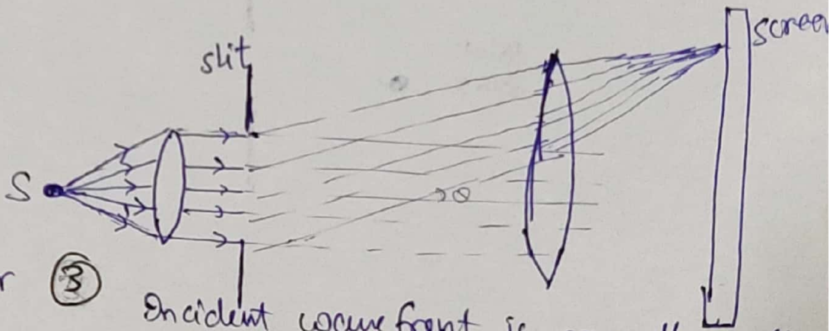
There are two kinds of diffraction

1) Fresnel Diffraction

2) Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
(1) In fresnel diffraction either the source of light or screen or both are at a finite distance from the obstacle. And thus distances are important.	(1) In fraunhofer diffraction, the source of light and screen are at infinite distances from the obstacle. In this class inclination is important.
(2) & observation phenomenon doesnot require any lenses.	(2) Two lenses are required one to make light from the source parallel & other to focus the light after diffraction on the screen. OR the incident rays becomell with a lens & diffracted beam is focus on the screen with another lens



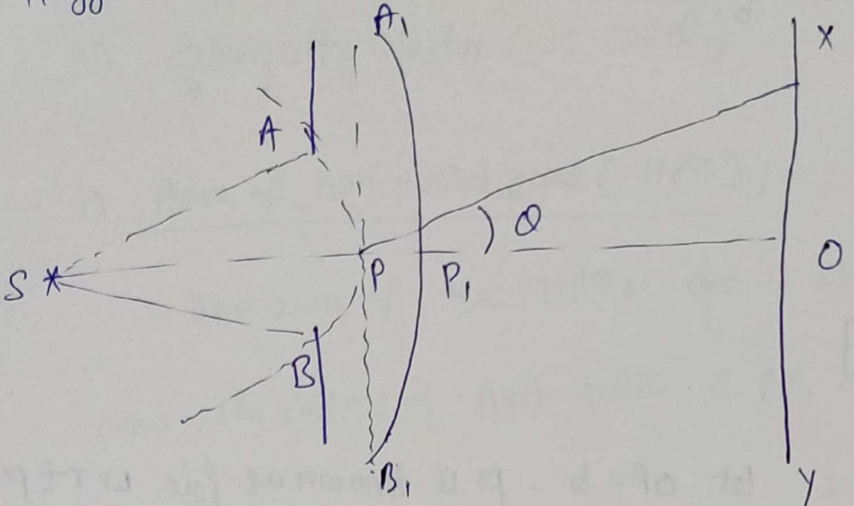


(3) The incident wavefront is either spherical or cylindrical	(3) Incident wavefront is generally ~~~~ plane
(4) The centre of diffraction pattern may be bright or dark depending upon the no. of fresnel's zones.	(4) The centre of diffraction pattern is always bright.
(5) The diffraction is studied with approximations i.e. zone plate	(5) It discussed with more accuracy in analytical terms.

Interference	Diffraction
① The interference occurs b/w two seperate wavefronts originating from two coherent sources	① It is the interference that occurs b/w secondary wavelets originating from the exposed part of the same wavefronts.
② The interference fringes are equally spaced.	② The diffraction fringes are never equally spaced.
③ The maxima are of the same intensity.	③ The intensity of central maximum is maximum & decrease on either side as the order of maximum increases.
④ The minima are perfectly dark.	④ The minima can never perfectly dark.

