Second order ordinary Diff. eqns with constant coefficients problems

(A) Homogeneous:
General form of 2nd order diff. eqn with const. coefficient:

$$ay'' + by' + cy = 0$$

Method: (1) We can solve a 2nd order homogeneous linear diff. eqn. with const. coefficients using exponential fun. The form of the soln depends on the roots of the characteristic eqn.

(2) When we have distinct real roots, there exists two linearly independent fun. of the form $\exp(m_1 x)$ and $\exp(m_2 x)$ and the general soln. is -

$$\boxed{y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x}}$$

(3) When we have real roots same:

$$\boxed{y(x) = (C_1 + C_2 x) e^{mx}}$$

(4) For complex roots -- $\alpha + i\beta$ and $\alpha - i\beta$

$$\boxed{y(x) = (C_1 \sin\beta x + C_2 \cos\beta x) \cdot e^{\alpha x}}$$

B: Non Homogeneous:
general form:

$$ay'' + by' + cy = g(x)$$

$g(x)$ is called forcing fun. which may be exponential fun (Ae^{kx}), Polynomial Ax^n ($n = 0, 1, 2, 3 \ldots$) (or) Trigonometric fun $A\sin kx$ or $A\cos kx$

Soln in Tabular form.

Forcing fun	Nature of the roots	Form of Particular integral
Ae^{kx}	When k is not a root k is single root k is double root	Ce^{kx} Cxe^{kx} Cx^2e^{kx}
polynomial Ax^n $(n=0,1,2)$	$k=0$ is not a root $k=0$ is a single root $k=0$ is a double root	$C_0+C_1x+C_2x^2+\cdots$ $x(C_0+C_1x+\cdots)$ $x^2(C_0+C_1x+C_2x^2+\cdots)$
$A\cos kx$ $A\sin kx$	ik is not a root ik is a single root	$C\cos kx+D\sin kx$ $x(C\cos kx+D\sin kx)$

Problems:

(i) $y''-y=x+\frac{x^2}{2}$

(ii) $y''+4y=2\sin 2x$

(iii) $y''+y=x^2$

(iv) $y''+4y=3\cos x$

(v) $y''+y'+2y=4e^x+2x^2$