12.3

Wednesday, 14 October 2020 1:07 PM

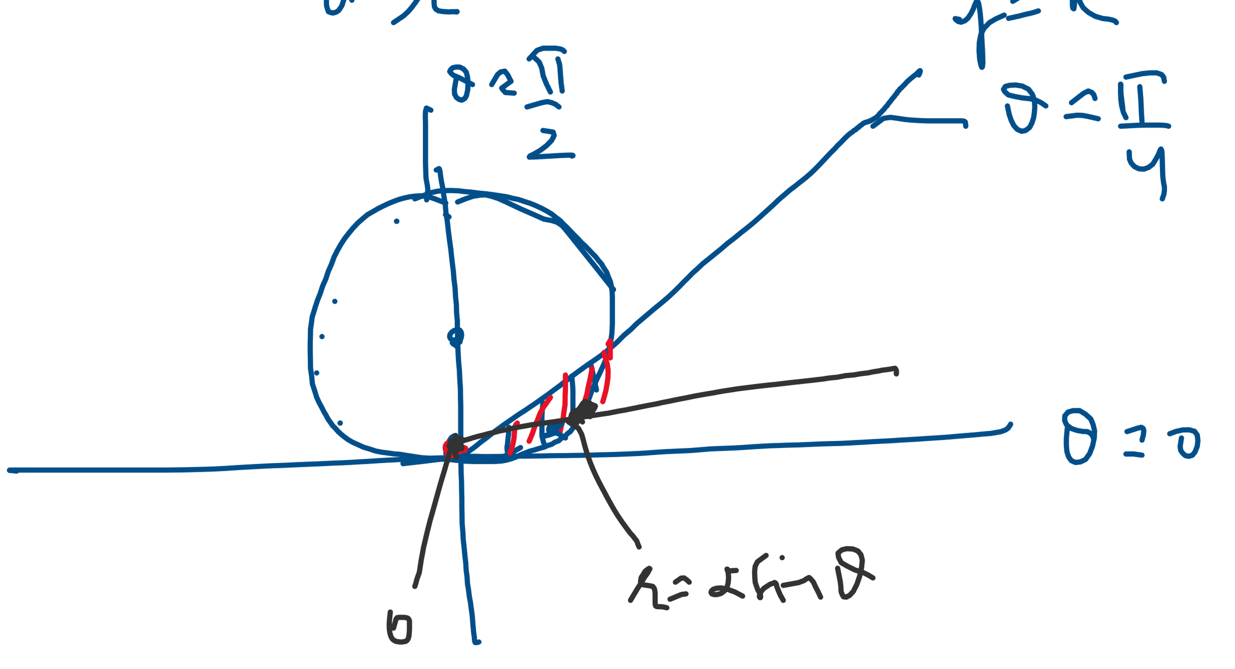
Double Integral in polar coordinates :