Fresnel's treatment of wavefront (1815)

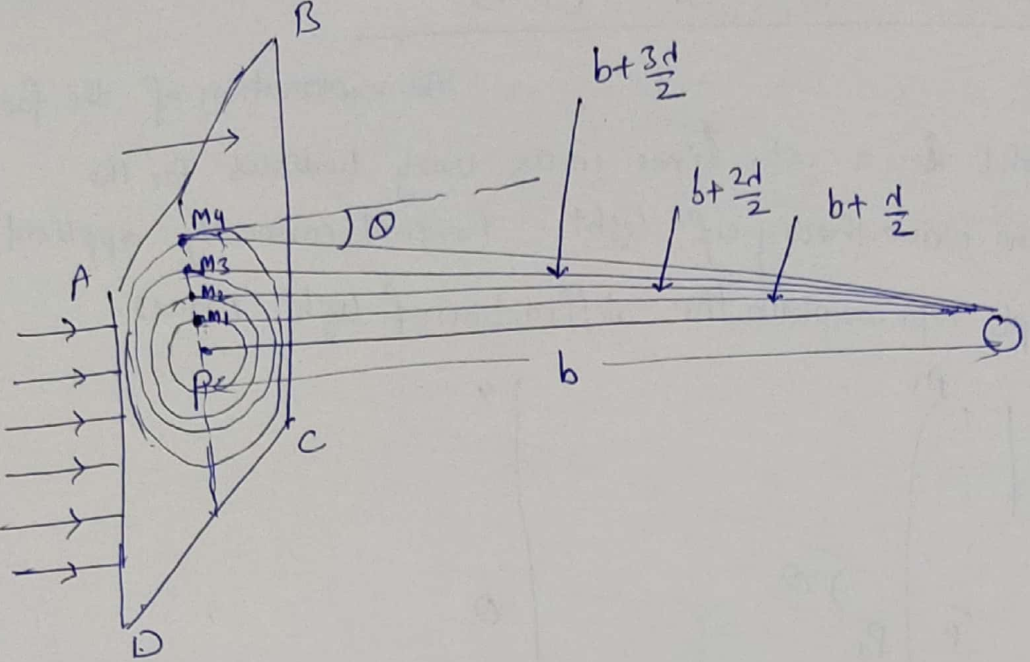
The explanation of the fact of propagation of light in a st. lines was very tedious for the supporters of the wave theory of light. Fresnel correctly applied Huygen's principle to explain the diffraction of light waves.



Let AB be an aperture on which light is incident from a source S, he assumed that the effect of each element of the wavefront APB in producing the next wavefront $A_1 P_1 B_1$ was confined to the apex of the secondary wave or that part of it which touches the envelope. This practically amounted to the assumption of the law of rectilinear propagation of light itself.

Fresnel's Explanation of Rectilinear Propagation of Light :

Let ABCD be a plane wavefront of monochromatic light of wavelength λ travelling from P to O. We want to find the effect of the wavefront at an external point O. Fresnel subdivided the wavefront ABCD into a large no. of elements or zones of small area (called half period zones) & that each of these elements was a source of secondary waves which were effective everywhere. From point O, drop perpendicular OP on the wavefront.



[Plane wave front]

Let $OP = b$. P is known as Pole w.r.t point O

With O as centre & radii

$$OM_1 = b + \frac{d}{2}$$

$$OM_2 = b + \frac{2d}{2}$$

$$OM_3 = b + \frac{3d}{2}$$

& so on.

draw concentric spheres. These spheres intersect the wavefront ABCD along concentric circles with O as centre, $OM_1, OM_2, OM_3 \text{----} OM_{n-1}, OM_n$ etc. are radii of the circles. The area enclosed b/w these circles etc. are known as fresnel's half period zone (h.p.z) or half period element.

The area enclosed by the first circle of radius OM_1 is called first half period zone.

The annular area b/w first & second circle is called second half period zone

" " " " $(n-1)^{th}$ & n^{th} " " " n^{th} half period zone

& so on.

Every zone differs from its nearest zone by a phase difference of π because there is a path difference of $d/2$ for any consecutive ray. Thus the successive zones differ by a phase difference of π or by a half period $(T/2)$.

Governing factors of Amplitude

The amplitude of the disturbances at O due to zone depends upon the following factors.

