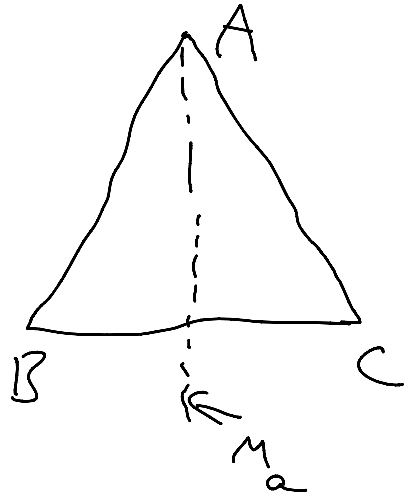
18 September 2021 11:07

## Symmetries of Isosceles Triangle:



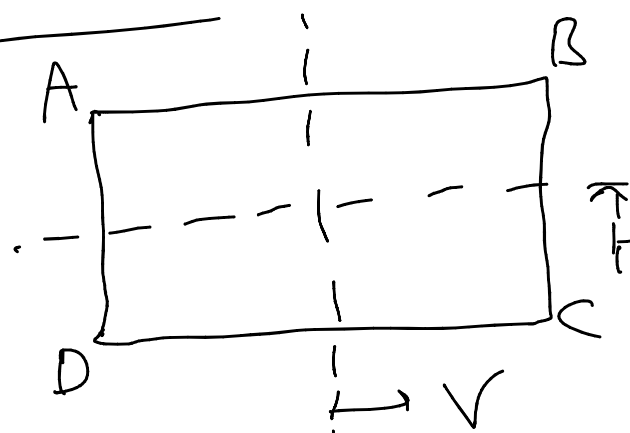

$G = \{R_0, M_a\}$

| $\circ$ | $R_0$ | $M_a$ |
|---|---|---|
| $R_0$ | $R_0$ | $M_a$ |
| $M_a$ | $M_a$ | $R_0$ |

Identity $\to R_0$, $(R_0)^{-1} = R_0$, $(M_a)^{-1} = M_a$.

Complete the theory.

## Symmetries of a rectangle:




$G = \{R_0, R_{180}, H, V\}$

| $\circ$ | $R_0$ | $R_{180}$ | $H$ | $V$ |
|---|---|---|---|---|
| $R_0$ | $R_0$ | $R_{180}$ | $H$ | $V$ |
| $R_{180}$ | $R_{180}$ | $R_0$ | $V$ | $H$ |
| $H$ | $H$ | $V$ | $R_0$ | $R_{180}$ |
| $V$ | $V$ | $H$ | $R_{180}$ | $R_0$ |

Identity $\to R_0$

$(R_0)^{-1} = R_0$, $(R_{180})^{-1} = R_{180}$,

$(H)^{-1} = H$, $(V)^{-1} = V$.

G is an abelian group.

## Center and Centralizer

$D_3 = \{R_0, R_{120}, R_{240}, M_a, M_b, M_c\}$

$D_4 = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$

$Z(D_4) = \{R_0, R_{180}\}$.

$C(R_{90}) = \{R_0, R_{90}, R_{180}, R_{270}\}$

$C(H) = \{R_0, R_{180}, H, V\}$

find all Centralizers in $D_4$.

## Cyclic Group:

A group $G$ is called cyclic if there is an element $a$ in $G$ such that

$$G = \{a^n \mid n \in \mathbb{Z}\}.$$

Such an element $a$ is called generator of $G$. and we can write $G = \langle a \rangle$.

**Example:** $U(10) = \{1, 3, 7, 9\}$ is a cyclic group.

$U(10) = \langle 3 \rangle$, $U(10) = \langle 7 \rangle$

**Result:** If $a$ is the generator of group $G$, then $a^{-1}$ will also the generator of $G$.

$(3)^{-1} = 7$ in $U(10)$.

$$\boxed{\begin{aligned} 7^1 &= 7,\ 7^2 = 49 \pmod{10} = 9, \\ 7^3 &= (9)(7) = 3,\ 7^4 = 1 \end{aligned}}$$

Q. Let $G$ be a group and let $a \in G$.

Prove that $\langle a^{-1} \rangle = \langle a \rangle$.

**Proof:** $\because \langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$

$\therefore a^{-1} \in \langle a \rangle$

$\Rightarrow (a^{-1})^n \in \langle a \rangle \quad \forall n \in \mathbb{Z}$ (using closure property & $\langle a \rangle$ is subgroup.)

$\Rightarrow \langle a^{-1} \rangle \subseteq \langle a \rangle$ —— (1)

$\because \langle a^{-1} \rangle = \{(a^{-1})^n \mid n \in \mathbb{Z}\}$

$\therefore \quad (a^{-1})^{-1} \in \langle a^{-1} \rangle$

$\Rightarrow \quad a \in \langle a^{-1} \rangle$

$\Rightarrow \quad a^{n} \in \langle a^{-1} \rangle \quad \forall n \in \mathbb{Z}$ (using closure property)

$\Rightarrow \langle a \rangle \subseteq \langle a^{-1} \rangle$ ——— (2)

from eqn (1) & (2), $\langle a^{-1} \rangle = \langle a \rangle$

In Particular, if $G$ is cyclic and $a$ is the generator of $G$, then $G = \langle a \rangle$

$\& \langle a \rangle = \langle a^{-1} \rangle$

$\therefore \boxed{G = \langle a^{-1} \rangle}$

$\boxed{G = \langle a \rangle = \langle a^{-1} \rangle, \text{ if } G \text{ is cyclic}}$