$$\iint_D f(x,y)\,dy\,dx = \iint_{\theta=\ r=} f(r,\theta)\, r\,dr\,d\theta$$

Put $x = r\cos\theta$, $y = r\sin\theta$] $dA = r\,dr\,d\theta$

Ex2 compute area of D bdd above by $y=x$ and below by circle $x^2+y^2-2y=0$

$$\iint_D dy\,dx = \iint_{\theta\ r} r\,dr\,d\theta$$



$x^2+y^2-2y=0$

$\Rightarrow x^2+(y-1)^2=1$

$y=x$

$r\sin\theta = r\cos\theta$

$\tan\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

$r^2-2r\sin\theta=0$

$r^2 = 2r\sin\theta \Rightarrow r = 2\sin\theta$

$$\therefore A = \int_0^{\pi/4}\int_0^{2\sin\theta} r\,dr\,d\theta \ \ldots$$

$$= \int_0^{\pi/4}\int_0^{2\sin\theta} r\,dr\,d\theta$$

$$= \int_0^{\pi/4} \frac{r^2}{2}\Big|_0^{2\sin\theta} d\theta$$

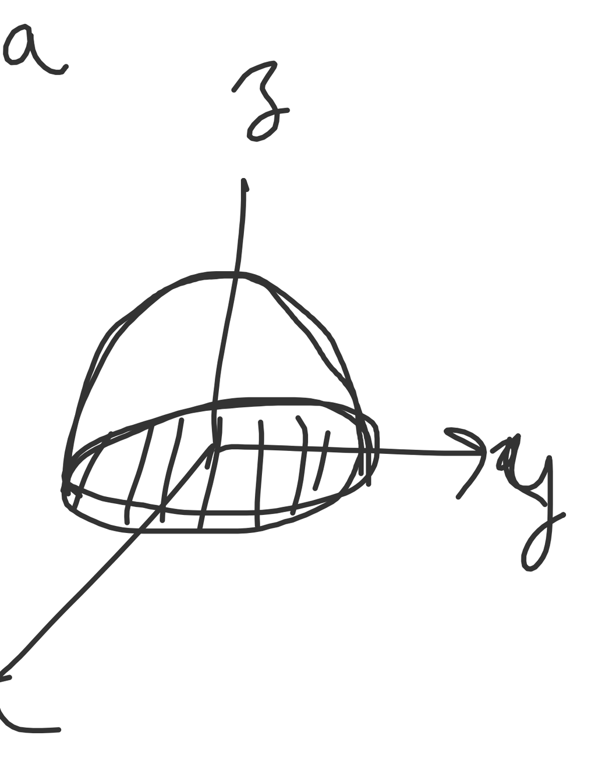
$$= \int_0^{\pi/4} \frac{4\sin^2\theta}{2}\, d\theta$$