1) Area of half period zone (HPZ.)
2) Average distance of half period zone from O &
3) obliquity factor $(1+\cos\theta)$

1) Area of half period zone (HPZ);

The area of the HPZs are $\pi \times (\text{radius})^2$

Now the radius of first HPZ is $PM_1 = \sqrt{OM_1^2 - OP^2} = \sqrt{\left(b+\frac{d}{2}\right)^2 - b^2}$

$$= \sqrt{\cancel{b^2} + \frac{d^2}{4} + bd - \cancel{b^2}}$$

$$= \sqrt{bd}$$

(only first approximation $\frac{d^2}{4}$ neglected as compared to bd

Similarly the radius of second HPZ is $PM_2 = \sqrt{OM_2^2 - OP^2}$

$$= \sqrt{(b+d)^2 - b^2}$$

$$= \sqrt{2bd} \quad (\text{approx.})$$

More generally, the radius of $(n-1)^{th}$ HPZ, $PM_{n-1} = \sqrt{\left[b+(n-1)\frac{d}{2}\right]^2 - b^2}$

$$= \sqrt{(n-1)bd} \quad (\text{approx})$$

& radius of the n^{th} zone. $P.M_n = \sqrt{\left(b+n\frac{d}{2}\right)^2 - b^2}$

$$= \sqrt{nbd} \quad (\text{approx.}).$$

Thus the radius of HPZ $\propto \sqrt{n}$, when $n = 1, 2, 3 - - -$.

so the area of first HPZ $= \pi PM_1^2 = \pi b d$

The area of first two HPZs $= \pi PM_2^2 = 2\pi b d$

$\therefore$ The area of the second HPZ $= \pi(PM_2^2 - PM_1^2) = \pi(2bd - bd)$

$= \pi b d$

Also the area of nth zone $=$ area of first n zones $-$ area of first $(n-1)$ zones

$= \pi PM_n^2 - \pi PM_{n-1}^2$

$= \pi(nbd - (n-1)bd)$

$= \pi b d$

Hence we can say that the area of each HPZ is $\pi b d$

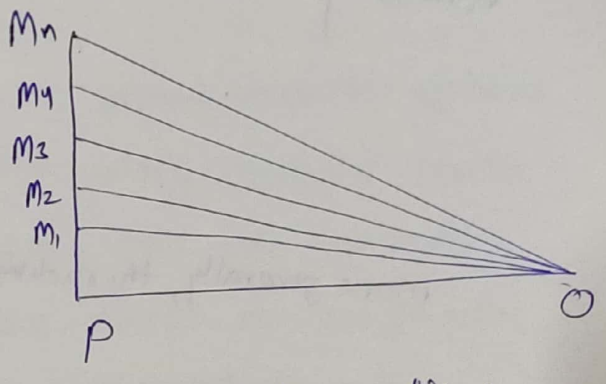
So we conclude that the area of HPZ is directly proportional to

(i) d [wavelength of incident light]

(ii) b [distance of point O from the wave front]

(2) Average distance of H.P.Z from O:

{Greater the distance of the zone, the smaller is the amplitude at O.}



The amplitude of distance is inversely proportional to the average distance of the zone from O. If A_1, A_2 represents the amplitudes at O due to 1st, 2nd .... HPZs, then A_1 is slightly greater than A_2, A_2 than A_3 ... A_{n-1} than A_n such that the amplitude of any zone may be taken as arithmatic mean b/w the preceding & the succeeding zones. Thus to a close approximation

$A_2 = \frac{A_1 + A_3}{2}$, $A_4 = \frac{A_3 + A_5}{2}$ & so on.

Thus greater the distance of zone, the smaller is the amplitude of vibration reaching O.

obliquity factor $(1+\cos\theta)$:

The amplitude is max. in a direction radially outward from P & falls to zero in opp. direction. Thus this variation with angle (θ) is called obliquity function or inclination factor.

Infact its effect along any direction is proportional to $(1+\cos\theta)$

It is max. along PO is $\theta = 0$ hence $\cos 0 = 1$,

half along PA_1 since $\cos 90^\circ = 0$ &

zero along PS. since $\cos 180^\circ = -1$ Since the effect at the rear. of the wave is zero, there willbe evidently be no "back wave"

So. $A_n \propto (1+\cos\theta_n)$

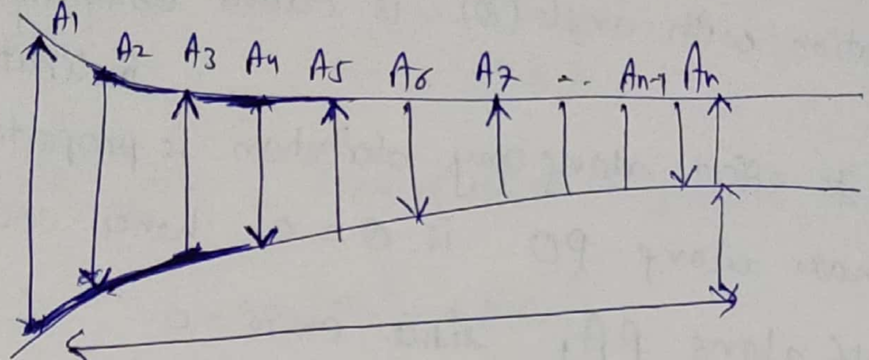
or. $$A_n = \frac{\pi\left[bd + \frac{d^2}{4}(n-1)\right]}{b + (2n-1)\frac{d}{4}}(1+\cos\theta_n)$$

$$\boxed{A_n = Knd(1+\cos\theta_n)}$$

K = constt of proportionality.