$$= \int_0^{\pi/4} (1-\cos 2\theta)\, d\theta$$

$$= \theta - \frac{\sin 2\theta}{2}\Big|_0^{\pi/4}$$

$$= \frac{\pi-2}{4}$$

Ex3 Volume of a sphere of radius a



$x^2+y^2+z^2=a^2$

$z = \sqrt{a^2-x^2-y^2} = f(x,y)$

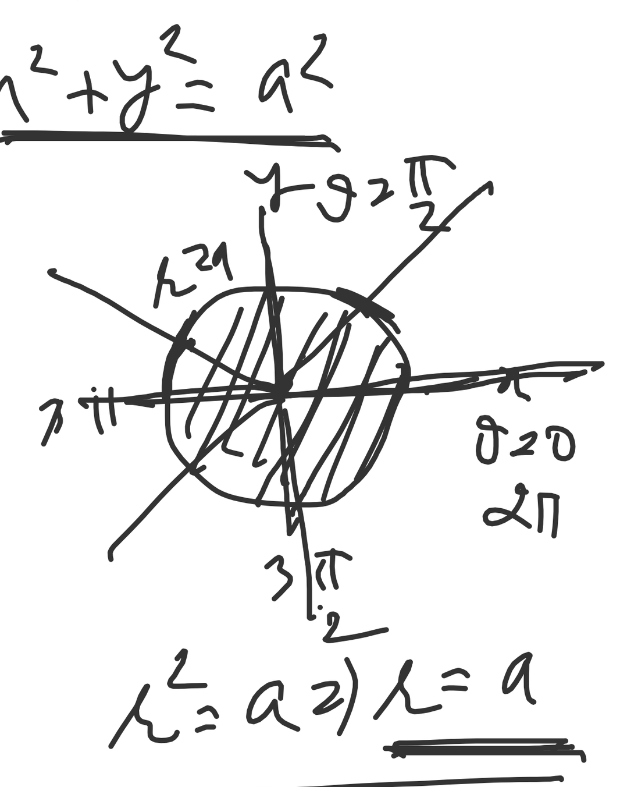
$$V = 2\iint f(x,y)\,dy\,dx$$

inside circle $x^2+y^2=a^2$

Put $z=0$

$x^2+y^2=a^2$

$$V = 2\int_0^{2\pi}\int_0^{a} \sqrt{a^2-r^2}\; r\,dr\,d\theta$$

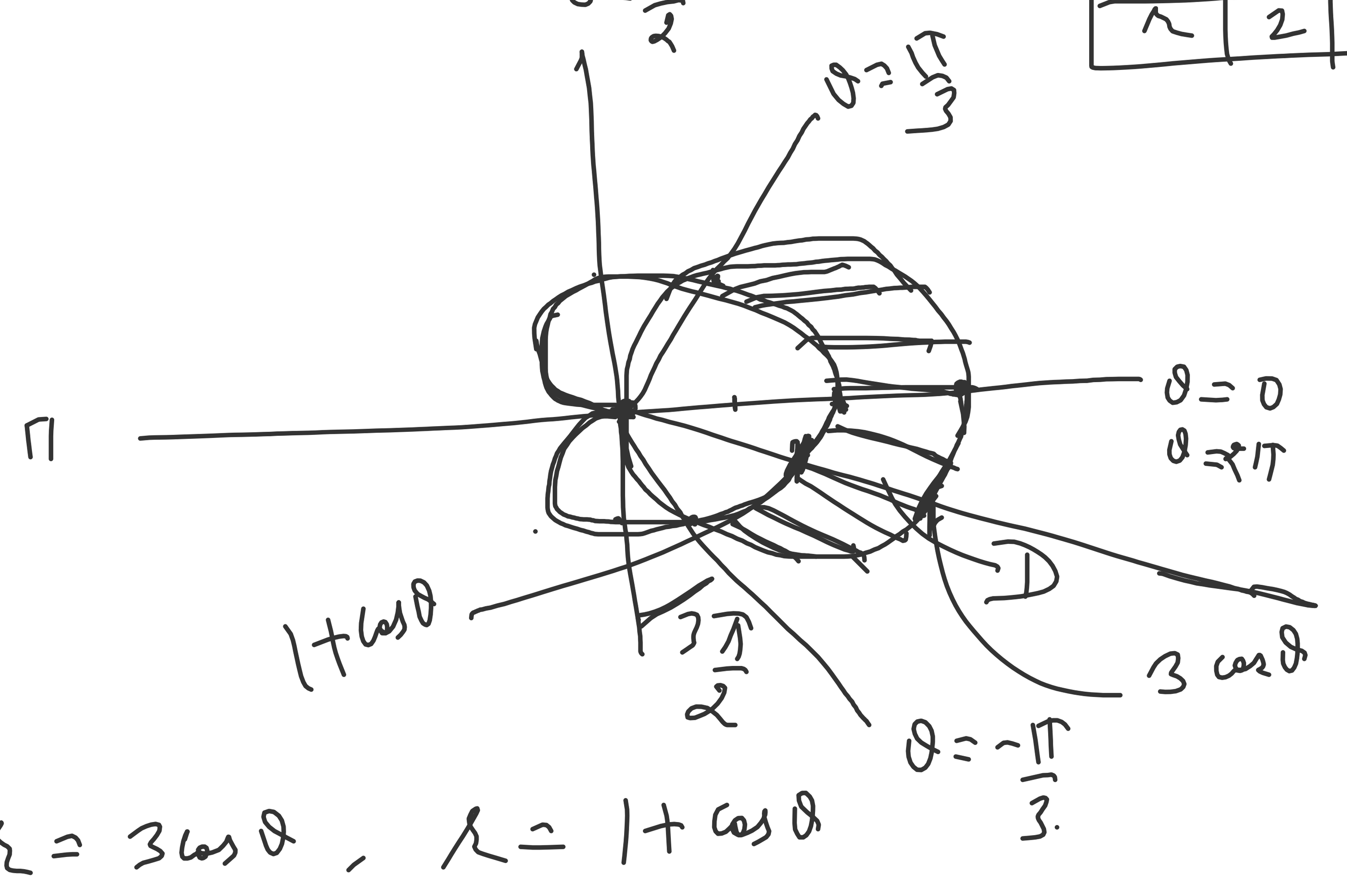


$$= \frac{4}{3}\pi a^3$$

$r^2=a^2 \Rightarrow r=a$

Ex4 Evaluate $\iint_D \frac{1}{x}\,dA$, where D is region inside $r=3\cos\theta$ and outside cardiode $r=1+\cos\theta$

θ	0	π/2	π	3π/2	2π
r	2	1	0	1	2



$r = 3\cos\theta$, $r = 1+\cos\theta$

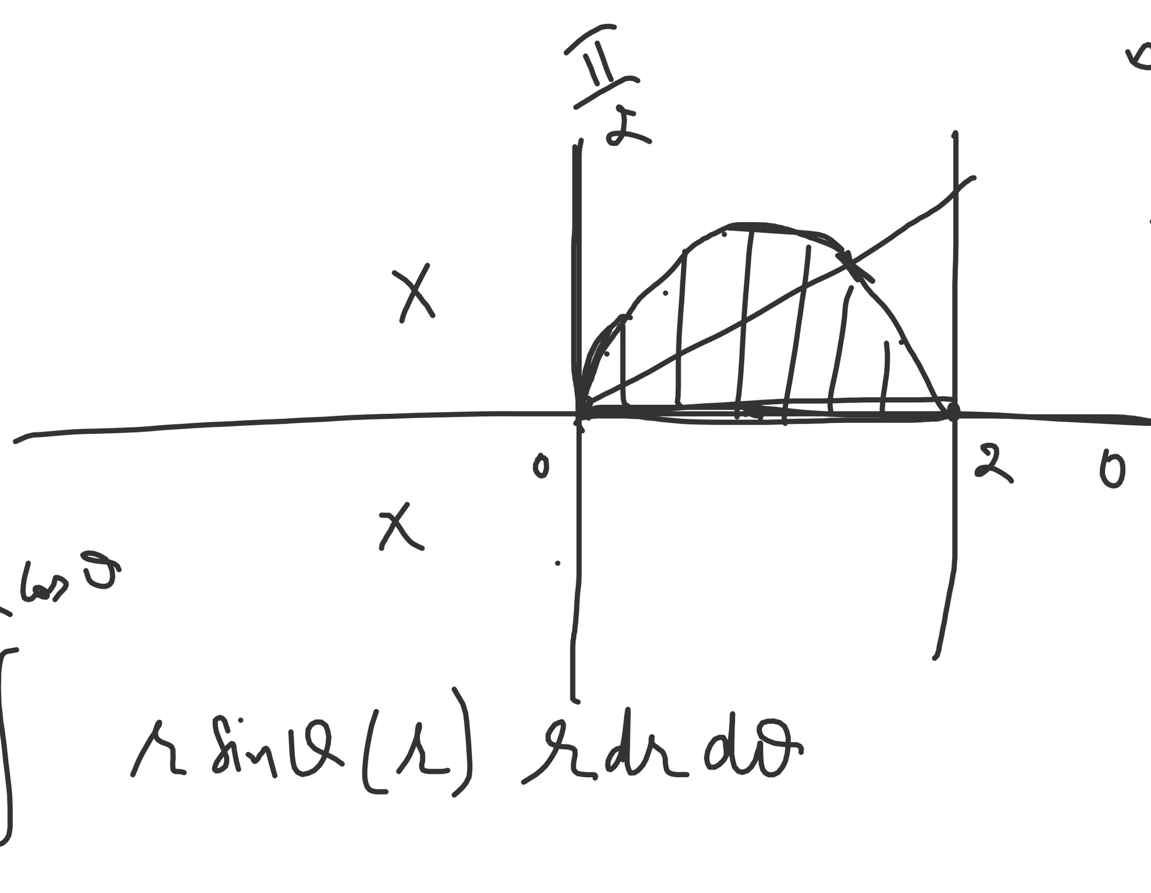
$3\cos\theta = 1+\cos\theta$

$\Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ | $\theta = -\frac{\pi}{3}$]