Now the resultant amplitude at O due to the Whole Wavefront.

Suppose $A_1, A_2 -- A_n$ are the amplitudes of the secondary waves originating from 1st, 2nd, --- nth HPZs.



Here area's of HPZs are equal. The distance of the zone from O increases with the order of zones; displacement $\propto \frac{1}{\text{distance}}$

Hence A_1 is slightly greater than A_2
A_2 " " " " A_3
.
A_{n-1} " " " " A_n

$\therefore A_1 > A_2 > A_3$ --------

Now

$$A_2 = \frac{A_1 + A_3}{2}$$

$$A_3 = \frac{A_2 + A_4}{2} \quad \text{\& so on}$$

It is clear that the successive amplitude have reverse directions as there is a phase difference of π b/w two consecutive zones.

Hence the resultant amplitude at O is

$$A = A_1 - A_2 + A_3 - A_4 + A_5 \text{ ------ } + A_n$$

$$= \frac{A_1}{2} + \left[\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right] + \left[\frac{A_3}{2} - A_4 + \frac{A_5}{2}\right] + \text{ --- } + \frac{A_n}{2}$$ (for n is odd)

$$= \frac{A_1}{2} + \left[\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right] + \left[\frac{A_3}{2} - A_4 + \frac{A_5}{2}\right] + \text{ --- } A_n$$ (for n is even)

but $A_2 = \frac{A_1 + A_3}{2}$ & $A_4 = \frac{A_3 + A_5}{2}$

$$\therefore \boxed{A = \frac{A_1}{2} + \frac{A_n}{2}} \quad [\text{if } n \text{ is odd}]$$

$$\& \quad \boxed{A = \frac{A_1}{2} - A_n} \quad [\text{if } n \text{ is even}]$$

If n is quite large ie $n \to \infty$ [then the effect due to $(n-1)^{th}$ or n^{th} zone is almost negligible on account of the distance & obliquity factor ie A_n & $A_{n-2} \to 0$] $\{A_n \to 0\}$

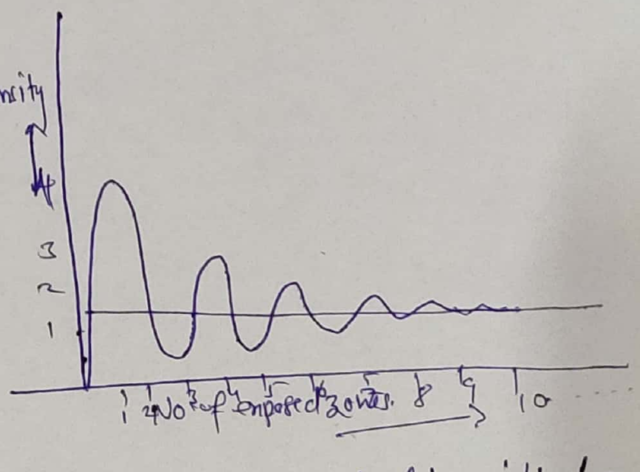
$\therefore$ resultant amplitude at O due to whole wavefront is reduces to $\frac{A_1}{2}$

ie $A = \frac{A_1}{2}$

Now, the Intensity is $\propto A^2$

$$\therefore \boxed{I = \frac{A_1^2}{4}}$$

Thus the intensity at O due to the whole wavefront is only one fourth of that due to the first HPZ alone.



Consider an opaque body interposed b/w wavefront & O. Since d is very small, area of each HPZ is $\pi b d$ is very small. Hence even a small obstacle cut off a considerable no of central effective h.p. zone. So no light will be recieved at O. The point O is thus practically dark. This explain the darkness observed at O. Thus Fresnel's wave theory accounts for the approximate rectilinear propagation of light.