$$\int_{-\pi/3}^{\pi/3}\int_{1+\cos\theta}^{3\cos\theta} \frac{1}{r\cos\theta}\, r\,dr\,d\theta$$

$$= \frac{4\pi}{3} - 2\ln(2+\sqrt{3})$$

Ex5 $\int_0^2\int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\; dy\,dx$ by coverting to polar



$0 \le x \le 2$ ✓

$0 \le y \le \sqrt{2x-x^2}$ ✓

$\Rightarrow y^2 = 2x-x^2$

$\Rightarrow y^2+x^2-2x+1 = 0+1$

$y^2+(x-1)^2=1$

$x^2+y^2=2x$

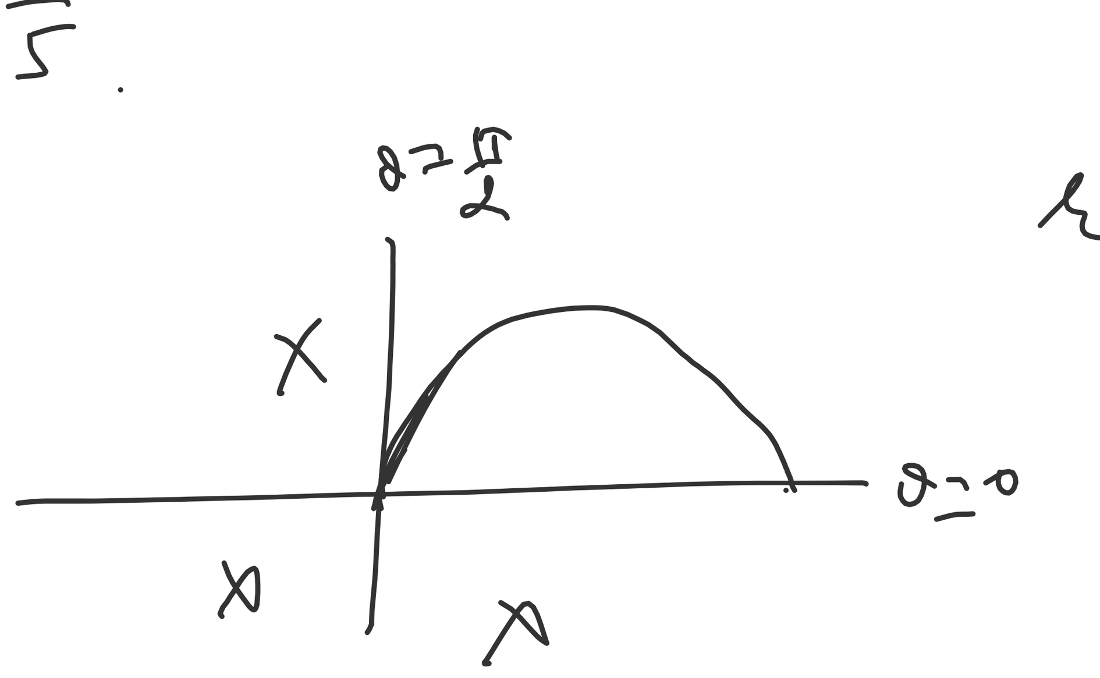
$r^2=2r\cos\theta$

$r = 2\cos\theta$ ✓

$$\int_0^{\pi/2}\int_0^{2\cos\theta} r\sin\theta\,(r)\; r\,dr\,d\theta$$

θ	0	π/2	π/2+3π/2
r	2	0	—

$$= \frac{4}{5}$$



$r = 2\cos\theta$

$\theta: 0 \text{ —— } \frac{\pi}{2}$